Industrial Applications of Computational Mechanics Cables and Beams

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FEM and Model Generation

Real Model	(Building)
↓ Classification	↑ Build
Structural Model	(Plates, Columns, Slabs)
↓ Abstraction	↑ Design
Mechanical Model	(Kirchhoff/Reisner)
↓ Discretisation	↑ Evaluation
Numerical Model	(FEM)
↓ Programming	↑ Post processing
Software Model	(FEM-Program)
↓ Implementation	↑ Calculate
Hardware Model	(Computer)

What is FEM ?

- FEM = Restriction = Method of Projection
  The space of possible deformations of the structure is restricted. The FEM-Solution is the shadow of the real solution into the selected solution space.
- FEM = Method of equivalent loadings
  The real loading is replaced by a loading which is equivalent with respect to the work.
- FEM = Method of Minimum of Energy
  FE-Program has its roots and possibilities in work and energy. Forces which do not contribute to the total work do not existent for the method.
- FEM = Method of approximate influence functions
  An element and the mesh build with it is as precise as the influence function for a selected result may be modelled with the mesh.

FEM = Projection

Cable

-H w‘‘(x) = p(x)

Cable Force ⬄ Bending Stiffness

Vertical Force ⬄ Shear Force

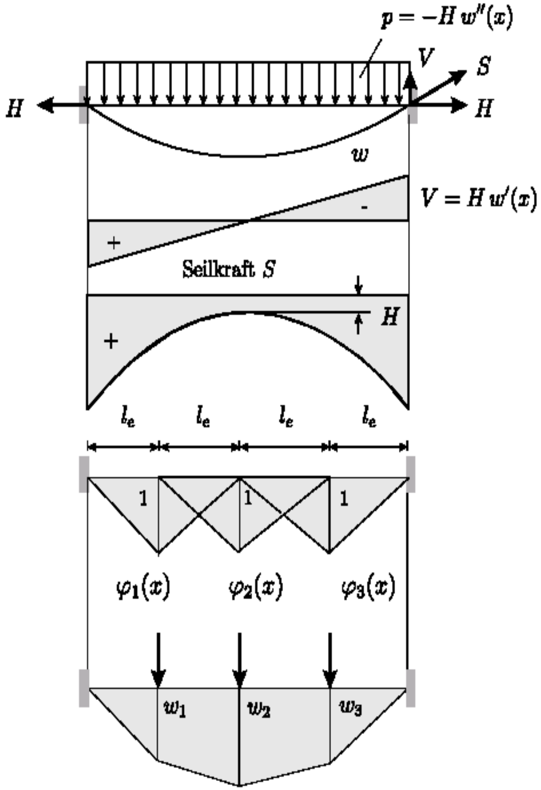
Solution for uniform load: quadr. Parabola

Cable element

Linear geometry of cable between nodes

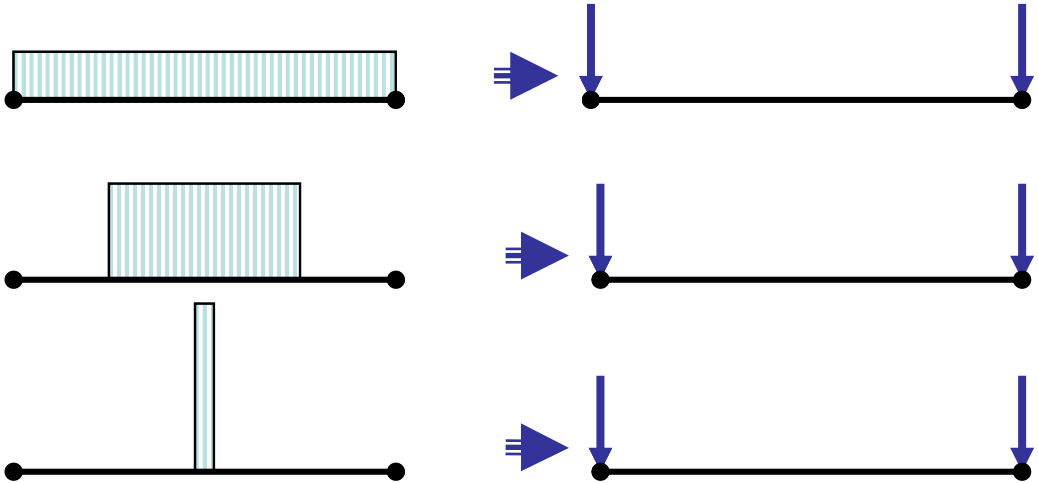
Solution space: polygon displacements

$$w_h(x) = w_1\varphi_1(x) + w_2\varphi_2(x) + w_3\varphi_3(x)$$



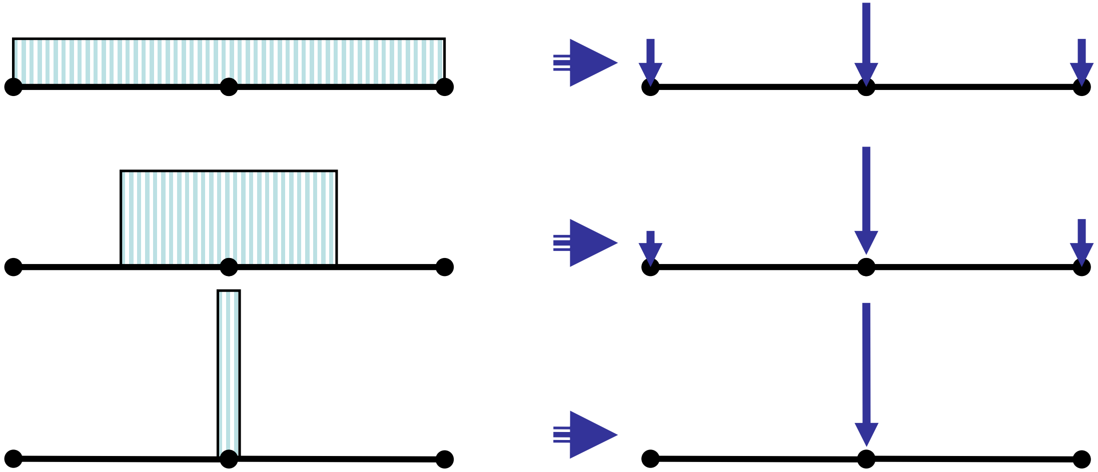
FEM = Equivalent Loadings

- Nodal loads are not point loads



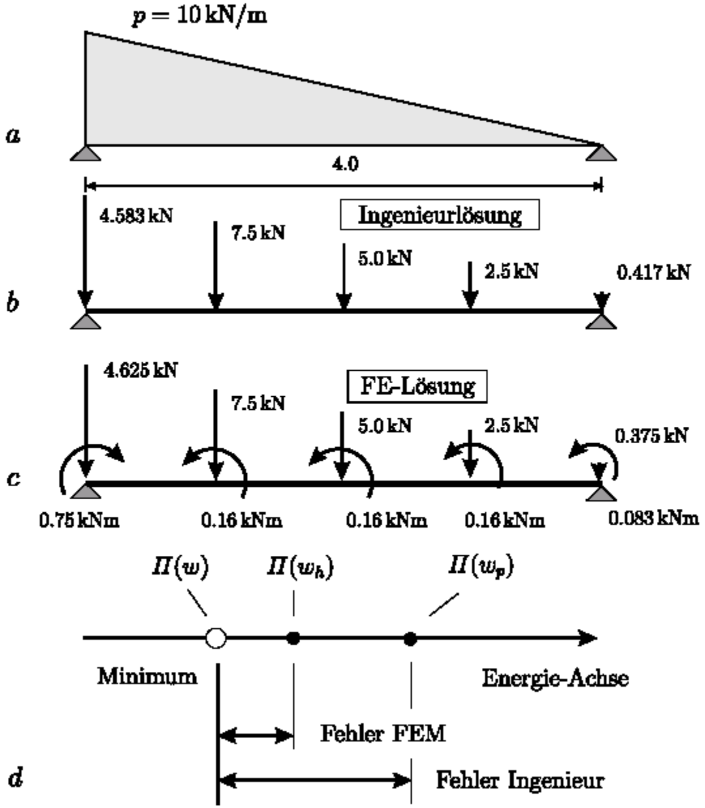
FEM = Equivalent Loadings

- Resolution of a mesh for loads



FEM – Loadings are more precise than engineering loads !

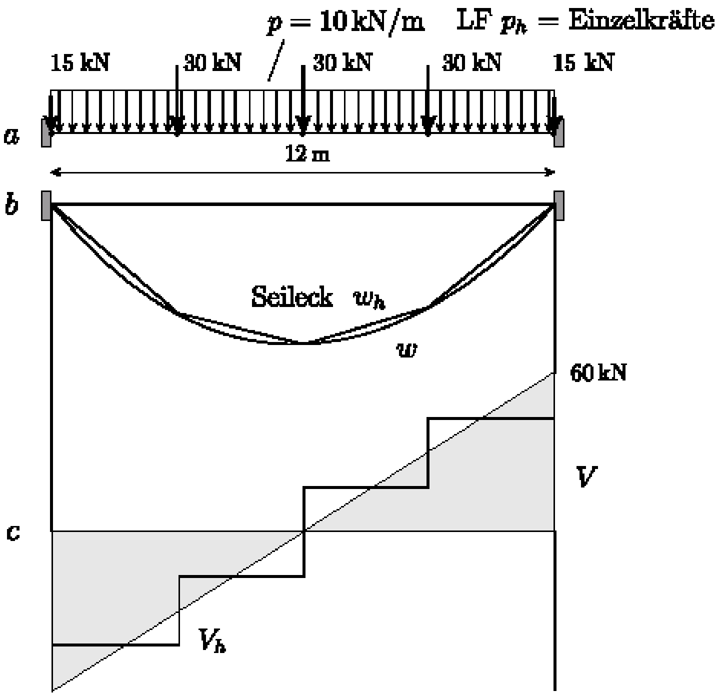
- Isoparametric Elements
- Drilling Degrees of Freedom



FEM = Minimum of Energy

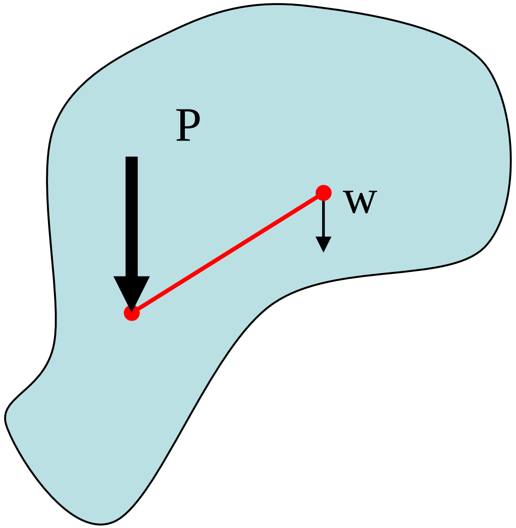
$$\int_0^l (V - V_h)^2 dx = \int_0^l H(w' - w'_h)^2 dx \Rightarrow Minimum$$

- Error:
  - Support (exact)
  - Loadings (Nodal loads)
  - Displacements (good)
  - Forces (constant)
- Exact Equilibrium if the forces act only at the nodes!
- A large error in the loads yields via integration a smaller error in the forces and an even smaller error in the displacements.



FEM = Method of Influence functions

- There is a simple fundamental solution for a singular load (e.g. Point force or single Moment etc.).
- Essential property of this fundamental solution $G_0(x,x_0)$ is, that all boundary conditions of the displacements are fulfilled and that there are no other singularities besides the point x_0 where the point load is located.
- The function $G_0(x,x_0)$ is called Greens Function and is the influence function for a displacement.



$$w = \frac{P}{2\pi G} \ln(z - z_0)$$

Integration of Greens Function

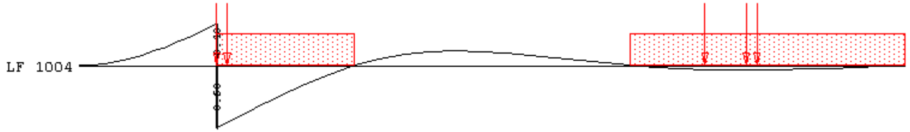
- If the point load is replaced by a differential loading (e.g. p(y)dy), the solution for any distributed loading is just an integral of this fundamental function over an area or a line.

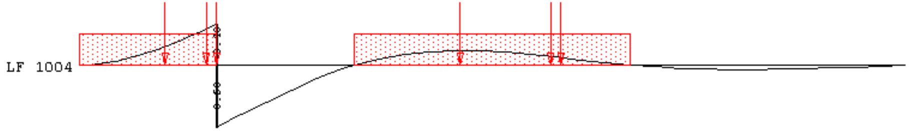
$$w(x) = \int_0^l G_0(x,y)\,p(y)\,dy$$

- As Greens function is symmetric, $G_0(y;x) = G_0(x;y)$, (Law from Maxwell), it is of no importance if we integrate along x or y.

FEM = Method of Influence functions

- Influence function for the displacement x_i of a cable is the polygon cable representing the solution for a load P=1 at point i.
- Influence function for the moment of a beam is the deformation obtained by a unit bend of size 1 at point i.





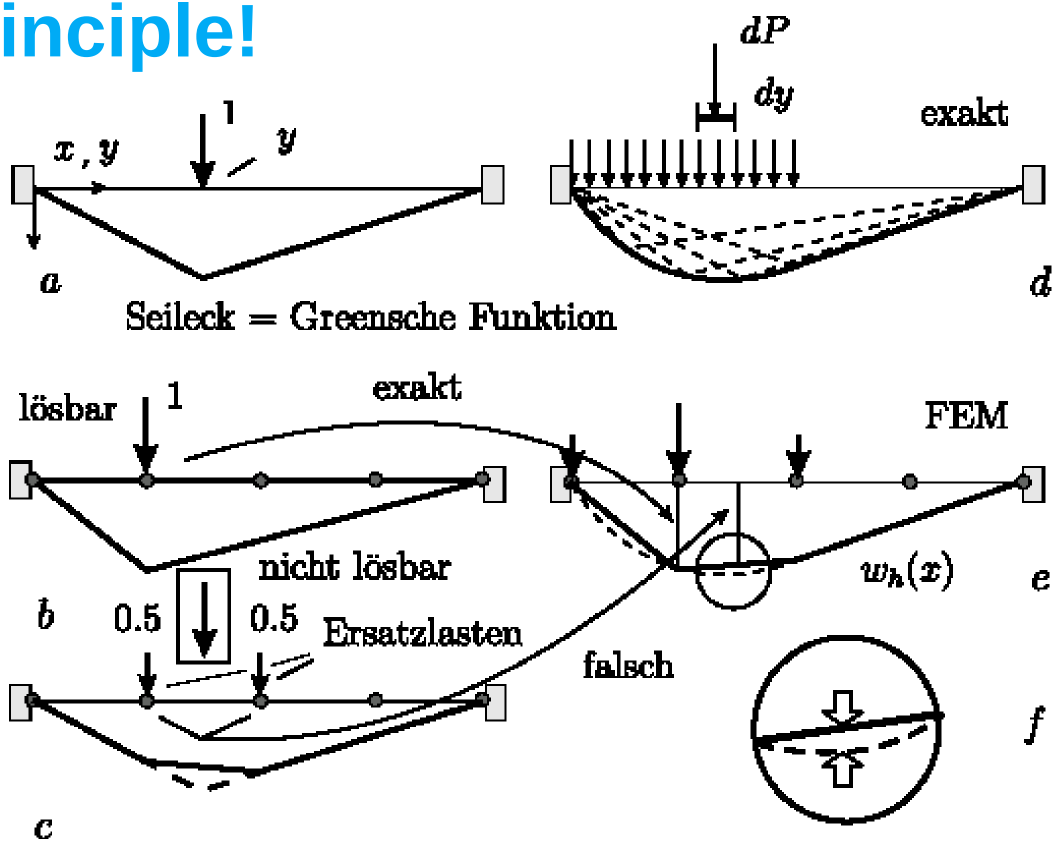
FEM = Method of Influence functions

- If the FE-system is able to represent the influence line exactly, the solution will be exact.
- In all other cases, if the influence function is only approximated, we get an approximate solution.
- The difference between the real and the approximate solution may be used to estimate the error with rather high precision.

An easy principle!

- A FE-Program calculates approximate results, because it is using approximate Greens functions.
- As a mesh consisting of linear, quadratic or even cubic elements may represent only a few selected displacements, these elements will not allow the structure to deform in the required correct shape of the true Greens function.
- A FE-Program thus is not able to achieve an exact result in most cases.

An easy principle!

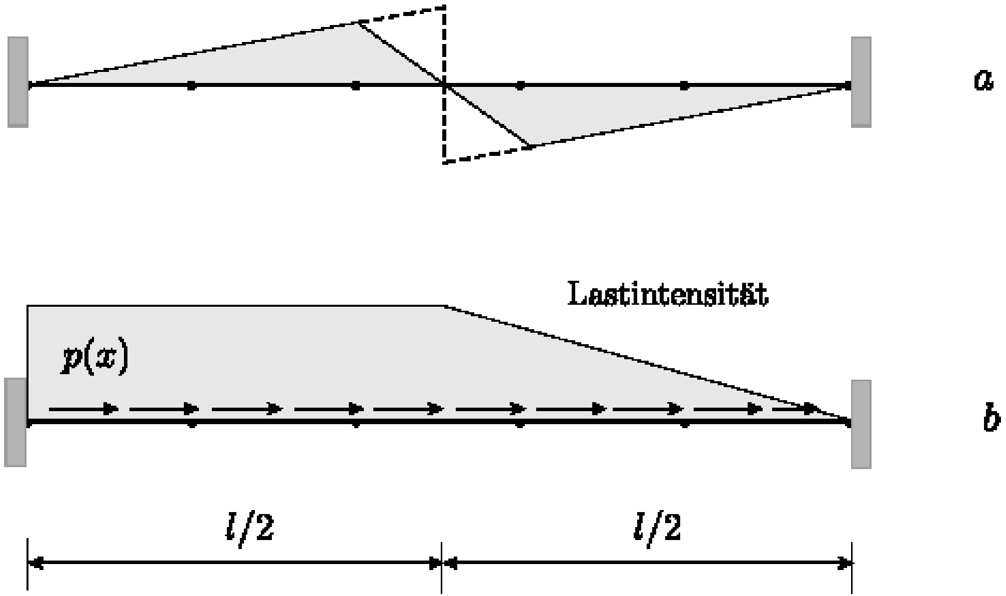


Other Conclusions

- For all results having their influence functions within the solution space of the FE-mesh, i.e. their Greens functions may be represented exactly, the FE solution w_h is exact.
- The total result value is obtained by integrating the deformation of the influence function with the given loading p(y).
- All results are obtained just with that value as if they would have been calculated with the above methodology.
- As the true Greens functions for stresses have always a singularity, it is evident that stresses within a FE program may be only exact if the singularity is reduced by the integration process of the load.

Error of forces

G_h^1 = FE-Näherung der EF für N in Stabmitte

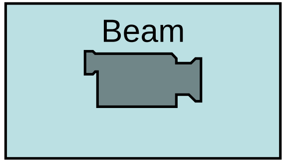


What are Beam Elements ?

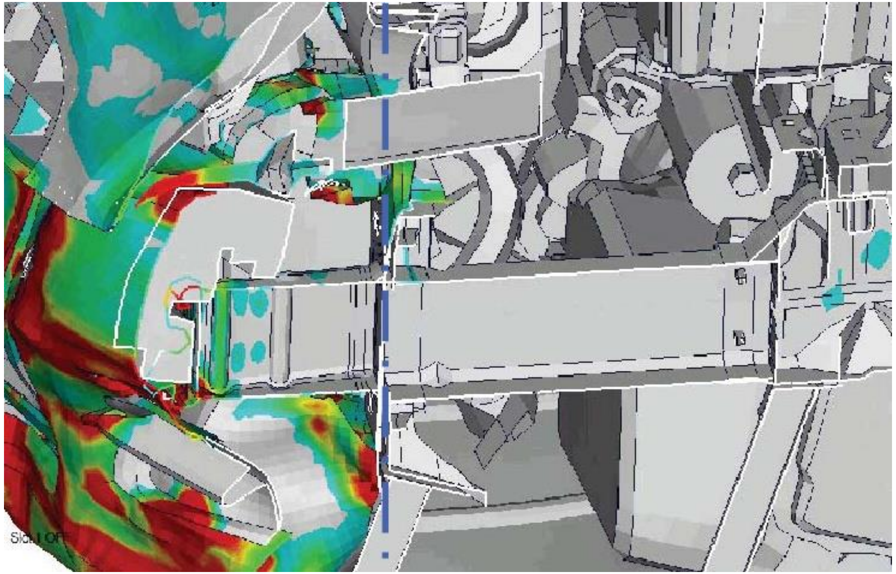
- 3D Continua with Length >> width/height
- Simplification of possible deformations (Bernoulli-Hypothesis and shape of section does not change)
- Some Simplification for manual analysis (Elastic centre, principal axis, shear centre)
- Myth:
  - Beam elements are simple
  - Beam elements are exact

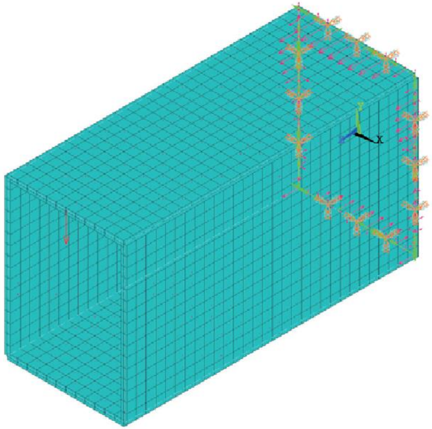
Still using beam elements ?

- Contra Beam Elements
  - Old fashioned
  - FE-Model is more general
  - Problems for D-Regions
- Pro Beam Elements
  - Engineering Concept
  - Advantage in Computing

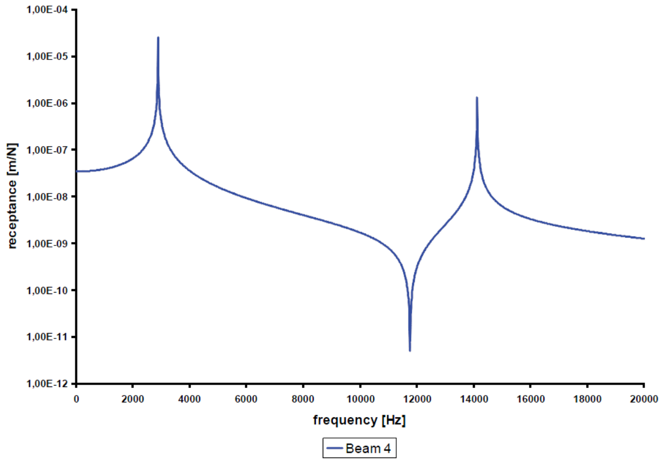


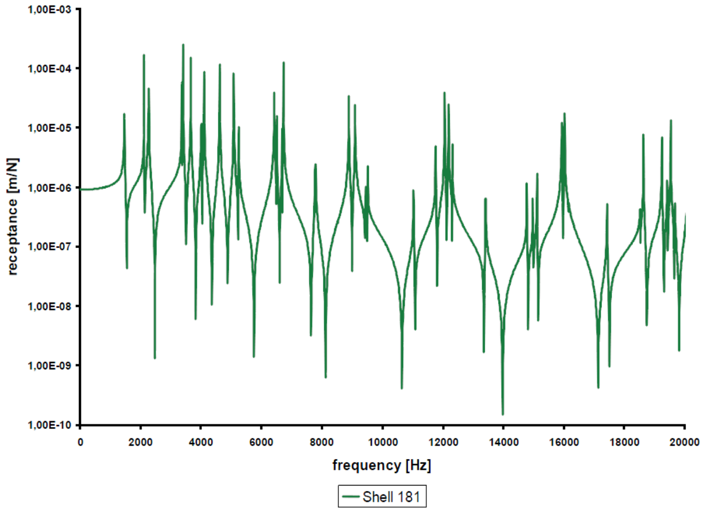
Other Limits for Beams



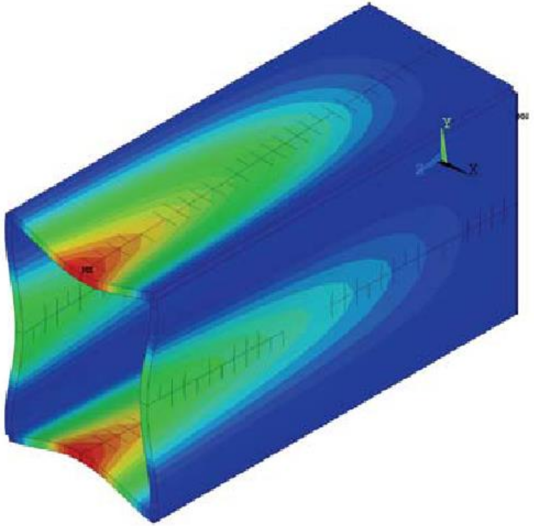


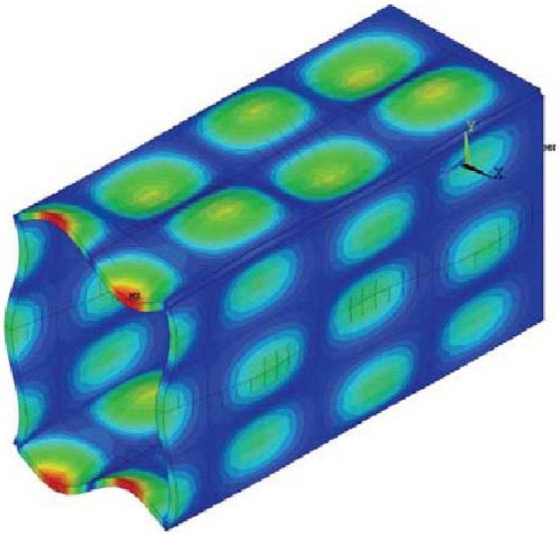
Frequency response beam / shell model





Local eigenforms: not a beam structure





Section of a beam element

- Planar section of deformation

$$u = u_0 + \varphi_y z - \varphi_z y$$

$$\sigma_x = E\varepsilon_x = E\frac{\partial u}{\partial x} = E\left[\frac{\partial u}{\partial x} + \frac{\partial \varphi_y}{\partial x} z - \frac{\partial \varphi_z}{\partial x} y\right]$$

Sections

- A section is thus a substructure of the beam

$$N = \int_A \sigma_x = EA\frac{\partial u}{\partial x} + EA_z\frac{\partial \varphi_y}{\partial x} - EA_y\frac{\partial \varphi_z}{\partial x}$$

$$M_y = \int_A z\sigma_x = EA_z\frac{\partial u}{\partial x} + EA_{zz}\frac{\partial \varphi_y}{\partial x} - EA_{yz}\frac{\partial \varphi_z}{\partial x}$$

$$M_z = \int_A y\sigma_x = EA_y\frac{\partial u}{\partial x} + EA_{yz}\frac{\partial \varphi_y}{\partial x} - EA_{yy}\frac{\partial \varphi_z}{\partial x}$$

Differential equation system

$$\begin{vmatrix} EA_x & EA_y & EA_z \\ EA_y & EI_y & EI_{yz} \\ EA_z & EI_{yz} & EI_z \end{vmatrix} * \begin{vmatrix} v_x^{II} \\ v_y^{IV} \\ v_z^{IV} \end{vmatrix} = \begin{vmatrix} p_x \\ p_y \\ p_z \end{vmatrix}$$

- Bending with respect to principal axis,
  $EI_{yz}=0$
- Normal force referenced on gravity centre
  $A_y=A_z=0$
- Shear forces referenced to the shear centre y_{sc}, z_{sc}

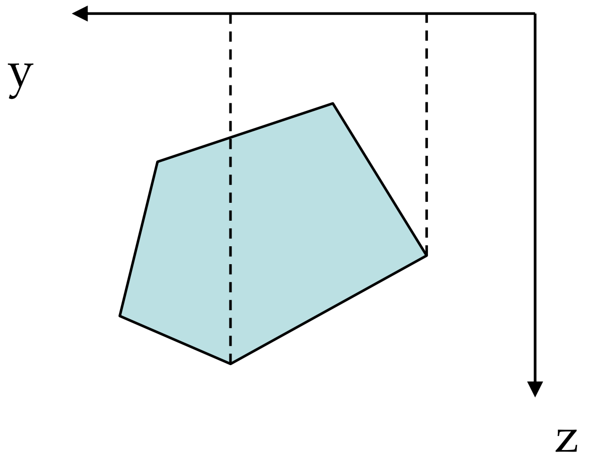
Sectional values

- Values may be calculated in advance:

$$A = \int_A dA$$

$$I_y = A_{zz} = \int_A z^2 dA$$

$$I_z = A_{yy} = \int_A y^2 dA$$



$$A = \sum_{i=0}^{n} \frac{1}{2}\left(z_{i+1} + z_i\right)\left(y_{i+1} - y_i\right)$$

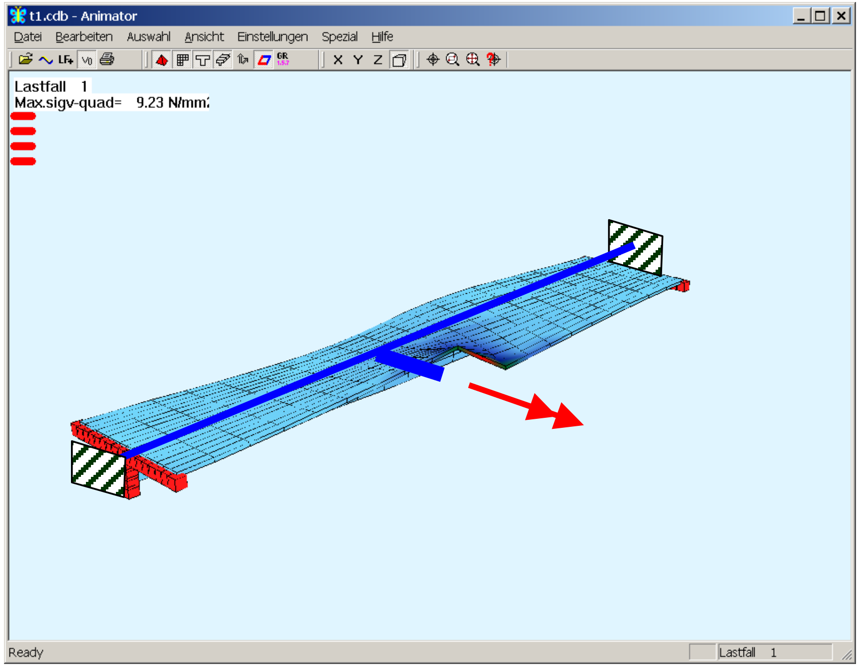
$$I_y = \sum_{i=0}^{n} \frac{1}{12}\left(z_{i+1} + z_i\right)\left(y_{i+1} - y_i\right)\left(z_{i+1}^2 + z_i^2\right)$$

Remark on effective width

- For some cases where the Bernoulli-Hypothesis is not really fulfilled, people introduce effective widths
  - For the sectional values i.e. stiffness
  - For the design itself
- Very difficult for biaxial bending
- The treatment of prestress loadings is prone for many discussions, as the normal force is acting on another effective section than the bending stress

A strange limit for Beam Theory

- Eccentric Moment creates Torsion



How to get the matrix of a Beam Element

- Closed solution for a prismatic beam:
  - No shear deformations
  - No warping
  - No second order effects
- Closed solution also possible for some special cases
  - Warping Torsion for prismatic beam
  - Initial stress for constant axial forces
- Closed Solution possible but not easy for some potential series of properties

Numerical Formulations

- Variational Method (FE-Method)
  - Solution space based on deformations
  - Strains calculated from deformations
  - Minimum of deformation energy
- Integration of differential equations
  - System of differential equations
  - Integration according Runge-Kutta

Integration method

- We calculate a transfer matrix either by exact or numerical integration of the D.E.:

$$\begin{bmatrix} w \\ \varphi \\ M \\ V \end{bmatrix}_e = \begin{bmatrix} a_{ij} & a_{ij} & a_{ij} & a_{ij} \\ & a_{ij} & a_{ij} & a_{ij} \\ & & a_{ij} & a_{ij} \\ & & & a_{ij} \end{bmatrix} \cdot \begin{bmatrix} w \\ \varphi \\ M \\ V \end{bmatrix}_a + \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}_L$$

- This matrix will be inverted yielding a stiffness matrix
- This element is a real substructure capable to deal with any type of loading or structural properties.

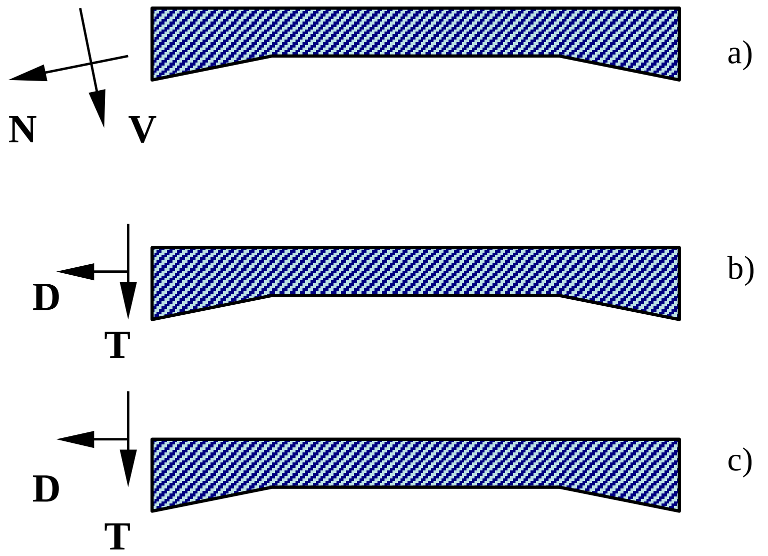
Dealing with off diagonal Inertias

- Program knows only principal axis (?)
- Rotation of Solution into the system of principal axis
- Complete treatment of Integrals with EI_{yz}
- The independent principal axis of shear deformations allow only the latter complete approach.

Elastic Center (Gravity axis)

- Elastic centre is not a constructional element. Beams are aligned with outer faces.
- Haunches create bends in the centre axis
- Haunches create skewed length of beams
- Centre changes with construction stages (Cast in situ concrete)
- So what else ?

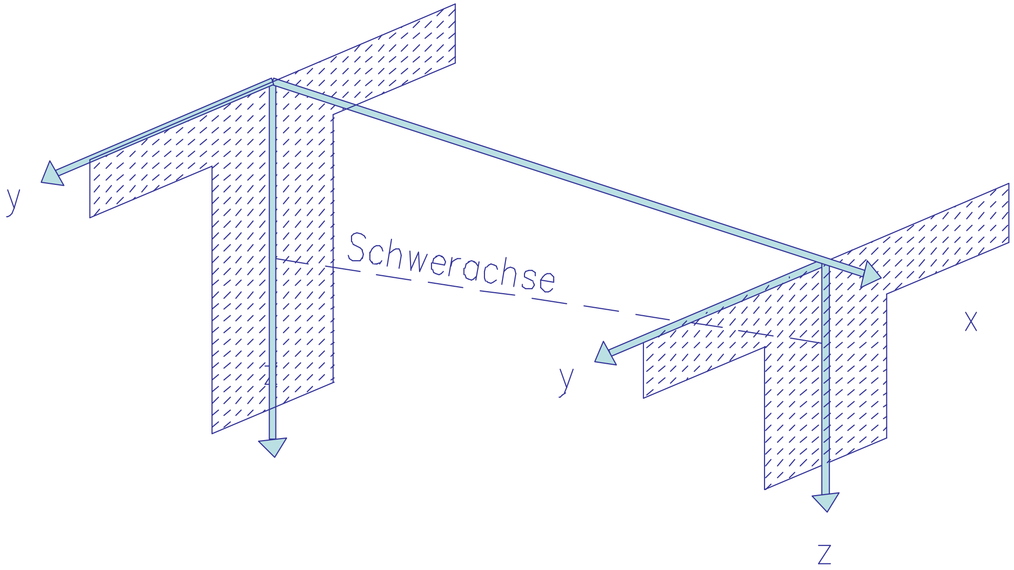
Reference axis



Reference axis

- Elastic centre axis with N + V
  - Results are directly applicable for design
- General reference axis
  - Not easy to control or understand
  - Superposition of actions
- Elastic centre with D + T
  - Similar to 2nd Order Theory
  - Superposition of forces is possible

Beam with a Haunch



Variational Method

$$Ansatzfunctions \qquad v = \sum H_i W_i$$

$$Strains \qquad \varepsilon = \frac{\partial u}{\partial x} + \ldots$$

$$Potential \qquad \Pi = \int pw\,dx + \int E\varepsilon^2 dV$$

Deformation Shape Functions

- Eccentricity at the endpoints

$$u_{0i} = u_i + \varphi_{yi}\Delta z_i - \varphi_{zi}\Delta y_i$$

$$u_{0j} = uj + \varphi_{yj}\Delta z_j - \varphi_{zj}\Delta y_j$$

- Interpolation u_0 linear, v,w,φ_x,φ_y cubic
- Displacement in Section

$$u = u_0 + \varphi_y\left(z - z_s\right) - \varphi_z\left(y - y_s\right)$$

Evaluation of Strains

- Location of elastic centre is NOT constant:

$$\varepsilon_x = \frac{\partial u}{\partial x} = u_o' + \varphi_y'(z - z_s) - \varphi_z'(y - y_s)$$

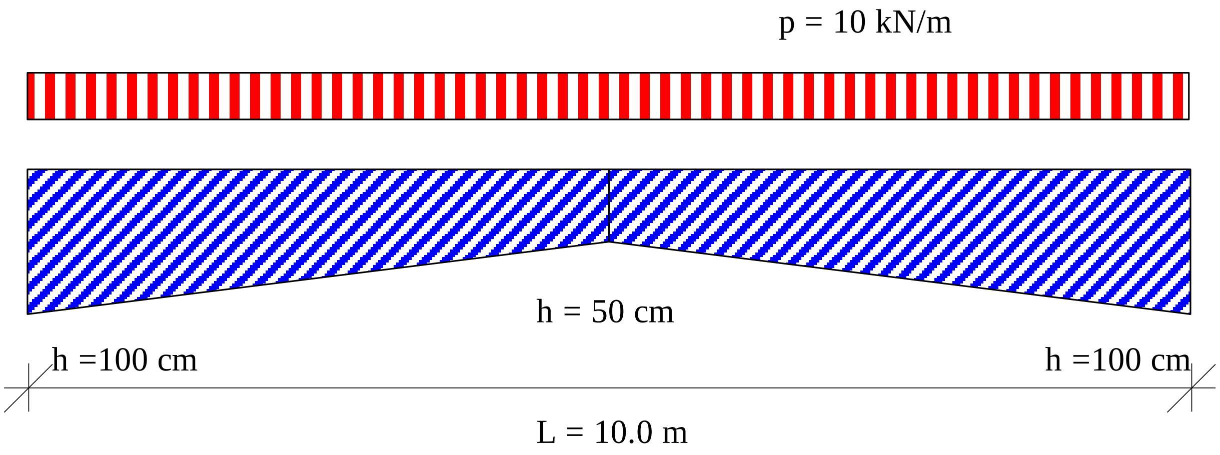
$$- \varphi_y z_s' + \varphi_z y_s'$$

Work of internal stress

$$\Pi_i = \int E\varepsilon_x^2 dV = \begin{aligned} & EA\left[u_o'^2 - 2u_o' \left[\varphi_y z_s' - \varphi_z y_s'\right]\right] + \\ & EA\left[\varphi_y{}^2 z_s'^2 + \varphi_z{}^2 y_s'^2 - 2\varphi_y z_s' \varphi_z y_s'\right] + \\ & EI_y \varphi_y'^2 + EI_z \varphi_z'^2 - 2EI_{yz} \varphi_y' \varphi_z' \end{aligned}$$

- Haunch creates normal force for a horizontal reference axis
- A haunch yields a variant moment for a constant axial force
- There is an additional coupling of primary and secondary bending if the inclination is not the same

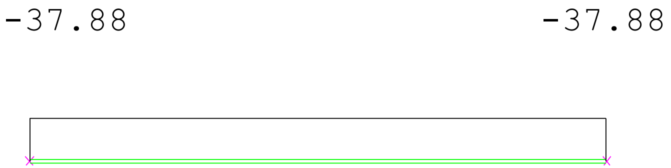
A Haunched Beam

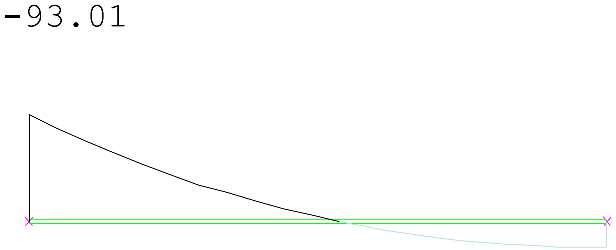


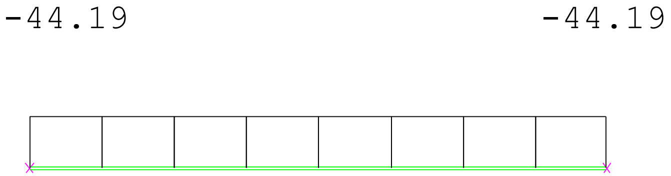
Results

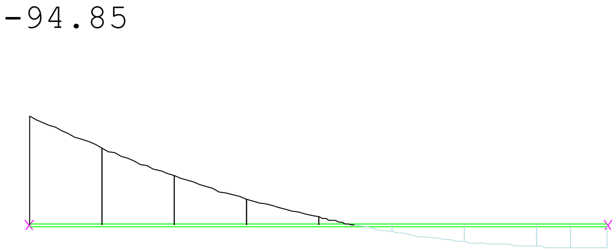
	w[mm]	Ne[kN]	Nm[kN]	Mye[kNm]	Mym[kNm]
Inclined axis					
CLOSED (1 element)	0,397	-80,50	-78,00	-73.58	31,91
CLOSED (8 elements)	0.208	-46,30	-43,80	-94,87	19,17
VAR (1 elements)	0,172	-39,80	-37,30	-93,65	22,00
VAR (8 elements)	0.206	-45,80	-43,30	-95,02	19,14
Horiz. Reference axis					
VAR (1 element)	0.168	-37,90	-37,90	-93.01	22,52
VAR (8 elements)	0.204	-44,20	-44,20	-94.85	19,10

Axial Force + Moments









Shear Deformations

- Not contained in the prerequisites
- Reduction of the bending stiffness by a comparison of deformations
- Theory Timoshenko/Marguerre
  separate deformation modes with a compatibility request
  $V = dM/dx$
- Inversion of the flexibility
  - Most general approach
  - For prismatic beam possible with a closed form

Non conforming element based on the variational method

$$\varphi = \varphi_i \cdot (1-\xi) + \varphi_j \cdot (\xi) + \varphi_m \cdot (4\xi(1-\xi))$$

$$M = -\frac{EI}{L} \cdot \left[\left(\varphi_j - \varphi_i \right) + 1.5 \cdot \varphi_m \left(8\xi - 4 \right) \right]$$

$$V = -GA \cdot \theta = -GA \cdot \left[\frac{\left(\varphi_i + \varphi_j \right)}{2} + \varphi_m - \frac{\left(u_j - u_i \right)}{L} \right]$$

- Non-conforming Ansatz for Θ yields
  - Concise Beam Element
  - Including effects of haunches
  - Including all shear deformations

Winkler Assumption

$$k_{ij} = \int_0^L EI \cdot \frac{d^2 Np_i}{dx^2} \frac{d^2 Np_j}{dx^2} + C \cdot Np_i \cdot Np_j \cdot dx$$

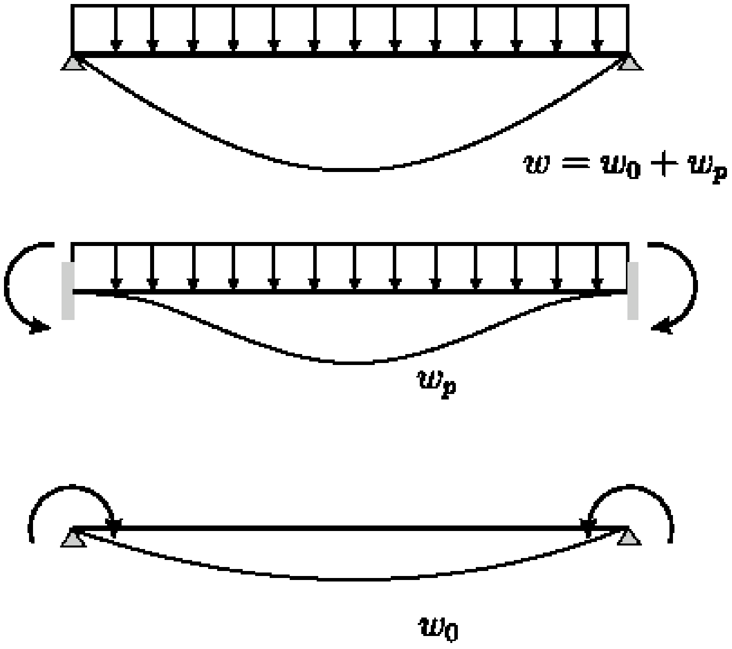
$$K = \frac{EI}{L^3} \begin{vmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{vmatrix} + \frac{C}{420} \begin{vmatrix} 156L & -22L^2 & 54L & -13L^2 \\ -22L^2 & 4L^3 & 13L^2 & -3L^3 \\ 54L & 13L^2 & 156L & 22L^2 \\ -13L^2 & -3L^3 & 22L^2 & 4L^3 \end{vmatrix}$$

Disappointing Example

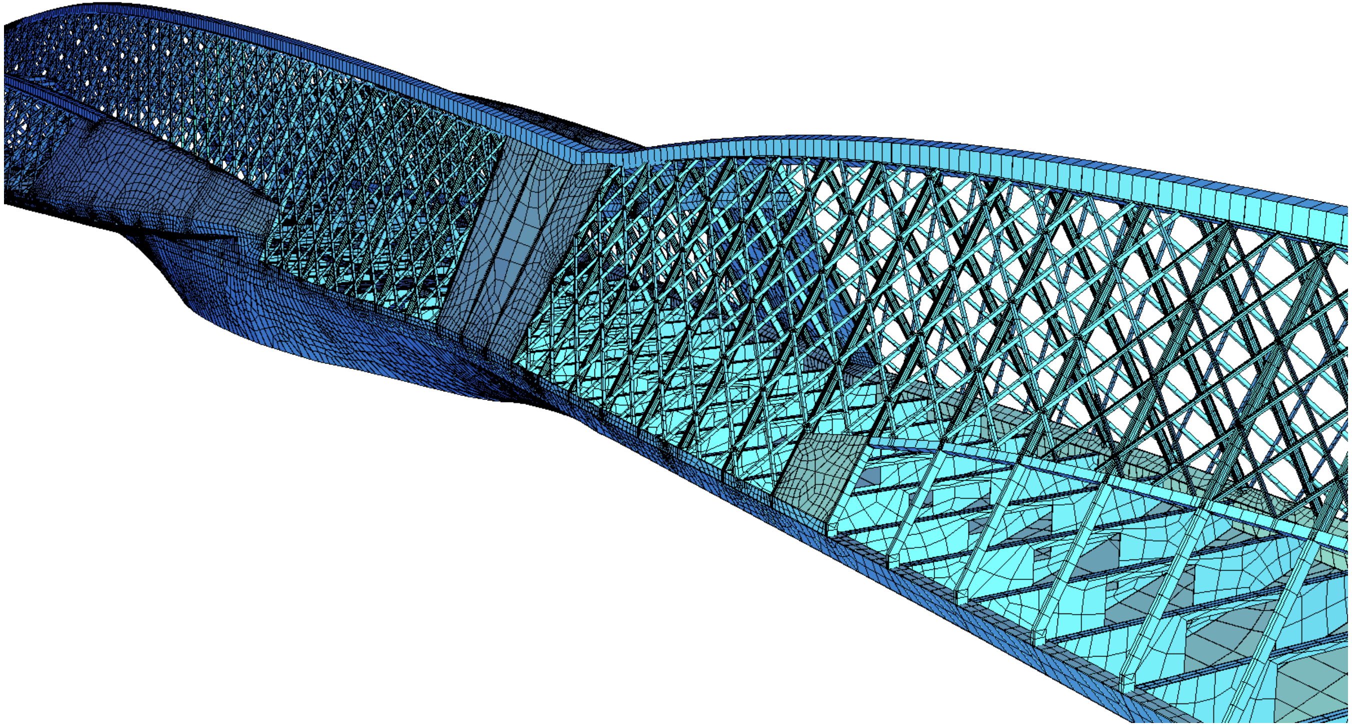
- FE-Beam element is a very powerful element, but it is a Finite Element.
- Displacements are only cubic parabolas.
- Simple span beam with uniform loading
  - cubic coefficients is zero (symmetric solution)
  -
  -

	Exact	1 element	2 elements	
Max. moment	281.25	281.25	281.25	kNm
End rotations	41.147	41.147	41.147	
Center deflection	19.2876	15.4301	19.2876	mm

Particular solution needed



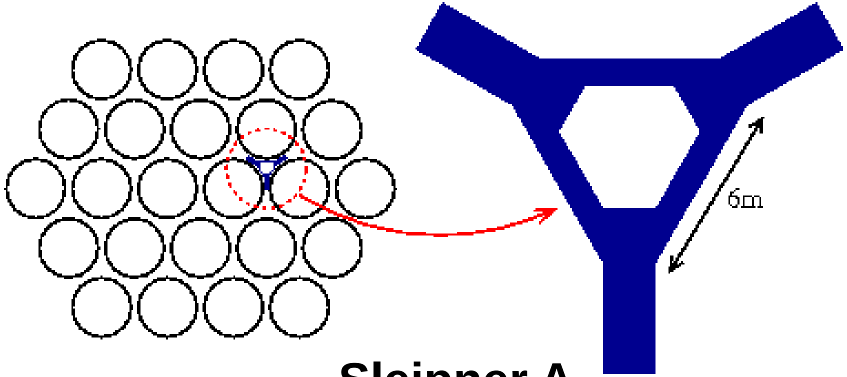
Real World (Saragossa Bridge-Pavillon)



Nearly Everything can be designed today

- Is it possible to build it ?
- If we do an analysis at the total system, do we cover all details?

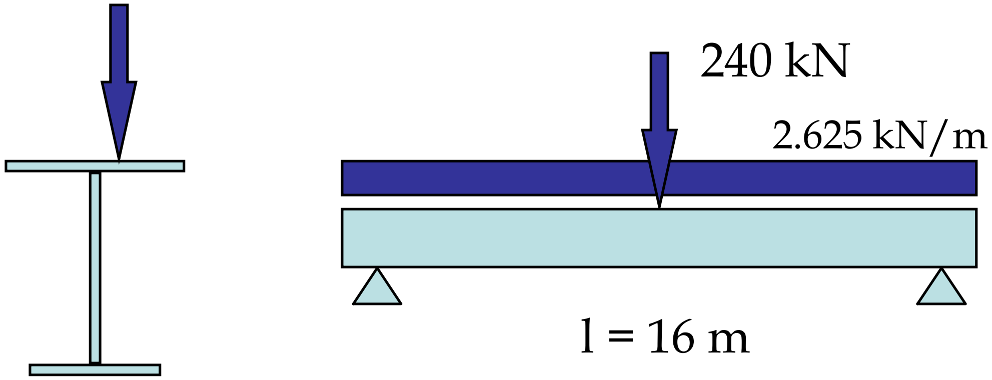
https://en.wikipedia.org/wiki/Sleipner_A



Sleipner A

Effects of Warping Torsion

- Petersen Stabilität page. 757 b)



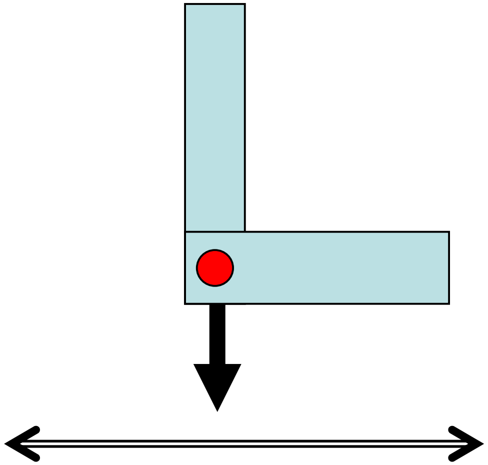
Effect of warping Torsion

- Bending stress $\sigma = 8.43$ kN/cm^2
- 2nd order Torsional Buckling $\sigma = 13.61$ kN/cm^2
- Warping stress $\sigma = 8.28$ kN/cm^2

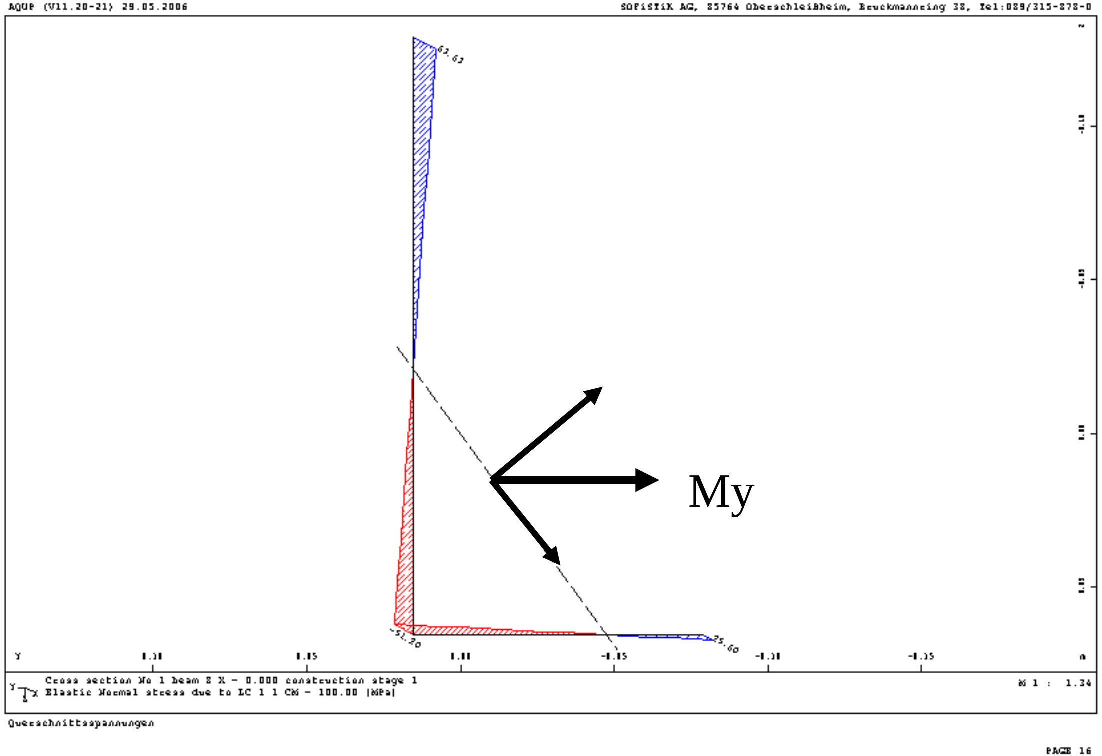
caveat:

- Dischinger-Factor Mb = 69.47 kNm^2
- Petersen with Formula 4 Mb = 38.03 kNm^2
- Petersen with Formula 9 Mb = 55.04 kNm^2
- FE-Element SOFiSTiK Mb = 54.77 kNm^2

A Question



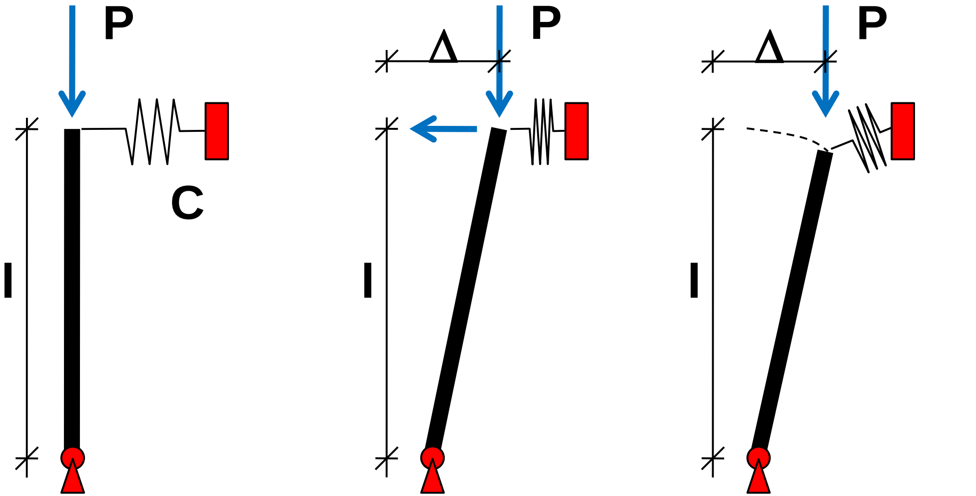
Look at the stresses



Two views for the stability problem

- The undeformed system is in an unstable equilibrium.
- Smallest deviations (imperfection) lead to a collapse.
- Two possible approaches:
  - Eigen value or bifurcation problem:
    Sudden failure from a differential deformed shape
  - Deformation problem:
    Non linear increase of deformations caused be the negative geometric (initial stress stiffness) Stiffness
    „Elasticity + Stress*Geometry“

Stability - Buckling



Equilibrium:

$$P \cdot \Delta - C \cdot \Delta \cdot l \leq 0 \quad \Rightarrow \quad C \geq \frac{P}{l}$$

Differential equation = check list of parameters

$$\left(EI(x)\cdot v_z''\right)''+D(x)\cdot\left(v_z''+v_{z0}''\right)+C\cdot v_z+p_x(x)\cdot v_z'=p_z(x)$$

- Transverse Deformation v_z & stress free imperfection v_{z0}
- Bending stiffness EI(x)
- Longitudinal force D(x)
- Bedding in transverse direction C
- Load in longitudinal direction $p_x(x)$
- Load in transverse direction $p_z(x)$

Warping Torsion

- 7. Degree of freedom + Warping Moment Mb
- secondary torsional moment Mt2

$$M_t = M_{tv} + M_{t2} = G I_T \vartheta' - E C_M \vartheta'''$$

- Hermitian Functions of 2nd Degree

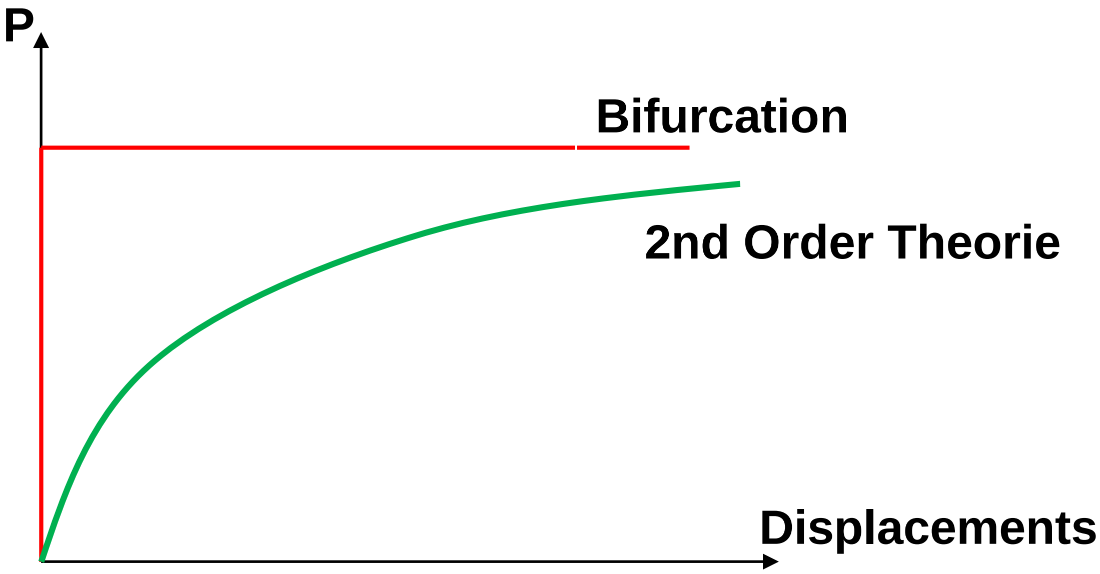
$$\Pi_{i2} = \int_L E C_M \vartheta''^2 + G I_t \vartheta'^2 +$$

$$N\left[2\vartheta' z_m v'_m + 2\vartheta' y_m w'_m + v'^2_m + w'^2_m + i_m \vartheta'^2\right] +$$

$$M_y\left[-2\vartheta w''_m + r_{My}\vartheta'^2\right] + M_z\left[2\vartheta v''_m + r_{Mz}\vartheta'^2\right] +$$

$$M_b\left[r_{Mw}\vartheta'^2\right] + M_t\left[v'_m w''_m - v''_m w'_m\right]$$

System behaviour



Solution methods

- Inhomogeneous Equation
  (2nd Order Theory, general nonlinear analysis)

$$\left[K_{lin} + K_{geo}(\sigma)\right] \cdot u = p$$

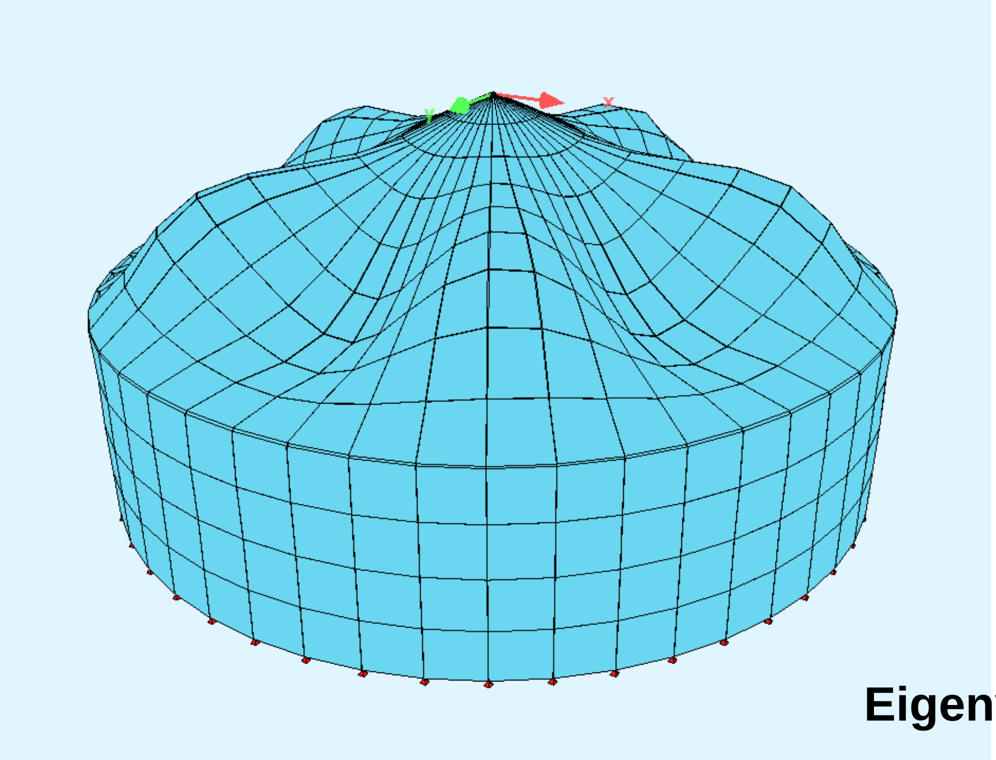
- Homogeneous equation (Stability Eigen value)

$$\left[K_{lin} + \lambda \cdot K_{geo}(\sigma_{prim})\right] \cdot X = 0$$

Frequencies

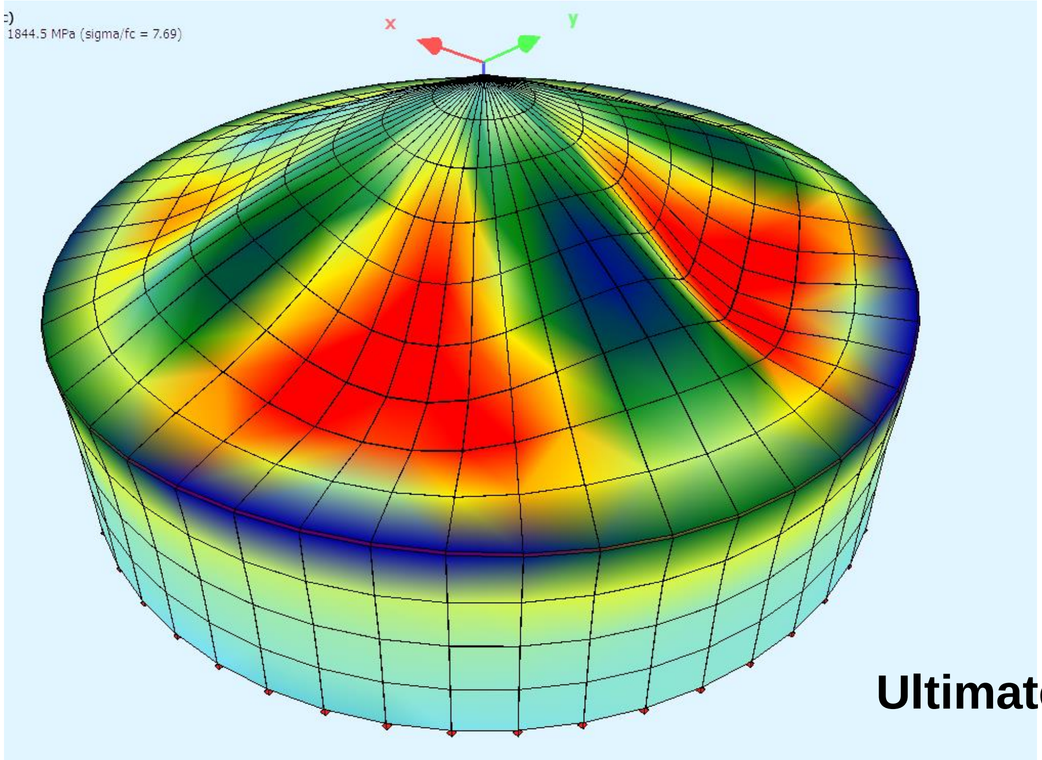
- String of a Guitar
  - Increase of tension increases the frequency
- Compressive Members
  - Increase of compression decreases the freuqncy
  - For a certain compressive force the frequency will become zero
  - i.e. The structure will collapse once and will never recover (The period becomes infinity)

Asymptotic Behaviour ?



Eigenvalue: 1.65

Asymptotic Behaviour ?



Ultimate load: 1.08

Solution methods

- Closed Solutions for special cases
  (using trigonometric or hyperbolic functions)
- FE-Ansatz with Ansatz functions and a variational principle
- Numerical Integration of the differential equation for beam elements

Variational Approach for Geometric Stiffness

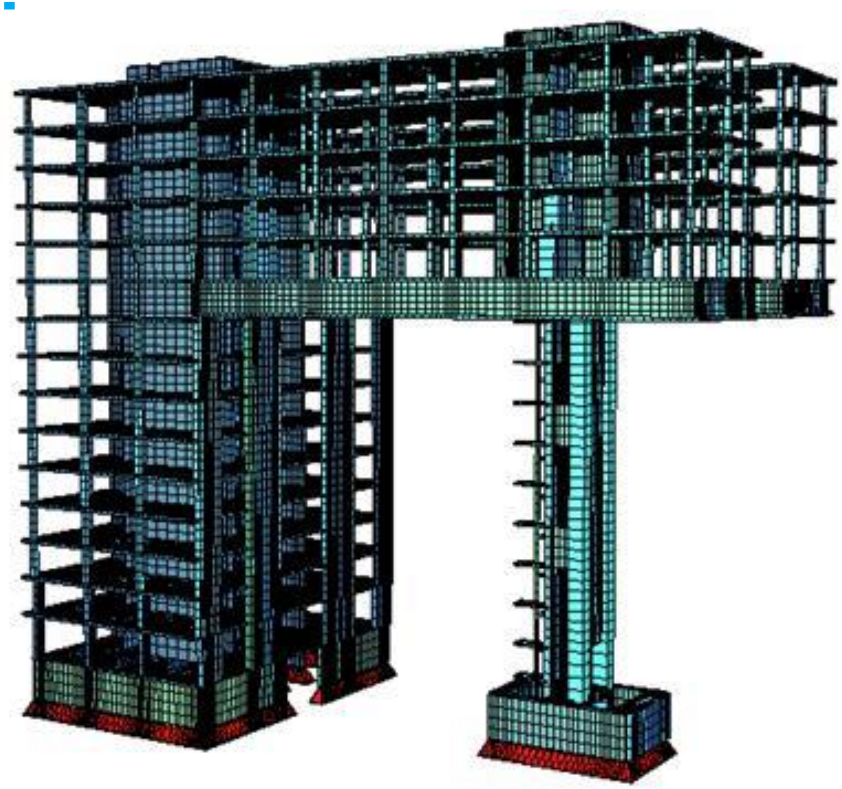
$$k_{ij} = \int_0^L EI \cdot \frac{d^2 Np_i}{dx^2} \frac{d^2 Np_j}{dx^2} + P \cdot \frac{dNp_i}{dx} \cdot \frac{dNp_j}{dx} \cdot dx$$

$$K = \frac{EI}{L^3} \begin{vmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{vmatrix} + \frac{P}{30L} \begin{vmatrix} 36 & -3L & -36 & -3L \\ -3L & 4L^2 & 3L & -L^2 \\ -36 & 3L & 36 & 3L \\ -3L & -L^2 & 3L & 4L^2 \end{vmatrix}$$

Same Ansatz problem with a single element for buckling Eigenvalues

Euler case for prismat. beam	Theoretical	Eigenvalue
I (1 element)	3303	3328
I (2 elements)		3305
I (4 elements)		3303
I (8 elements)		3303
II (1 element)	13212	16065
II (2 elements)		13312
II (4 elements)		13219
II (8 elements)		13213

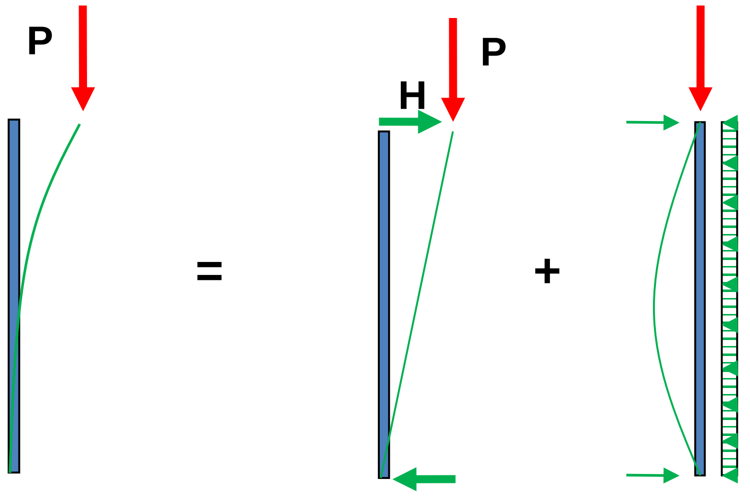
And here ?



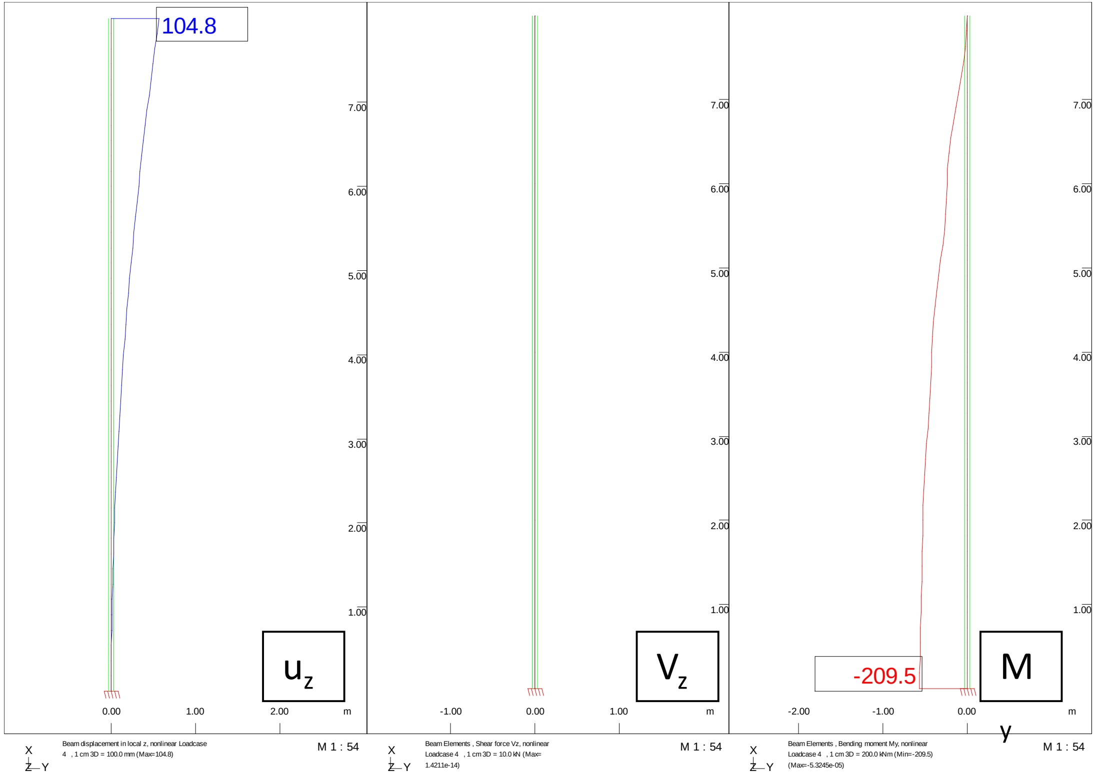
Combined Strategy

- Evaluation of the total stability with the total system and imperfections.
- As we do not model every single beam with it's own imperfection and at least two elements we do local checks based on the representative beam for
  - Deformations / Buckling transverse to the structure
  - Truss elements
  - Lateral torsional buckling
  - But: Stiffness at start and end node is difficult to obtain!

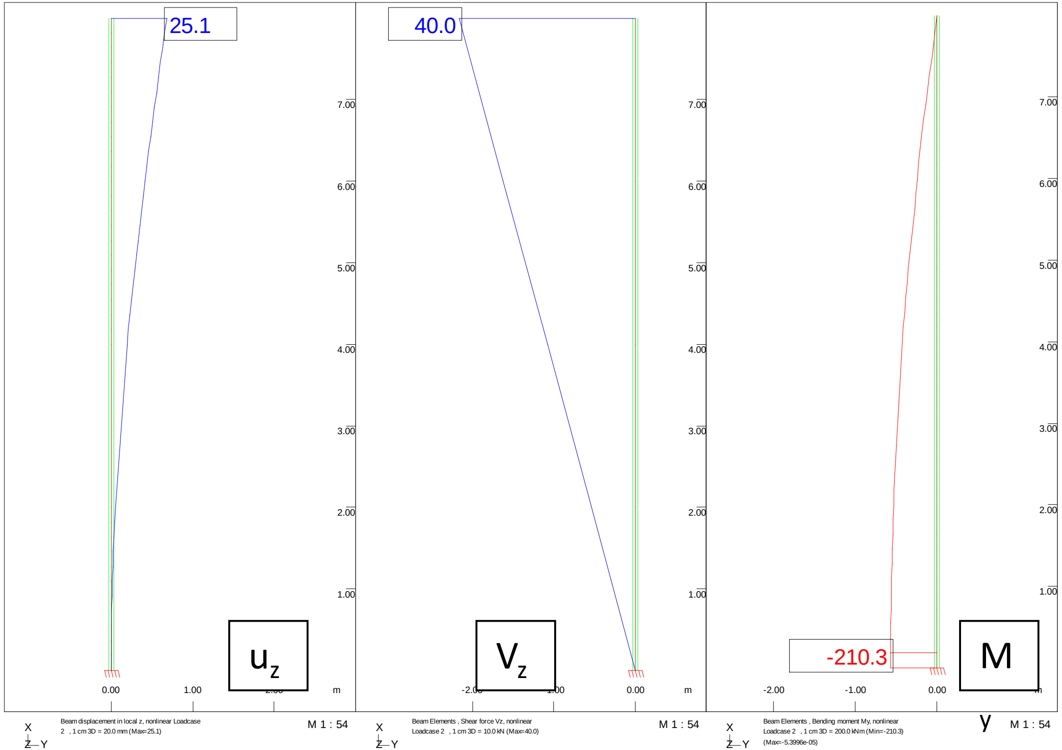
Imperfection or Equivalent Loads ?



Quadratic Imperfection of 80 mm, P=2000kN



Equivalent forces according Design codes



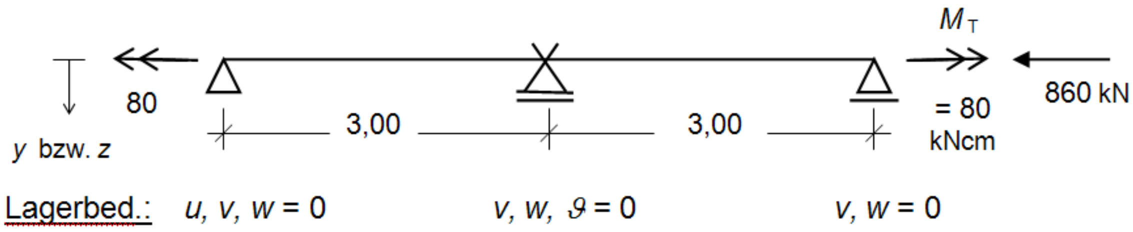
Cantilever L = 8 m, P = 2000 kN, $e_u = s_k/200$

- Imperfection
  - Total deformation 104.8 = 80 + 24.8 mm
  - Bending moment = 2000 * 0.1048 = 209.6 kNm
  - Transversal force 0, shear force at top: 52.4 kN
- Equivalent force H = 20+20 kN + q = 5 kN
  - Load deformation 25.1 mm
  - Bending moment 210.3 kNm
  - Shear at top = 40*cos(0.36)+2000*sin(0.36)= 52.5 kN

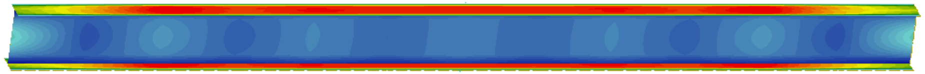
Imperfections versus equivalent loadings

- Using the equivalent forces there is a transversal force of 40 kN at the top reduced to zero at the bottom.
- Using imperfections the transverse force is 0 kN
- Shear deformations are caused by shear forces, but have been calculated with the transverse force. For columns the effect is small in general.
- Geometric non linear (GMNAI is ok)

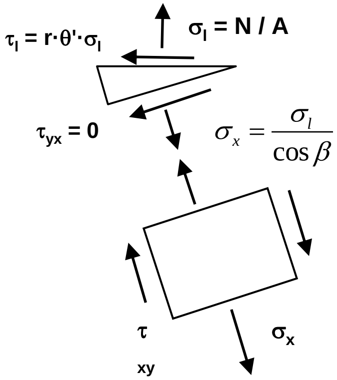
Torsion 2nd Order Theory



- Torsional buckling load 1185 kN
- Rotation 265 mrad
- Primary torsional moment 292 kNcm
- Torsional moment $N \cdot i_p^2 \cdot \theta'$ -212 kNcm



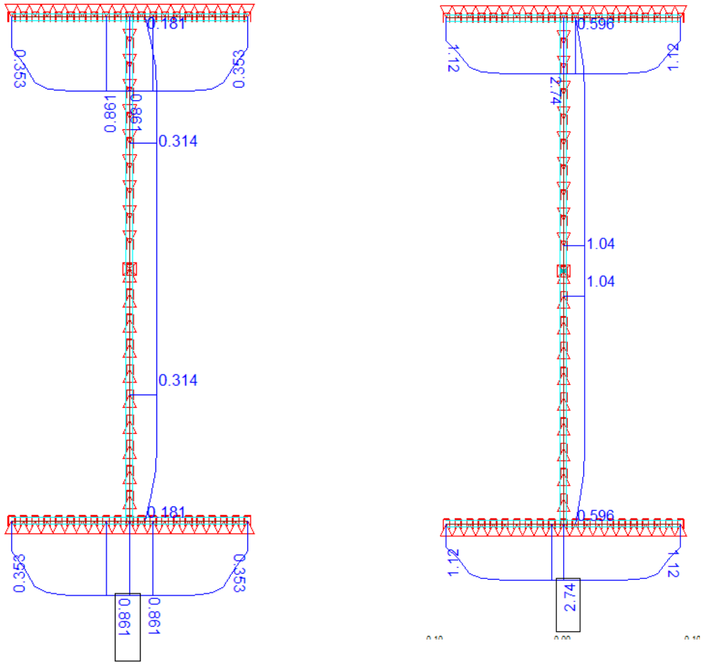
Stresses from 2nd order Torsion ?



$$\sigma_x = \frac{\sigma_l}{\cos\beta} \approx \frac{\sigma_l}{1-(r\Theta')^2}$$

- No stresses like primary torsion
- No stresses like secondary torsion
- => Shear stresses distributed similar to the normal stresses

Components of drilling moments in FE-System 1st and 2nd order theory



Higher Geometric Nonlinear Effects

- horizontal cable with a length of 10.0 m, a sectional area of 0.84 cm^2 and a prestress of 1 kN with self weight.
  Taking into account the sagging of approx. 8 o/oo will increase the normal force by a factor of 2 and has considerable influence on deformation and Frequency:

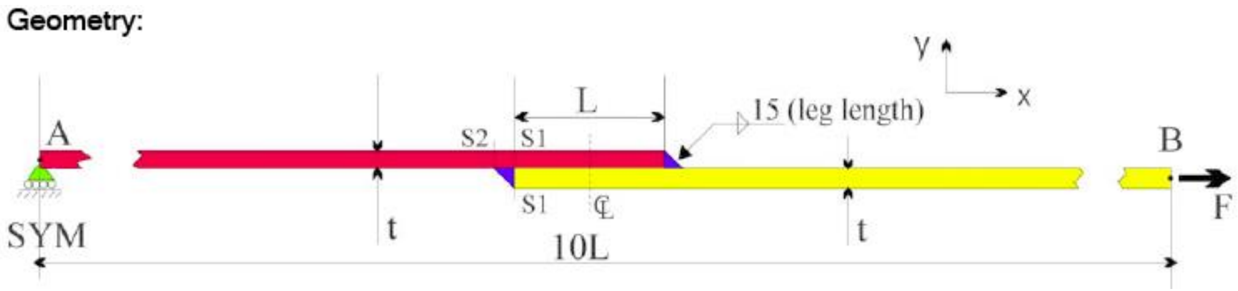
	N [kN]	u [mm]
Without slack	**1.0**	**83.36**
With slack	**1.9**	**43.95**

	f1	f2	f3
Without slack	**1.928**	**3.809**	**5.596**
With slack	**2.374 / 3.468**	**4.690**	**6.890**

Movement up and down different

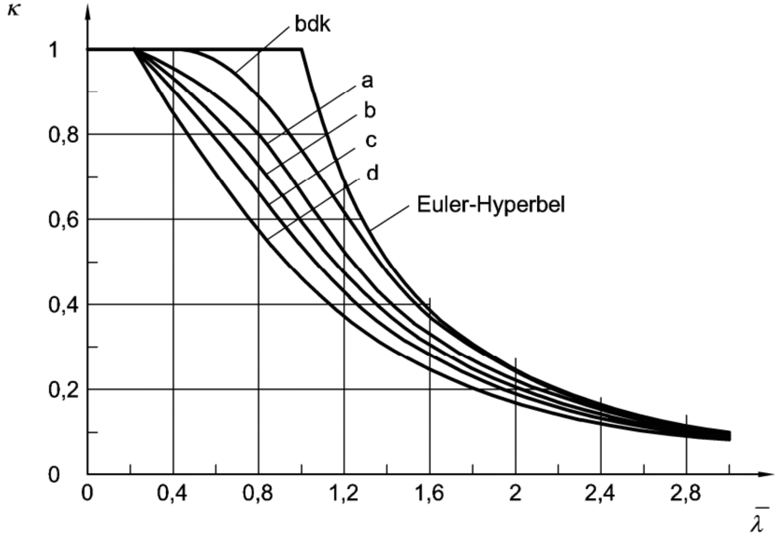
3^{rd} order Theory

- Load excentricity will be reduced by geometric non linear effects and thus the bending stress is reduced by a factor of 2
- Only one participant taking part in that benchmark has recognized this effect !



Buckling Design on a representative beam

- Buckling curves



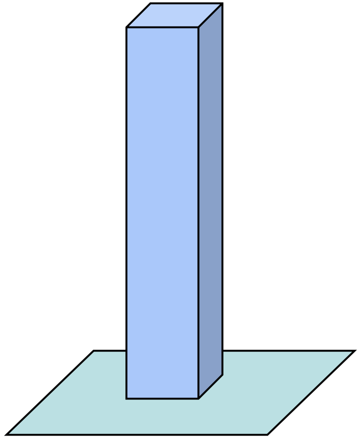
Slendernes requires Buckling length

- Useful results are only obtained by using 2nd order Theory and imperfections.!
- e.g. EN 1993 : Buckling in transverse direction is handled with a slenderness ratio λ_{opt}
- But this is not always wanted, as the number of imperfection cases may increase dramatically for complex spatial structures. And the superposition is only possible for cases with the same normal force.
- AISC and BS do not recommend to use 2nd order theory (there are also some differences to PI-delta-analysis) !
- => People want to do the design check on a single beam with a buckling length

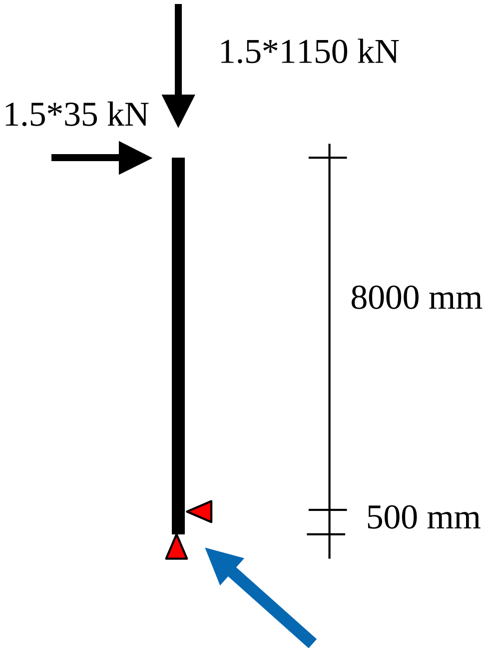
$$s_K = \pi \sqrt{\frac{EI}{\nu_{Ki} \cdot D}}$$

Buckling length ?

- It is not a geometrical size ! And it is not related to any mesh density of a finite element structure !
- It is rather easy to get eigenvalues of the loading at the total system based on the geometric stiffness (there are more than one eigenvalue!)
- How to convert the global buckling factor to the local single beam ?
- General assumption:
  If all local eigenvalues are above the global one, everything is ok.



Flag pole



HEB 500 - S 235

σ_N = N/A = 72.3 Mpa

σ_M = M/W = 97.8 Mpa

Theory II. Order (e_u = 8000/250):

$\sigma = \sigma_N + \sigma_M^{II} = 72.3 + 134.9 = 207.4$

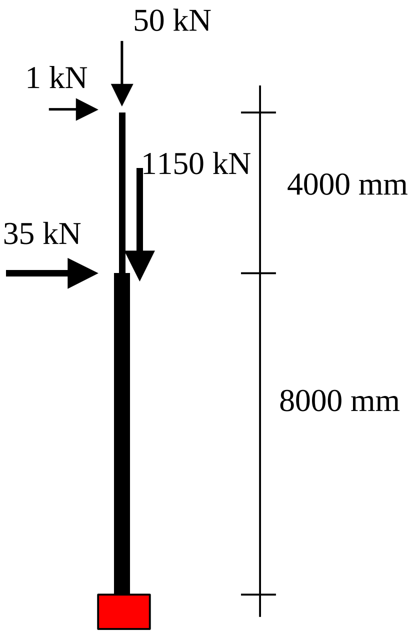
Centr. Buckling: 8339 kN, s_k=16.331 m

$\lambda = 77$, $\omega = 1.50$

$\sigma = \omega\ \sigma_N + 0.9\ \sigma_M = 196.5$ MPa

Beam with 500 mm length has a $\beta = 32.66$!

First Eigenvalue not critical

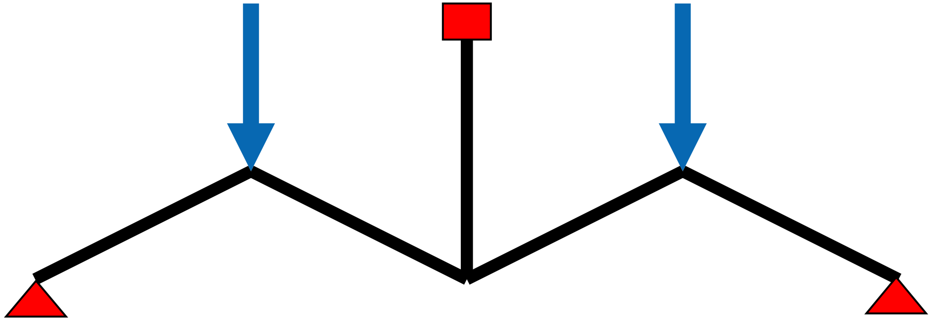


HEB 500 - S 235	
Buckling without antenna:	8687 kN
Classical from 1st Eigenform	3019 kN
Classical from 2nd Eigenform	8762 kN
Antenna tube 110/10	
Fully fixed reference:	128 kN
Classical from 1st Eigenform	126 kN
Classical from 2nd Eigenform	365 kN

„Da haben wir den Salat“

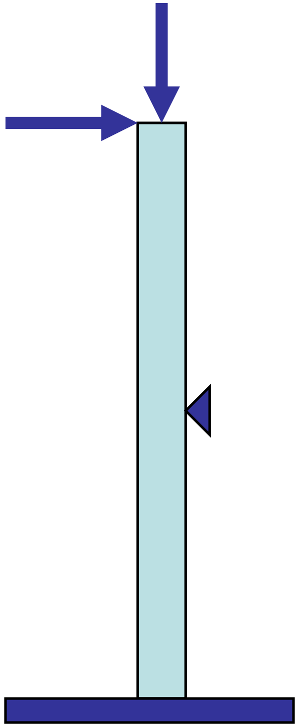
- Buckling length is not a suitable design method in all cases !
- Thus it is not possible to write a program which may be used as a black box for that purpose !
- But: For any design with buckling curves we need some value !

A Buckling Tensile Member



Material Nonlinear Analysis of reinforced concrete beam

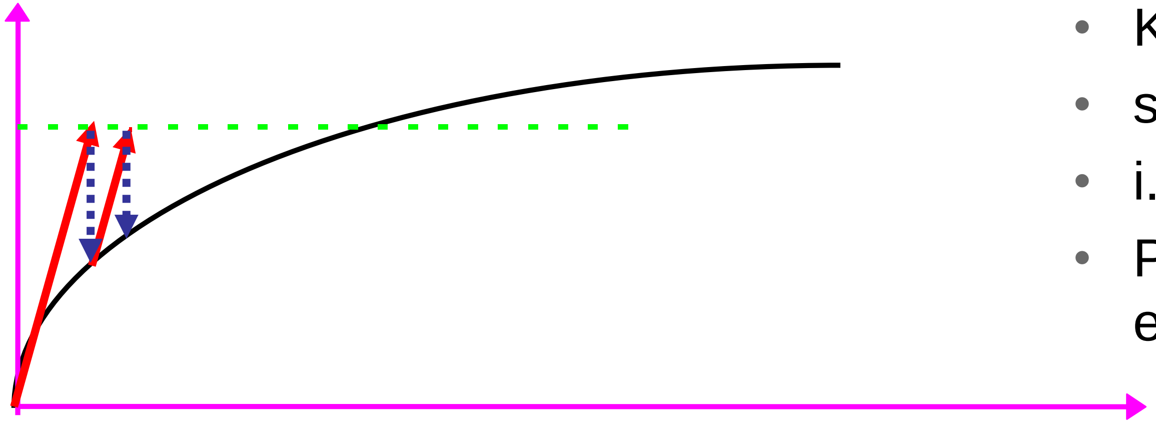
- The reinforcement is not known a priori
- Reinforcement may be staggered or not
- There is more than one possible solution even for the case with a given reinforcement



Basic Steps nonlinear

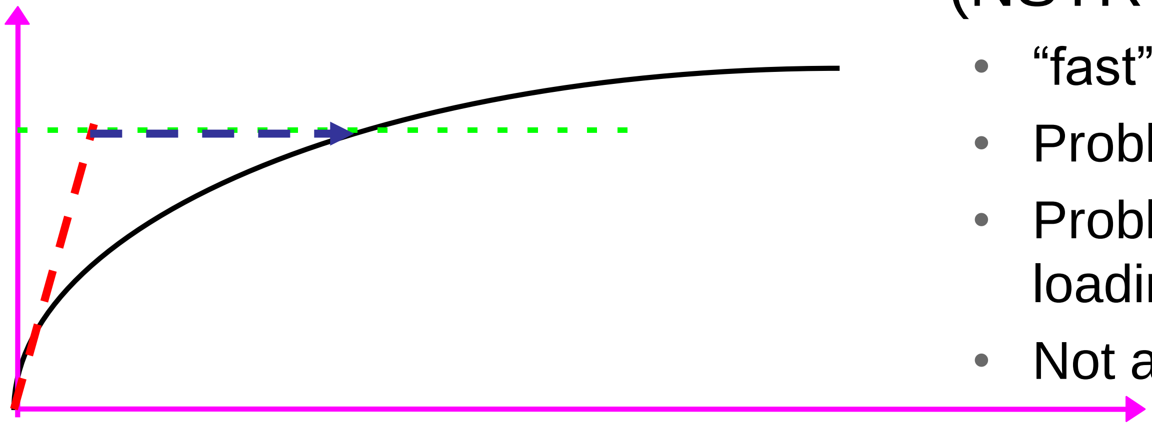
- Inner Iteration within section
  - Choose a strain distribution
  - Integration of stresses defined by the stress-strain law to forces and moments
- Corrective Residuas
  - Differences bettween inner / outer forces
  - Plastic Strains
  - Secant or Tangential stiffness

Strain or Residual based Evaluation ?



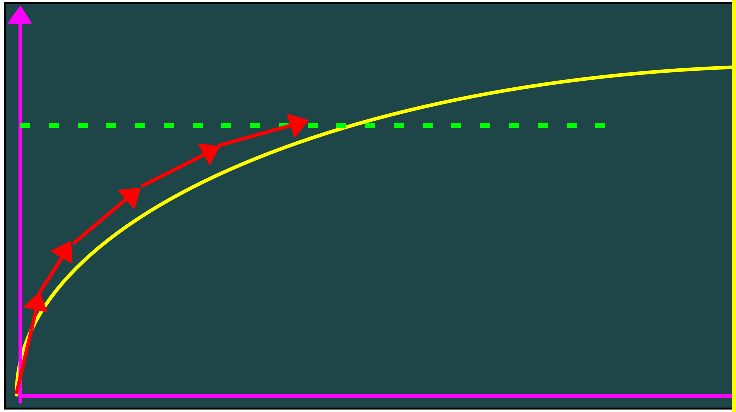
- Strain based approach
  - Konsistent to FE-Method
  - slow convergence
  - i.g. stable method
  - Problems with hardening effects

Strain or Residual based Evaluation ?



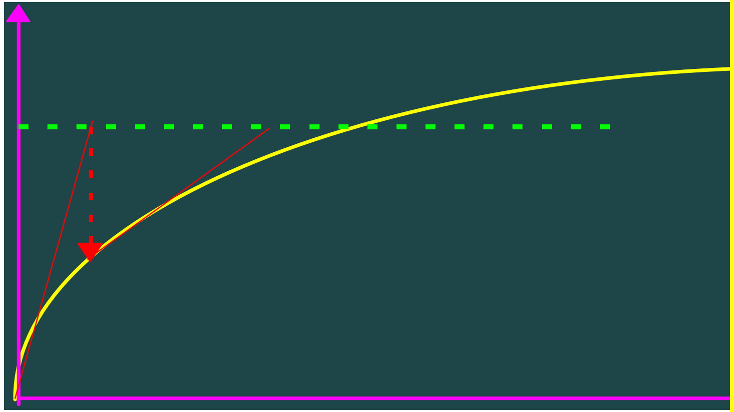
- Force based approach (NSTR SN)
  - “fast” convergence
  - Problems with saddle points
  - Problems with ultimate loadings
  - Not applicable in all occasions

Iteration Methods



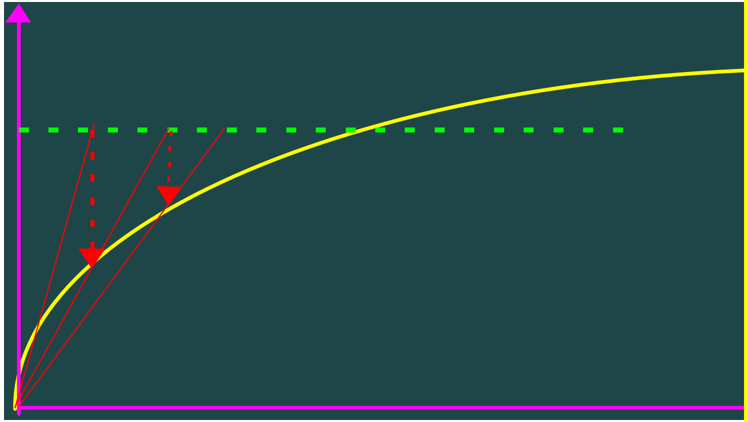
- Incremental Strategy with/without Iteration
  - High computational effort
  - Unique solutions
  - However: Precision not guaranteed
  - Runge-Kutta-Method

Iteration methods



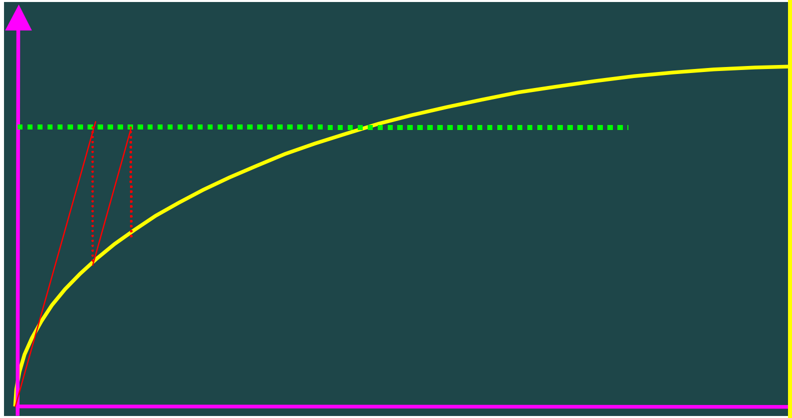
- Newton-Method
  - High computational effort
  - optimum quadratic convergence
  - Problems with limit values of Stiffness

Iteration Methods



- Secant-Method
  - Mean numerical effort
  - Rather fast convergence
  - No problems with limit values of stiffness

Iteration Methods



- Quasi-Newton-Method
  - Least numerical effort
  - Problems with non local behaviour
  - Crisfield + BFGS - Methods

New Stiffness or strains ?

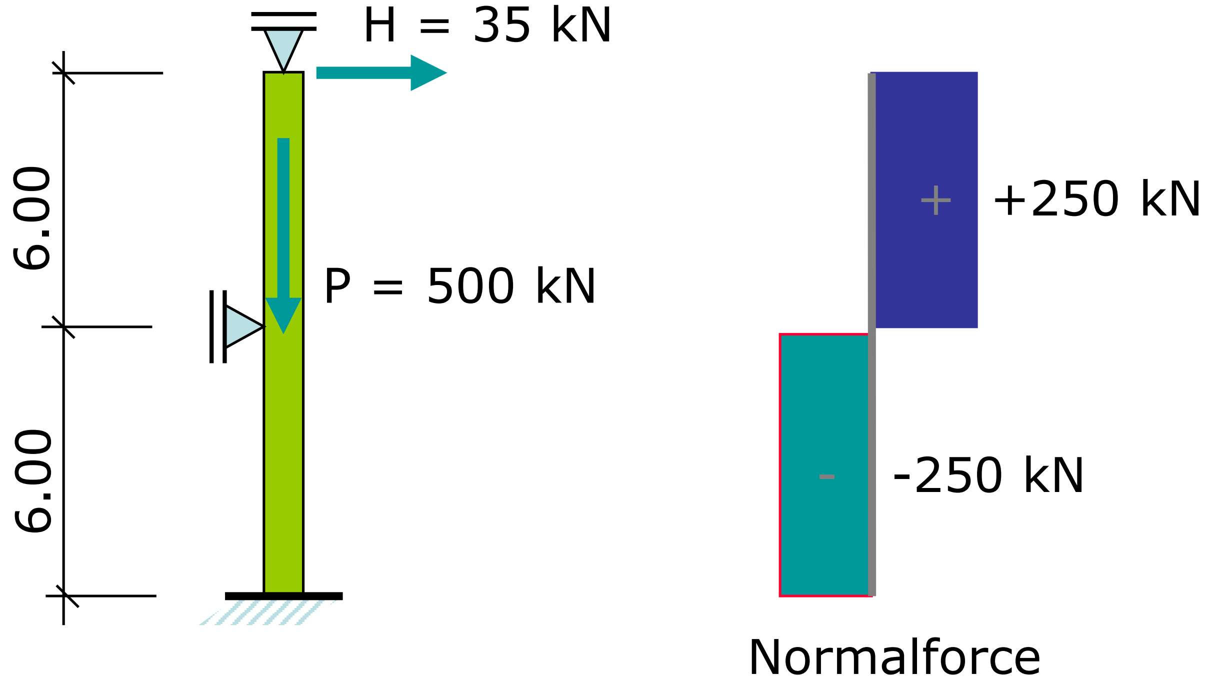
$$\begin{bmatrix} M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EI_y & EI_{yz} \\ EI_{yz} & EI_z \end{bmatrix} \cdot \begin{bmatrix} k_y - k_{y,pl} \\ k_z - k_{z,pl} \end{bmatrix}$$

- 2 Equations – 5 unknowns
- Solutions:
  - Calculate diagonal stiffness only
  - Keep Stiffness, change plastic strains
  - Select tangential stiffness,
    add corrective strains

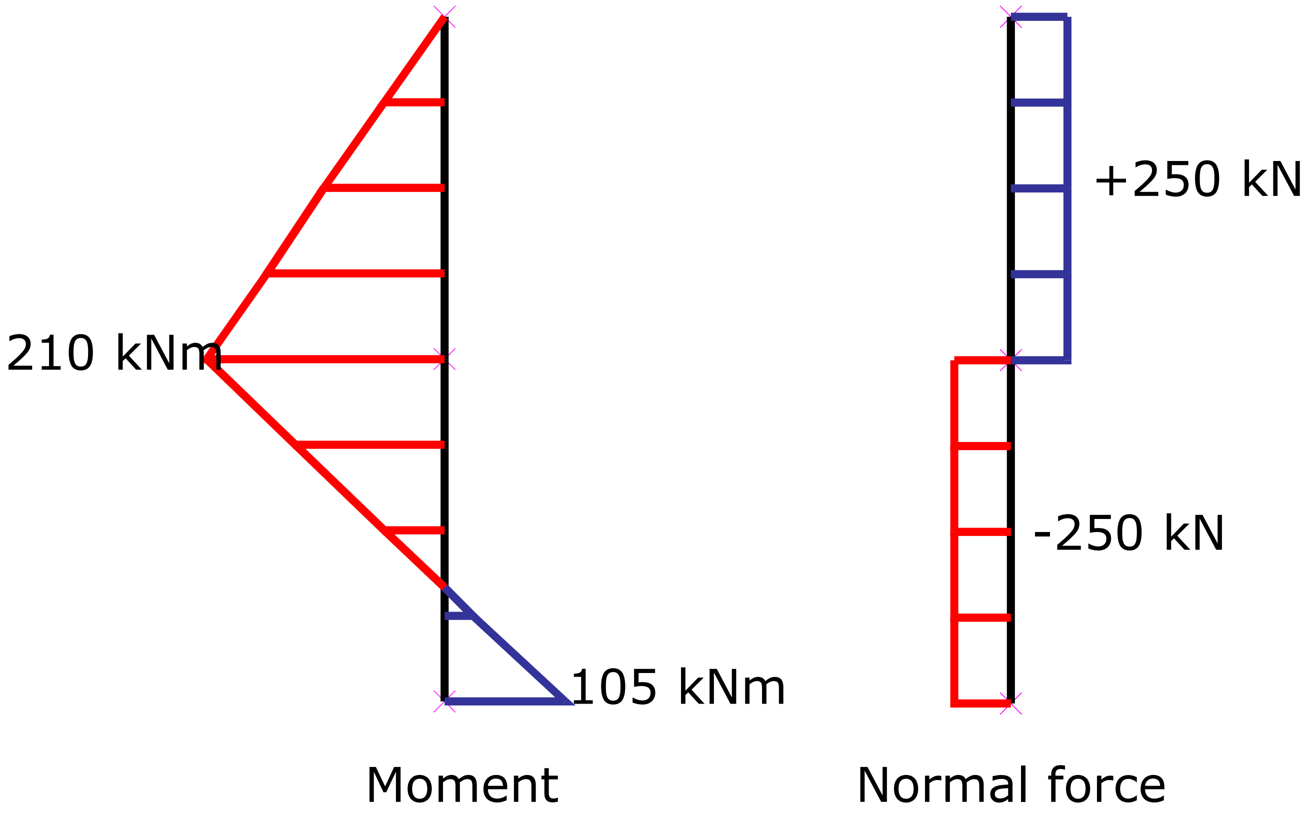
Torsion and Transverse Shear

- Reduction of deformation areas by some empirical method
- Real Energy equivalents

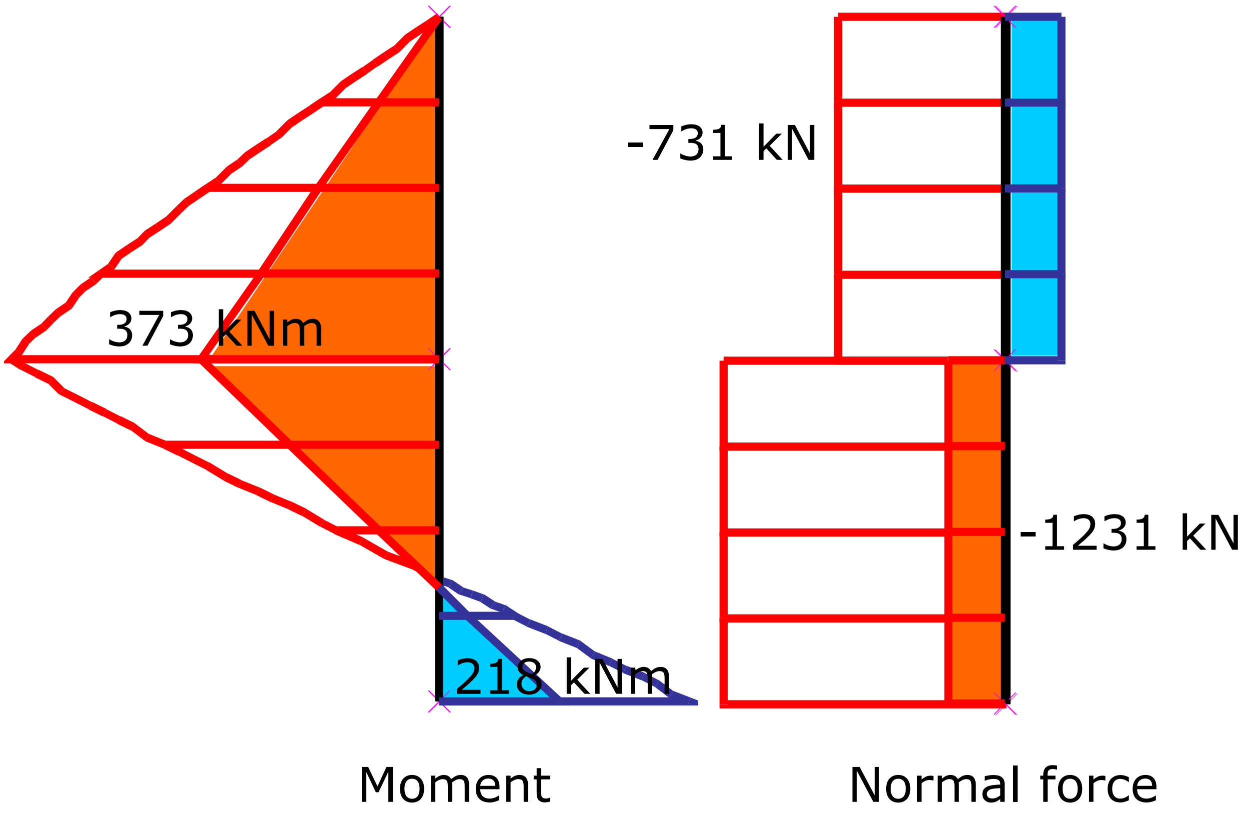
Unexpected Effects



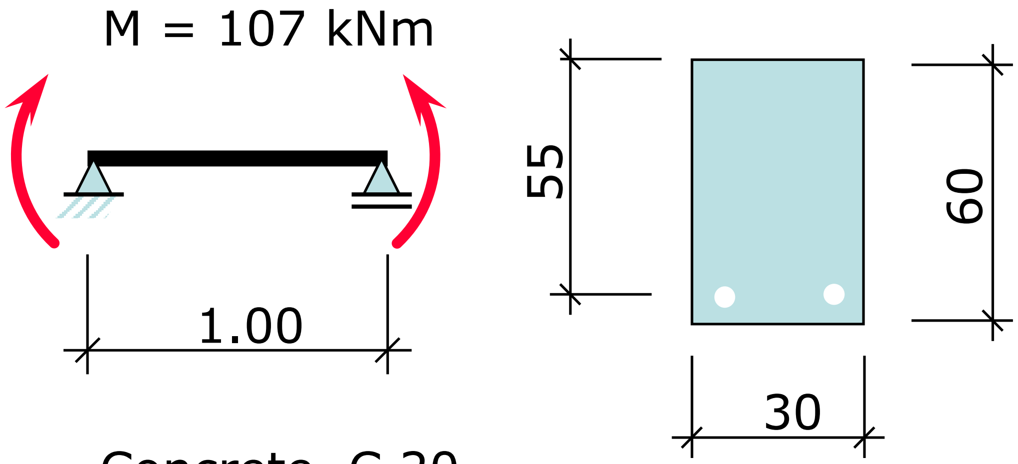
Linear Analysis



Non linear Analysis

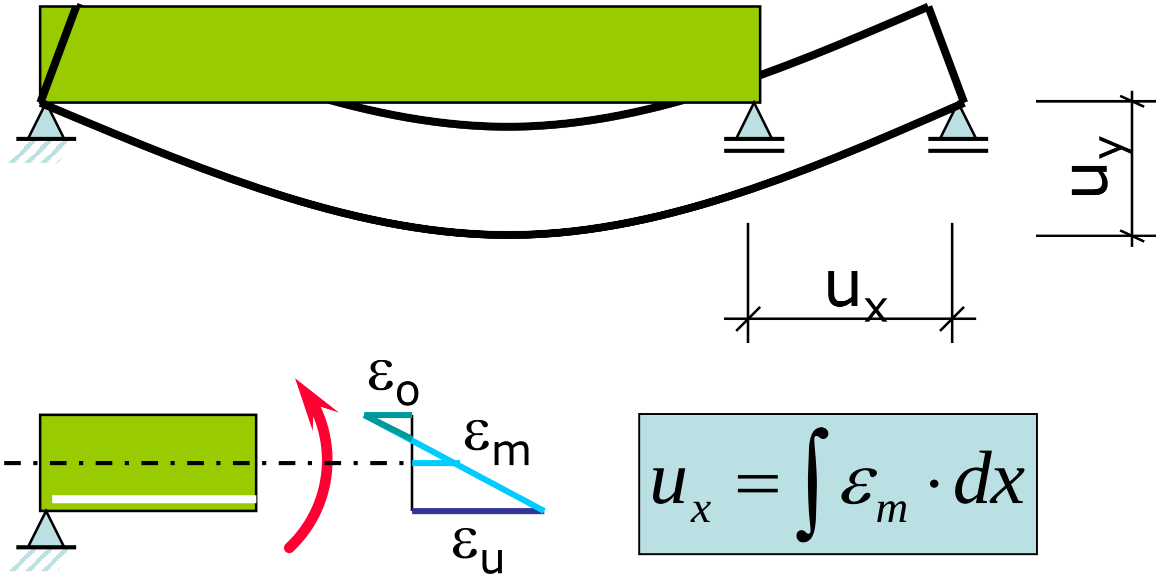


Simple Example



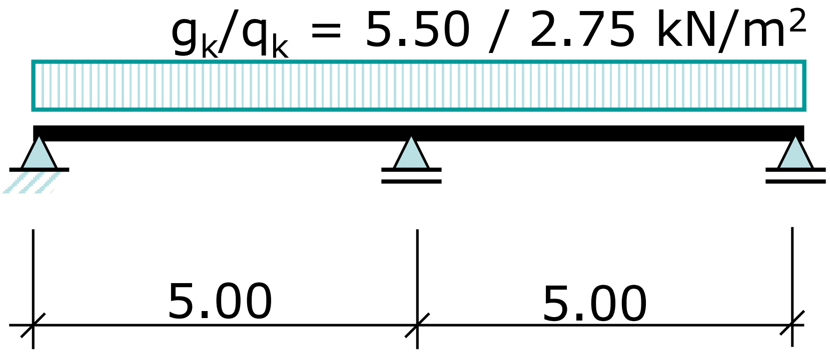
Concrete C 20
Reinforcement S 500

Deformed System



$$u_x = \int \varepsilon_m \cdot dx$$

Building Slab



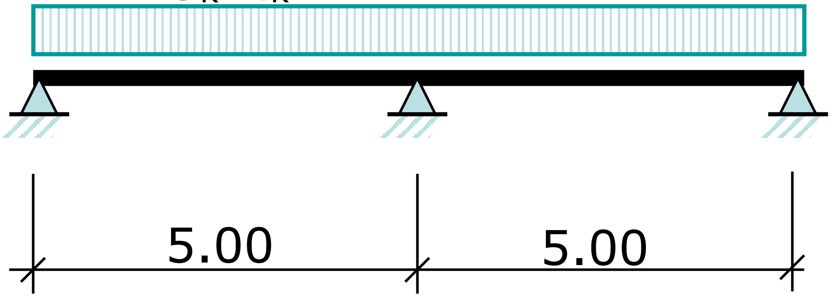
Concrete C 20
Reinforc. S 500
Slab Thickness
h = 16 cm

Design for deflections

Analysis according to	obtained deformation	allowed deformation	
uncracked	0.24	1.0	cm
fully cracked	1.79		cm
Incl. Tension stiffening	1.07		cm

Change support Condition

g_k/q_k = 5.50 / 2.75 kN/m²



Concrete C 20
Reinforc. S 500
Slab thickness
h = 16 cm

quasi permanent combination = 5.50 + 0.3 × 2.75 kN/m²

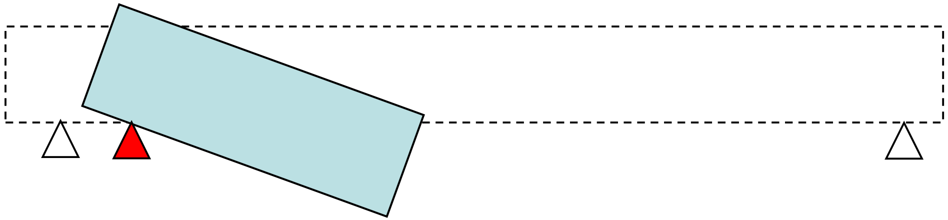
f = 0.51 cm

Support of an Elastic Beam

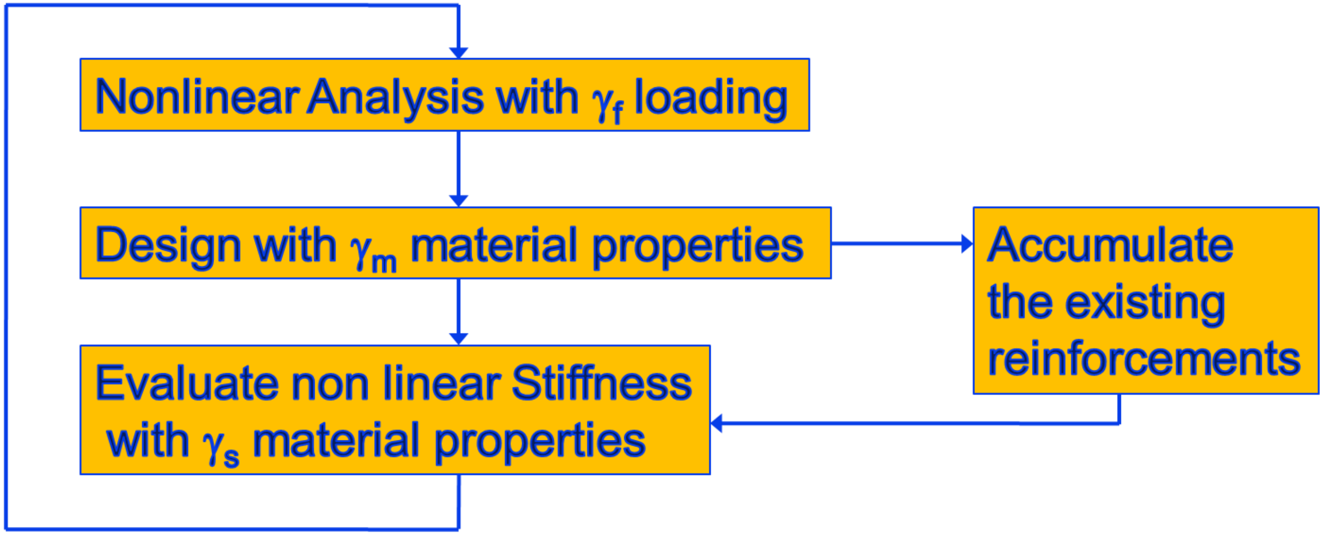
- General assumption: support in the neutral center axis
- Real world is a support at the lower side
- The curvature induces deformations of the supports
- If the support is fixed a normal force is introduced

$$F = q \cdot l^2 / 8 / h$$

reducing the sagging moment by a factor of 2 !



Flowchart of a non linear Analysis



Safety factors

- Partial safety coefficients can not be applied at an arbitrary location in a non linear analysis. There is a significant difference if they are applied on the load F or the resistance R !

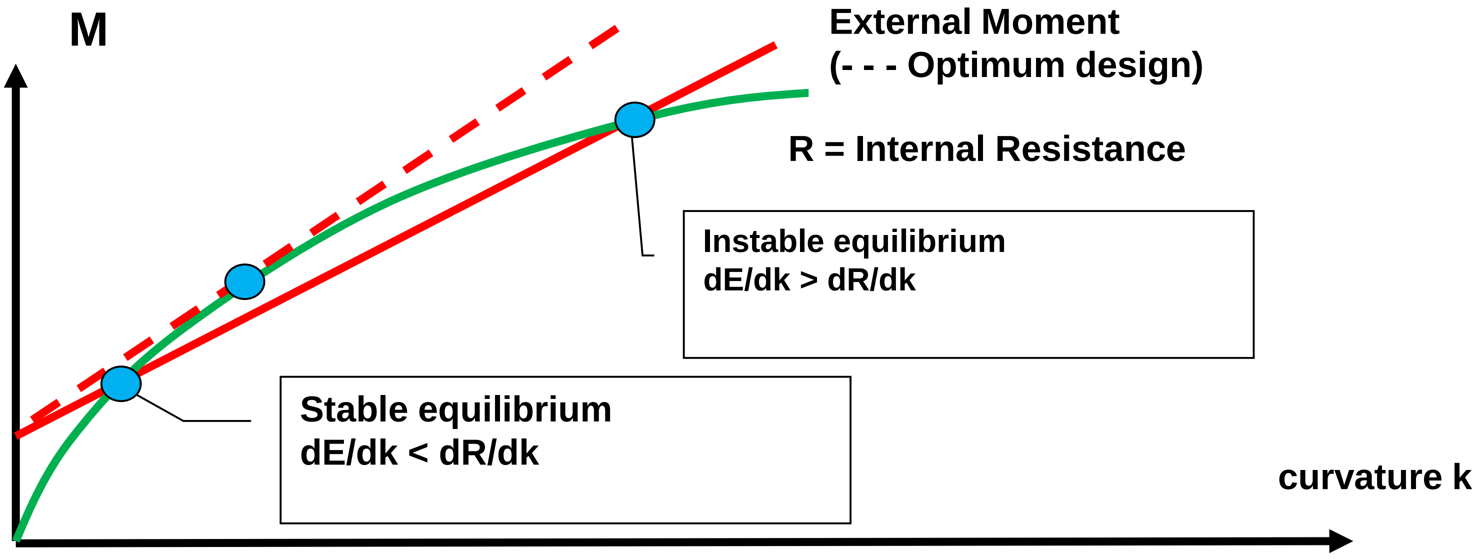
$$E(\gamma_f \cdot F) \leq R\left(\frac{f}{\gamma_M \cdot \gamma_r}\right)$$

$$\gamma_r \cdot E(\gamma_f \cdot F) \leq R\left(\frac{f}{\gamma_M}\right) \neq E(\gamma_r \cdot \gamma_f \cdot F) \leq R\left(\frac{f}{\gamma_M}\right)$$

Safety Factors γ_s on Stiffness ?

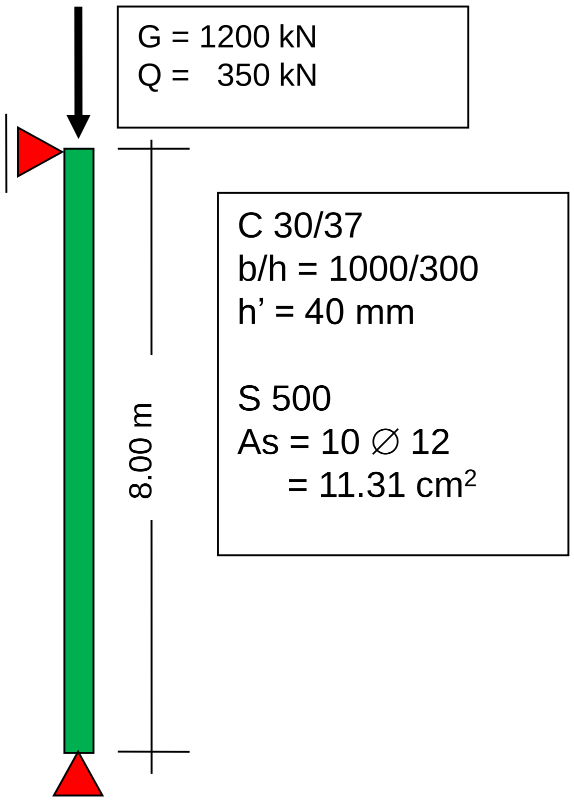
- In general, only mean values are known
- For stability problems a reduction is applied, sometimes.
- But for the stiffness there is no clear „on the safe side“ e.g. dynamics, settlements, thermal loadings, soil engineering.
- Thus, there is no generally accepted rule how to treat safety factors for the stiffness.
- Special treatment in Germany with a unified safety factor γ_r

Unique Solution ?



A Slender Example

- EN 1992 5.2. (7)
  (θ_o = l/200, a_h=0.707, a_m=1.0)
  e = 14.1 mm
- h = 240 mm
  A_s = 11.3 cm^2
  ε_s = 0.0 o/oo
- h = 230 mm
  A_s = 63,7 cm^2
  ε_s = 1.56 o/oo



Creep effects neglected !

- EN 1992-2004, clause 5.8.4. (4) states three conditions to be fulfilled when creep may be neglected:

$$\varphi_0 \leq 2$$

$$\lambda \leq 75$$

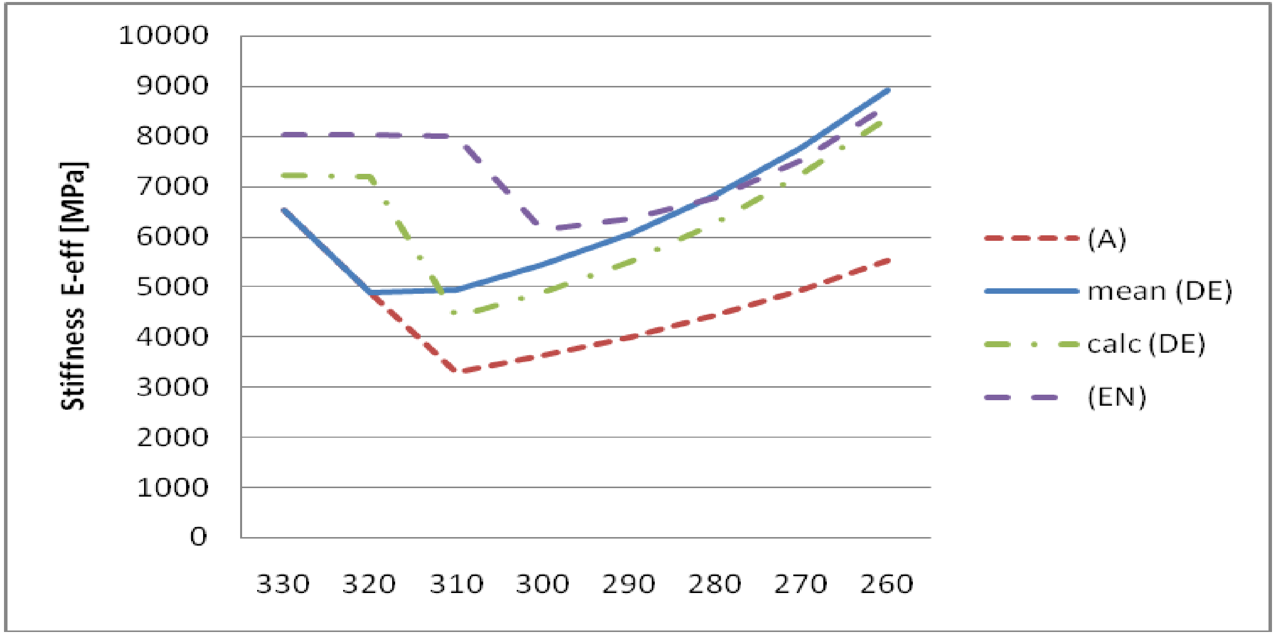
$$M/N \geq h$$

- All three are violated in this case !

Creep Effects included:

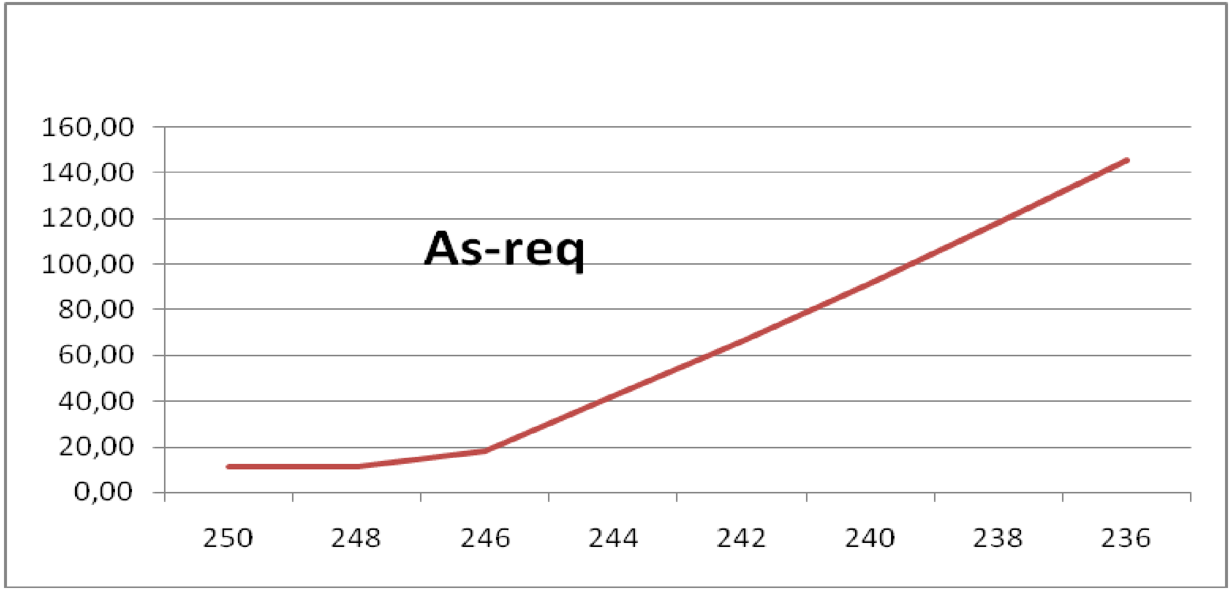
	Case A (deform)			Case B (complete)		
h [mm]	As [cm^2]	ε_s [o/oo]	E_{eff} [Mpa]	As [cm^2]	ε_s [o/oo]	E_{eff} [Mpa]
330	11,31	0,01	6517	11,31	-0,17	6517
320	11,31	0,61	4880	11,31	0,00	4880
310	27,80	2,50	3288	16,67	0,18	4941
300	34,08	2,26	3622	23,10	0,26	5433
290	40,41	2,10	3993	30,64	0,36	6054
280	48,04	2,00	4437	39,03	0,47	6829
270	56,33	1,92	4937	48,42	0,58	7763
260	66,25	1,85	5517	59,48	0,70	8909

Stiffness depending on height

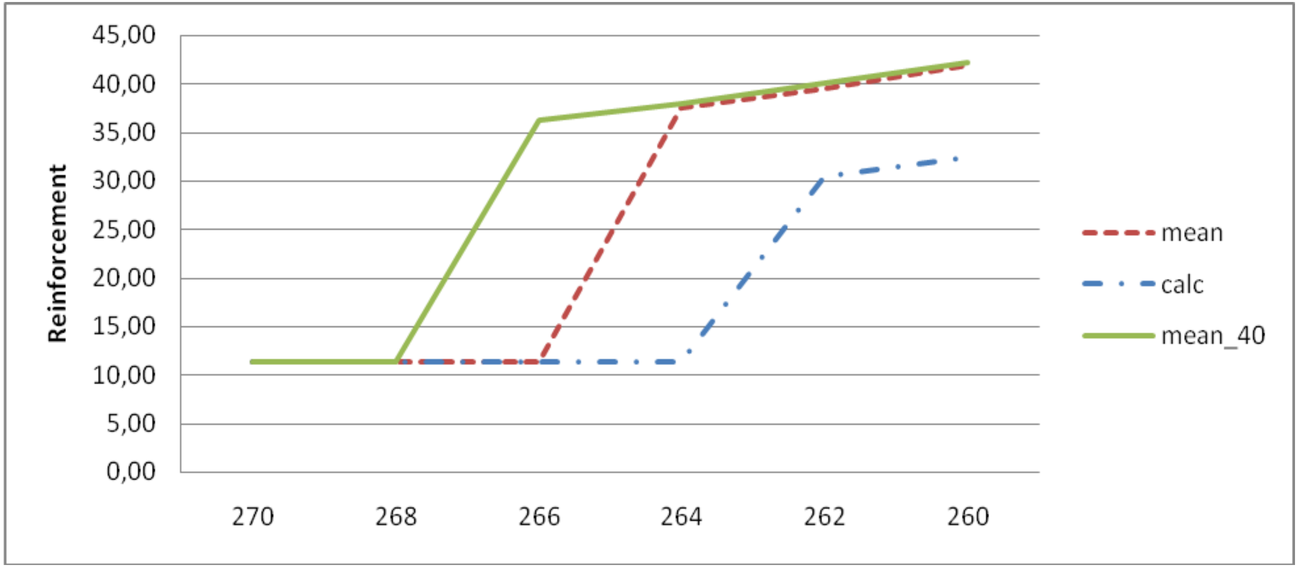


Linear Buckling requires E = 6182 MPa for h=300 mm!

Required Stiffness to prevent Buckling



Sensitive High Strength Concrete !



A cause of the unsteady behaviour

- For an uncracked section the deformations and 2nd Order effects are small, a low reinforcement is sufficient
- If a crack occurs, the stiffness drops suddenly, the limit condition is not the strength of the reinforcements but the stiffness to prevent linear buckling.
- Is that a problem for the reliability ?

Conclusion / Remedies ?

- There are/were provisions in some codes to prevent such strain distributions.
  - minimum excentricities including creep effects
  - minimum tensile strain (e.g. OEN)
  - maximum height of compressive zone (SNIP)
  - Higher safety factor for small strains
    (e.g. old DIN, ACI, AS)
  - Include Tension stiffening effects
- What should we do ?

Conclusion

- Beam and Cable elements facilitate the engineering judgement
- However they are neither easy nor exact
- Beware of Systems where beam theory is not adequate