Chair of Structural Mechanics
TUM School of Engineering and Design
Technical University of Munich

Bachelor's Thesis

Vibration Control through Periodic Structures: Analytical Modeling of Band Gap Mechanisms in Mass-Spring Systems

Background and Motivation

In many engineering cases, controlling vibrations is crucial – for instance, to reduce noise in machinery, protect structures from seismic waves, and shield the performance of precision measurement equipment. One promising way to acquire vibration control that is tailor-made for its application case is using engineered materials with a periodically ordered micro-structure, often called *metamaterials*.

These structures can exhibit special dynamic properties, most notably the appearance of *band gaps*. *Band gaps* are frequency regions where the wave propagation is blocked, theoretically resulting in complete vibrational isolation within the structure. They particularly arise in periodic structures, making them attractive for vibration isolation and wave filtering in mechanical systems. From a physical point of view, band gaps originate from different mechanisms, such as

- *Bragg scattering*, which arises from the destructive interference of waves scattered in the structure
- *Local resonance*, where small sub-structural elements couple with the wave motion in the host structure and absorb vibrational energy
- *Inertia amplification*, where the effective inertia of the vibrating system is increased without changing the physical mass by enabling internal relative motions through structural kinematics

Understanding how these mechanisms work and how structural parameters like mass and stiffness influence the band gap is key to designing efficient vibration control systems.

This thesis will analytically investigate these mechanisms using mass-spring chain models. By deriving the equations of motion and solving the resulting dispersion relations, the project aims to study how design choices influence wave propagation and band gap formation. Numerical tools such as Matlab® or Python will be used to solve the equations, visualize results, and explore the impact of mass and stiffness on the band gap width and location.

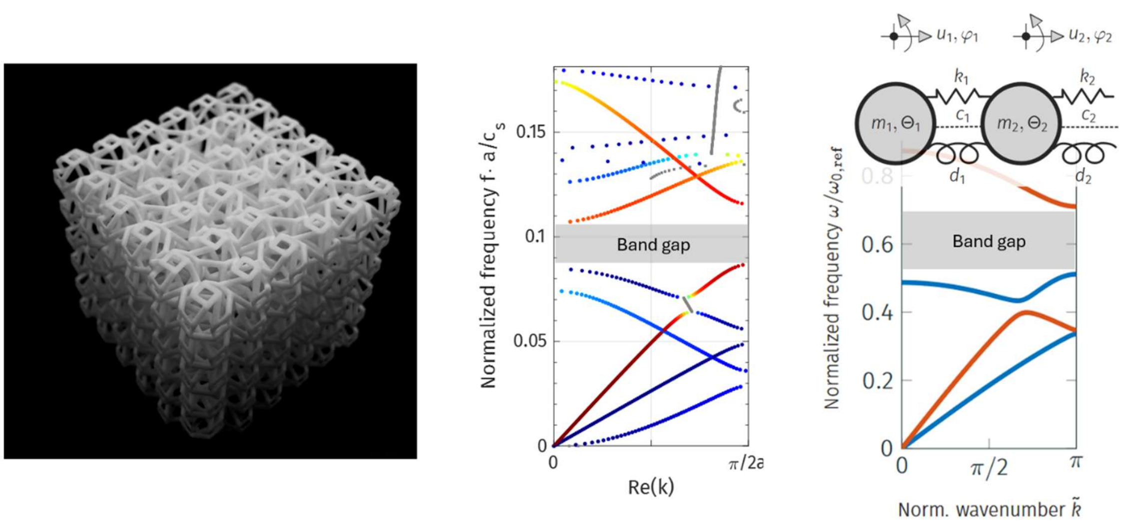

***Figure 1:** 3D-printed periodic structure (left) and a dispersion relation with a band gap (middle). The dispersion relation of the mass-spring-model (right) captures this band gap and the dispersion characteristics of the complicated real model, allowing an intuitive understanding of the band gap origin.*

Key research questions

- How do Bragg scattering, local resonance, and inertia amplification mechanisms influence the formation and location of band gaps in periodic mass-spring chains?
- What is the effect of structural parameters (mass, stiffness, periodicity) on the width and position of the band gaps?
- What are the trade-offs between design parameters (e.g., larger mass, spring stiffness, periodic spacing)? Which mechanism is most suited for different situations?
- Bonus: Can combining the mechanisms lead to enhanced band gap characteristics?

Project Tasks and Stages

1. Literature Review
    a. Review foundational literature on elastic metamaterials and phononic crystals in the context of vibration isolation
    b. Study the principles of Bragg scattering, local resonance, and inertia amplification as mechanisms for band gap formation.
    c. Get familiar with dispersion curves and how to read them
    d. Get familiar with Matlab® or Python
2. Analytical Modelling
    a. Identify and develop periodic mass-spring models for each of the band gap mechanisms
    b. Derive the equations of motions using Newtonian or Lagrangian methods
    c. Obtain the dispersion relations for each of the chosen model
3. Simulation
    - Implement the dispersion relations in a Matlab® live or Python/Jupyter script and solve them analytically or numerically if required
    - Conduct a parametric study to investigate the influence of mass, stiffness, and further structural parameters
    - Identify band gap locations, widths, and their shifts as design parameters vary
4. Interpretation
    - Compare the effects of the different mechanisms and their origin
    - Evaluate design trade-offs and best-suited configurations for different vibration conditions
    - Relate findings to practical design considerations for periodic structures for vibration control

Literature

[1] P. A. Deymier (2013): *Acoustic Metamaterials and Phononic Crystals*. Springer Berlin Heidelberg. DOI: 10.1007/978-3-642-31232-8

[2] Hussein, M. I., Leamy, M. J., & Ruzzene, M. (2014). Dynamics of phononic materials and structures: Historical origins, recent progress, and future outlook. In *Applied Mechanics Reviews*, 66(4).

[3] Frandsen, N. M. M., Bilal, O. R., Jensen, J. S., & Hussein, M. I. (2016). Inertial amplification of continuous structures: Large band gaps from small masses. In *Journal of Applied Physics, 119*(9).

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