

Software Lab:

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Exploring the effect of loss function in convergence behavior of FWI

Description

Full Waveform Inversion (FWI) is a high-resolution seismic imaging technique used to estimate subsurface properties by minimizing the difference between observed and modeled seismic data. It can also be used to detect holes in solid structure. Traditionally, this is framed as a non-linear least-squares optimization problem [1] using the L_2 norm. However, FWI is usually very ill-posed problem which results in significant noise in the reconstruction. This project investigates how alternative loss functions (such as L_2 , Huber, or Optimal Transport/Wasserstein metrics) influence the convergence landscape. By transitioning from traditional point-to-point comparisons to more global or robust measures, we aim to mitigate the non-linearity of the inverse problem and improve reconstruction accuracy in the presence of noise or poor initial models.

Task

The project is divided into five parts. The team will first develop and understand the Full waveform inversion task and develop the code together. After which, the team will be divided into subgroups to implement various loss functions to study their behavior in detail.

- Understanding the Full waveform inversion (FWI) code: First step is to understand the current FWI code that involves a forward wave equation solver and adjoint gradient calculation for gradient calculation with respect to the density field.
- Implementation of auto-diff based FWI code: If one wants to change the loss function, the adjoint gradient, which is based on the loss, also changes. Therefore, one would need to recalculate the gradient for every new loss function. To avoid this burden, students need to set up a differentiable script using a differentiable forward solver provided by us.
- Loss Function Library Implementation: Implement a library of new objective functions. This includes the standard L_2 norm, L_1 for outlier robustness, and at least one advanced metric like the Wasserstein distance (W_2) to test its convexity properties.
- Optimization & Convergence Analysis: Integrate local optimization algorithms (e.g., Conjugate Gradient or L-BFGS). Develop metrics to track convergence speed, residual reduction, and "model error" relative to the ground truth.
- Sensitivity Study & Benchmarking: Conduct tests to compare how different loss perform in finding out low density areas in the domain

Supervisor

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References

1. Burchner, T., Kopp, P., Kollmannsberger, S., & Rank, E. (2023). Immersed boundary parametrizations for full waveform inversion. *Computer Methods in Applied Mechanics and Engineering*, 406, 115893. <https://doi.org/10.1016/j.cma.2023.115893>