Chair of Computational Modeling and Simulation
TUM Department of Civil, Geo and Environmental Engineering
Technical University of Munich

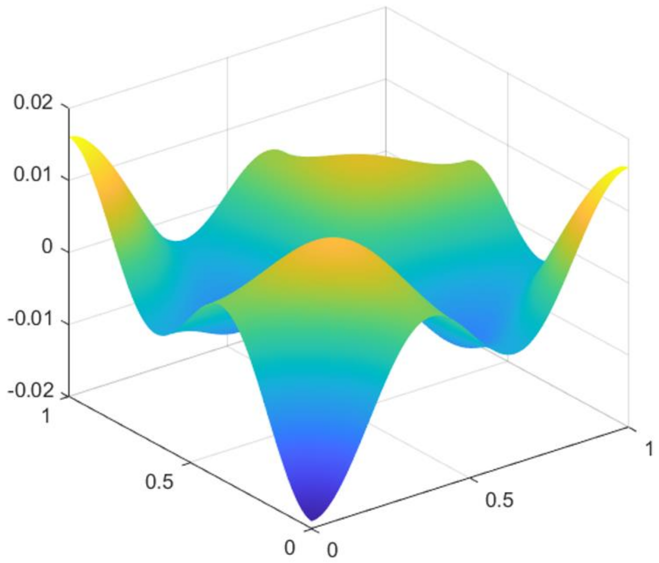
Modeling:
Mathematics:
Programming:
Science:

# Software Lab:

# High-order finite element discretizations of the scalar wave equation

### Description

The finite element method (FEM) is a numerical method for solving partial differential equations in engineering tasks. Instead intending to solve the governing equations directly, the weak formulation of the PDE is transferred in a discretized solution space. Starting from locally linear shape functions, many approaches have been introduced using different kinds of high-order shape functions. It has been shown that solution spaces with higher polynomial functions lead to a better convergence of the scalar wave equation. The spectral element method (SEM) uses regular Lagrange polynomials at GLL points, the p-version of FEM integrated Legendre polynomials and IGA B-splines.




The goal of this software lab topic is to write a simple finite element code for the scalar wave equation in Matlab or Python. The different approaches shall be tested with respect to their convergence and conditioning. To this end, the students should start with a one-dimensional example first and then continue with two-dimensional problems towards the end of the project. Also, different material distributions shall be considered.

### Task

- Write an boundary conforming finite element code in 1D
- Implement different approaches with high-order shape functions (e.g. [1], [2], [3])
- Compare the different approaches with respect to convergence, conditioning, etc.
- Extend implementations and comparison to two-dimensional problems

### Supervisor

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### References

[1] Düster A, Rank E, Szabó B (2018) The p-version of the finite element and finite cell methods, chap. 4. Wiley, New York, pp 1–55
[2] Dimitri Komatitsch, Jeroen Tromp (1999) Introduction to the spectral element method for three-dimensional seismic wave propagation, *Geophysical Journal International*, Volume 139, Issue 3, December 1999, Pages 806–822
[3] Thomas JR Hughes, John A Cottrell, Yuri Bazilevs (2005) Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Computer Methods in Applied Mechanics and Engineering, Volume 194, Issues 39–41, Pages 4135-4195