Chair of Structural Mechanics
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Master's Thesis

Extension of a Sparse Bayesian Learning Framework for Rational Polynomial Chaos Expansion

Motivation

Engineering problems often involve the design and analysis of system using numerical models which are associated with modelling uncertainty. This uncertainty might be related to incomplete knowledge about the model structure or the model parameters. In such cases uncertainty quantification methods can be applied to obtain knowledge about the uncertainty in the system performance. For computationally demanding simulation models, one often resorts to replacing the original model with a surrogate model that can be fast evaluated. Popular choices for surrogate models are neural networks or polynomial chaos expansion (PCE). Recently, a sparse Bayesian learning approach has been proposed for rational polynomial chaos expansion (rPCE), which is a surrogate model that is specifically suitable for frequency domain models in structural dynamics [1]. The sparse Bayesian learning approach can capture the model response with relatively few numbers of model evaluations. The implementation is based on a hierarchical prior structure for the model coefficients originally proposed in [2]. However, subsequently, other authors [3] have proposed alternative prior structures that are able to identify a sparser set of coefficients in comparison to the approach in [2].

Tasks

In the scope of this master's thesis, the effect of different hierarchical prior structures and corresponding hyperparameter updating strategies shall be investigated for PCE and rPCE.

- Carry out a literature study about sparse Bayesian learning and hierarchical prior construction.
- Implement the proposed methods for PCE and compare their performance in terms of generalization error and sparsity.
- Extend an existing implementation for rPCE with the previously considered hierarchical prior structures and compare to the existing implementation for several dynamic models.
- Investigate different hyperparameter estimation methods for the implemented problems.

References

[1] Schneider, F, Papaioannou, I, Müller, G. Sparse "Bayesian learning for complex-valued rational approximations." *Int J Numer Methods Eng*. 2023; 124(8): 1721– 1747. doi:10.1002/nme.7182
[2] Tipping, Michael E. "Sparse Bayesian learning and the relevance vector machine." *Journal of machine learning research* 1.Jun (2001): 211-244.
[3] S. D. Babacan, R. Molina and A. K. Katsaggelos, "Bayesian Compressive Sensing Using Laplace Priors," in IEEE *Transactions on Image Processing*, vol. 19, no. 1, pp. 53-63, Jan. 2010, doi: 10.1109/TIP.2009.2032894.

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