Chair of Structural Mechanics
TUM School of Engineering and Design
Technical University of Munich

Master's Thesis

Derivation of a Variational Bayesian Inference Approach for the Sparse Learning of Rational Polynomial Chaos Expansions

Motivation

Engineering problems often involve the design and analysis of system using numerical models which are associated with modelling uncertainty. This uncertainty might be related to incomplete knowledge about the model structure or the model parameters. In such cases uncertainty quantification methods can be applied to obtain knowledge about the uncertainty in the system performance. For computationally demanding simulation models, one often resorts to replacing the original model with a surrogate model that can be fast evaluated. Popular choices for surrogate models are neural networks or polynomial chaos expansion (PCE). Recently, a sparse Bayesian learning approach has been proposed for rational polynomial chaos expansion (rPCE), which is a surrogate model that is specifically suitable for frequency domain models in structural dynamics [1]. The sparse Bayesian learning approach can capture the model response with relatively few numbers of model evaluations. The implementation is based on a hierarchical prior structure for the model coefficients originally proposed in [2]. Therein a marginal likelihood approach for the hyperparameter estimation step is proposed. Alternatively, variational Bayesian approaches have been proposed for sparse Bayesian learning, e.g., in [3]. These transfer the problem of inference to an optimization problem.

Tasks

In the scope of this master's thesis, a variational Bayesian inference formulation shall be derived for the sparse Bayesian learning problem for rational polynomial chaos expansion.

- Carry out a literature study about sparse Bayesian learning and variational Bayesian inference.
- Propose a suitable parametric family of approximate distributions and derive the variational formulation of the problem.
- Implement the formulation and investigate suitable numerical techniques for the problem.
- Investigate the method for different mechanical problems and compare to an existing marginal-likelihood-based approach..

References

[1] Schneider, F, Papaioannou, I, Müller, G. Sparse "Bayesian learning for complex-valued rational approximations." *Int J Numer Methods Eng*. 2023; 124(8): 1721– 1747. doi:10.1002/nme.7182
[2] Tipping, Michael E. "Sparse Bayesian learning and the relevance vector machine." *Journal of machine learning research* 1.Jun (2001): 211-244.
[3] Christopher M. Bishop and Michael E. Tipping. 2000. "Variational relevance vector machines." *Proceedings of the Sixteenth conference on Uncertainty in artificial intelligence (UAI'00).*

Prerequisites

Students should be interested in working with mathematical derivations. Furthermore, students need to show a solid understanding of probability theory and stochastic modeling. Finally, solid implementation are beneficial.

Supervisors

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