Chair of Structural Mechanics
TUM School of Engineering and Design
Technical University of Munich

COME and Civil Eng. students welcome!

Master's Thesis

Bloch's theorem for screw and glide-symmetric periodic structures: Implementation & Application for Acoustic Metamaterials

Motivation

Acoustic metamaterials are rationally designed structures commonly obtained from periodic repetitions of unit cells. When periodically assembled, the resulting "*meta*-material" may exhibit phenomena such as inhibition of structural wave propagation in distinct frequency regimes [1]. For that reason, periodic cellular structures are attractive for vibroacoustic applications as they can provide effective and customizable insulation of structure-borne sound.

The wave propagation characteristics of periodic structures are strongly related to the dynamics and symmetries of their ingredient unit cells. Symmetry is exploitable to restrict the dynamic analysis of an infinite periodic structure to a single unit cell and an appropriate set of periodic boundary conditions [2]. This procedure is governed by *Bloch's theorem* and applies to infinite structures composed by periodically translating a unit cell.
Apart from such, spatial periodicity can also be imposed by other symmetry operations such as rotations ("screw") and reflections ("glide") of a unit cell (see *Figure 1*). While these operations may give rise to other interesting wave propagation effects, they complicate the dynamic analysis as the initial formulation of *Bloch's theorem* does not apply to other symmetries than translations. However, two publications demonstrated that revising the formulation of periodic conditions allows to extend Bloch's theorem to screw- and glide-symmetric structures, facilitating the convenient reduction to single unit cells in connection with the finite element method [3,4].

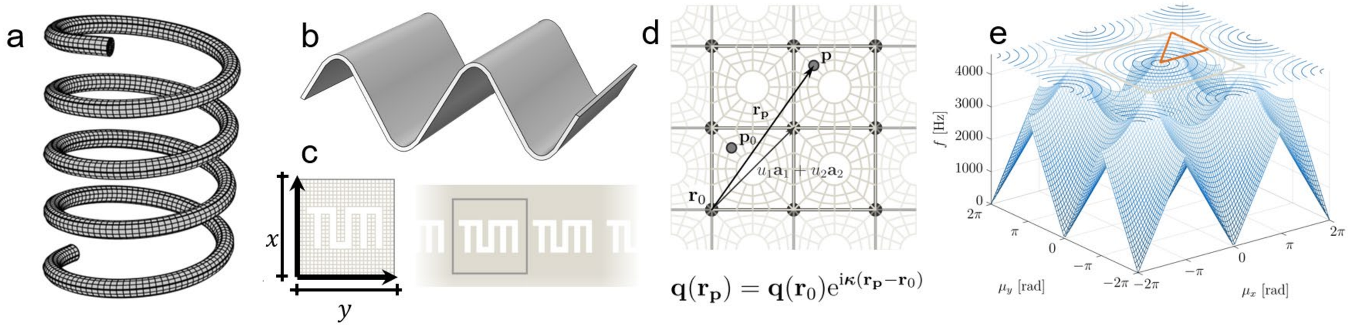

Figure 1: *Examples of periodic structures exhibiting screw (a), glide (b) and translational (c) symmetries. The latter are covered by the classical formulation of BLOCH's theorem (d), which can be exploited to investigate the acoustic dispersion in a unit cell (e).*

Tasks

In the scope of this master's thesis, the dynamic characteristics of metamaterial unit cells possessing glide and screw symmetries shall be analyzed using the Wave Finite Element method (WFEM). Particular interest shall be directed to the question on how to recover the dispersion relation of unit cells that cannot be repeated by pure translations only. To this end, the revised formulation of BLOCH'S theorem will be implemented into the Chair's in-house Matlab FE-framework *bm-fem,* starting with infinite periodicity in one-dimension. The implementation needs to be validated by comparing the results to predictions obtained from the classical formulation, when available. Further important questions of this project are the accuracy of the revised formulation compared to the classical approach as well as the computational demands.

Project Stages

- Familiarization with the topics *Acoustic Metamaterials* and the *Wave Finite Element* method
- Introduction into the Chair's Matlab-based code framework *bm-fem*
- Extension of the readily implemented Wave Finite Element method to glide and screw symmetric structures (assuming 1D-periodicity)
- Validation of the implementation based on suitable cases to be selected from literature studies
- *Extension of the method to 2D-periodic cases

Literature

[1] P. A. Deymier (2013): *Acoustic Metamaterials and Phononic Crystals*. Springer Berlin Heidelberg. DOI: 10.1007/978-3-642-31232-8

[2] B. Mace *et al.* (2005): Finite element prediction of wave motion in structural waveguides. In *The Journal of the Acoustical Society of America* Vol. 117, pp. 2835-2843. DOI: 10.1121/1.1887126

[3] F. Maurin (2016): Bloch theorem with revised boundary conditions applied to glide and screw symmetric, quasi-one-dimensional structures. In *Wave Motion* Vol. 61, pp. 20-39. DOI: 10.1016/j.wavemoti.2015.10.008

[4] F. Maurin *et al.* (2017): Bloch theorem with revised boundary conditions applied to glide, screw and rotational symmetric structures. In *Computer Methods in Applied Mechanics and Engineering* Vol. 381, pp. 497-513. DOI: 10.1016/j.cma.2017.01.034

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