Industrial Applications of Computational Mechanics
Post processing and Design codes

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SOFiSTiK AG

FE – Sections versus Classical Beam Theory

- The length is considerably greater than the height (slender members)
- Sections remain plane (Bernoulli-Hypothesis)
- Thus we have a „simple“ theory for static analysis
- There are cases where this theory is not sufficient

A beam as a 3D-solid substructure

$$u_x = u_{xo} + \varphi_{yo}(z - z_s) - \varphi_{zo}(y - y_s)$$

$$u_y = u_{yo} - \varphi_{xo}(z - z_m)$$

$$u_z = u_{zo} + \varphi_{xo}(y - y_m)$$

- Shape functions for beam-deformations
- Applying a variational principle on the strains and stresses derived from that yields a powerful beam element including haunches and eccentricities.

Missing functionalities

- all types of sectional deformations
  - Torsion / Warping torsion
  - Transverse shear, secondary torsion
  - Shape deformations
- transverse loading
- local effects (supports etc.)
- general temperature loadings
- Some of them may be added by extended theories

What is shear ?

- A force not perpendicular to a reference area.
- Thus shear depends on the selection of the coordinate system.
- Tensor invariants (e.g. principal stresses) may describe the problem more generally.
- Bernoulli-Hypothesis suppresses the effects of shear stresses.

The shear design problem

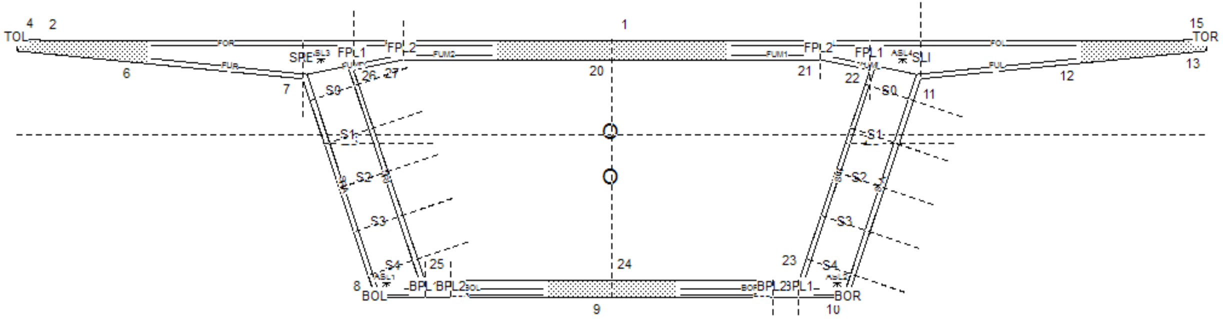
- Simplified (e.g. ACI 318-M)
  modified strength values for concrete and steel
  thinnest web thickness and distance of reinforcements d

$$\phi\left[V_c + V_s\right] \geq V_u$$

$$V_c = f_{cv} \cdot b_w \cdot d$$

$$V_s = f_y \cdot \frac{A_v}{s} \cdot d$$

Bridge section



- Design codes favour rectangular sections
- Bridge sections require extra considerations.

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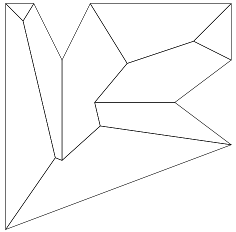
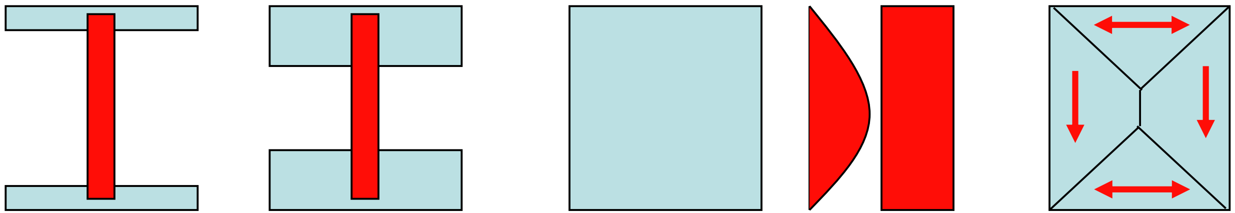
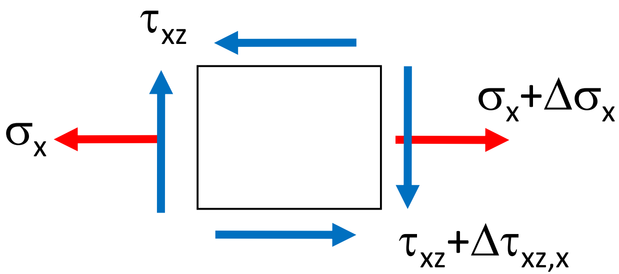
Maximum design force for steel sections

- e.g. web area of a double t-beam
- And for a rectangular section ?
- a constant value violates boundary conditions
- sand hill analogue yields not enough area





Equilibrium



$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \sigma_x}{\partial x} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + p = 0$$

Classical force dependant method

$$\sigma_x = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2} \cdot z - \frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2} \cdot y$$

$$T = \int -\frac{\partial \sigma_x}{\partial x} dA = \int \left(\frac{V_z I_z + V_y I_{yz}}{I_y I_z - I_{yz}^2} \cdot z + \frac{V_y I_y + V_z I_{yz}}{I_y I_z - I_{yz}^2} \cdot y \right) dA + T_0$$

$$\tau = \frac{T}{b} = \frac{V \cdot S}{I \cdot b}$$

- *Prismatic beam element, no further derivatives*
- *Shear stress is constant along the width b*
- *Integration constant T_o will become zero.*
- *Use the complete formula for biaxial bending.*

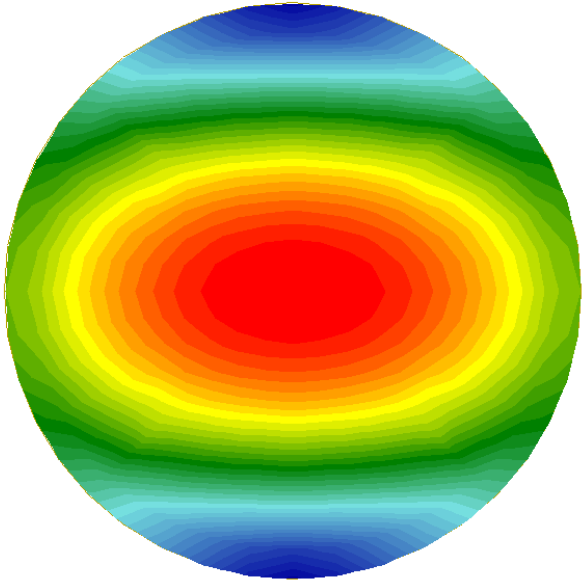
Deformation based method (FEM/BIEM)

$$\tau_{xy} = G\left(\frac{\partial w}{\partial y} - z\frac{\partial \Theta_x}{\partial x}\right)$$

$$\tau_{xz} = G\left(\frac{\partial w}{\partial z} + y\frac{\partial \Theta_x}{\partial x}\right)$$

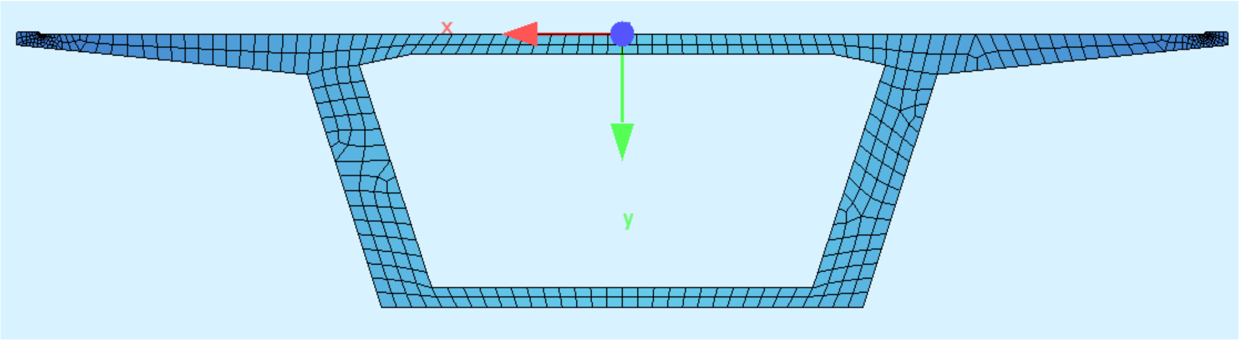
$$G\Delta w = G\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = -\frac{\partial \sigma_x}{\partial x}$$

$$Boundary\,condition: \quad \tau_{xy}n_y + \tau_{xz}n_z = 0$$



Transverse shear

FEM - section



QNR 1 MNR 1 MBW 2 BEZ 'Hohlquerschnitt'

QPOL U MNR 1 SMAX 0.40 ; QP '1' -6.448 0.307 ….

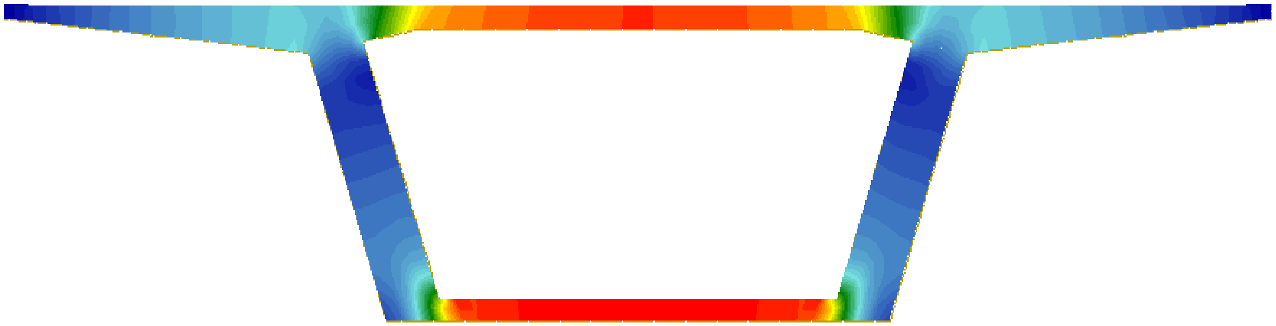
QPOL A MNR 1 SMAX 0.40 ; QP '20' 0.000 3.360 ….

QNR 2 MNR 1 MBW 2 BEZ 'Hohlquerschnitt' FEM 'bridge_fem' LTAU 9900

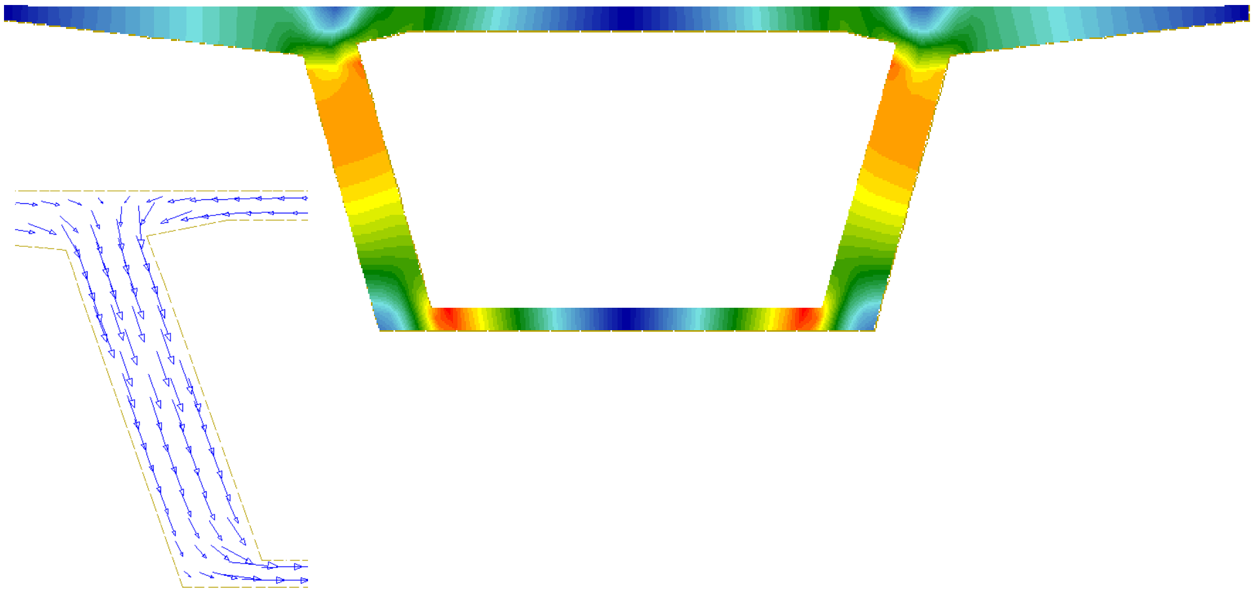
QPOL GRP 0

Shear due to Mt

Shear due to Vy

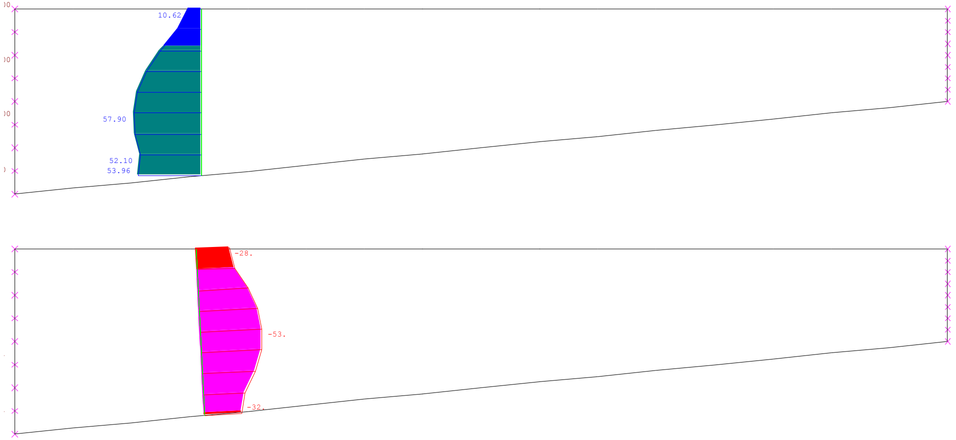


Shear due to Vz

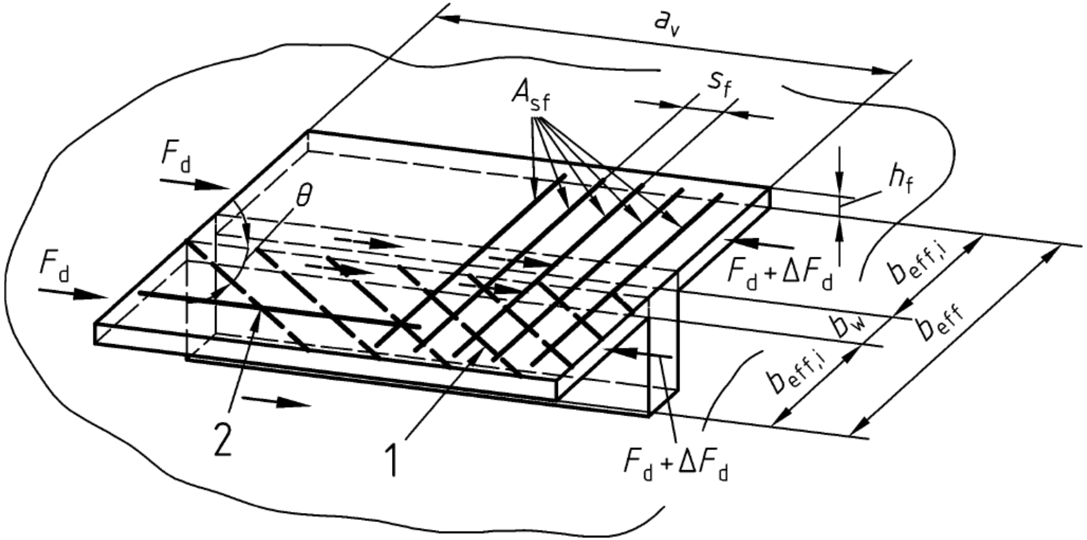


Shear stress defficencies

- They are „fictious" values, depending on the orientation of the local coordinate system
- Material behaviour is generally defined in principal stresses (Tensor invariants)



Shear Flux



$$T = \tau \cdot b = z.B. \frac{V}{z} \; or \; \frac{\Delta F_d}{a_v} \; or \; \frac{M_t}{2 \cdot A_k}$$

Applications in composite structures

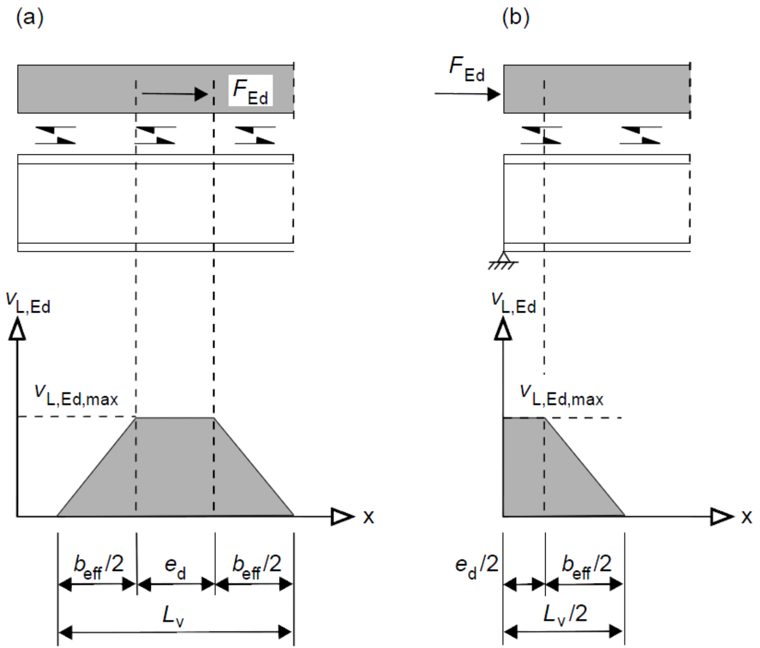
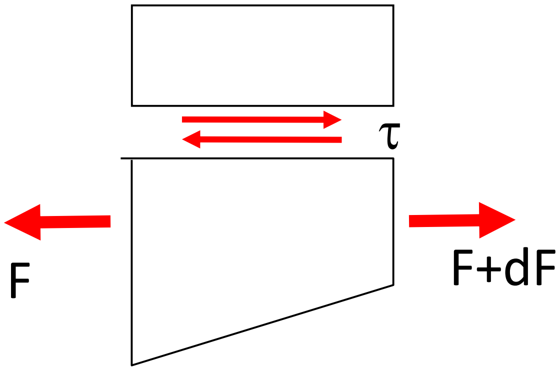


Bild 6.3 Verteilung der Längsschubkraft entlang der Verbundfuge

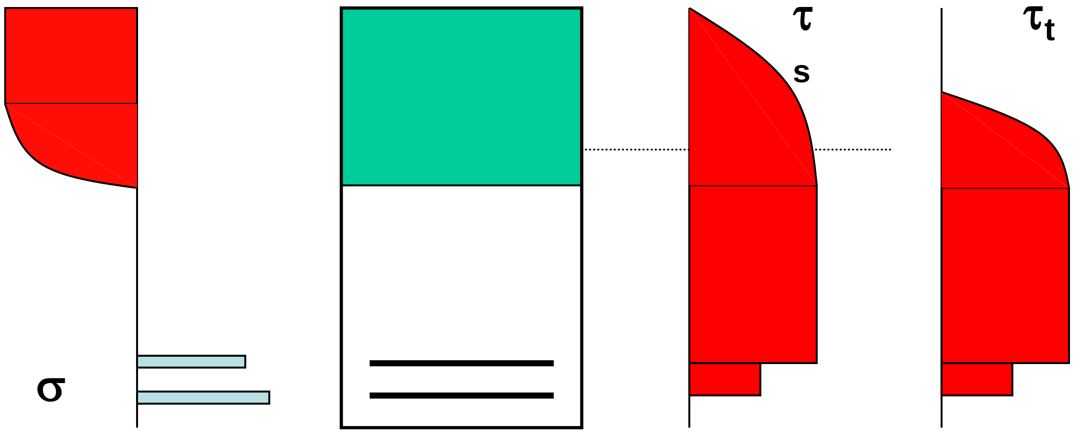
General: Difference of longitudinal forces



- Well known approach (BK)
- Applicable for haunched beams
- No butts: equilibrium !

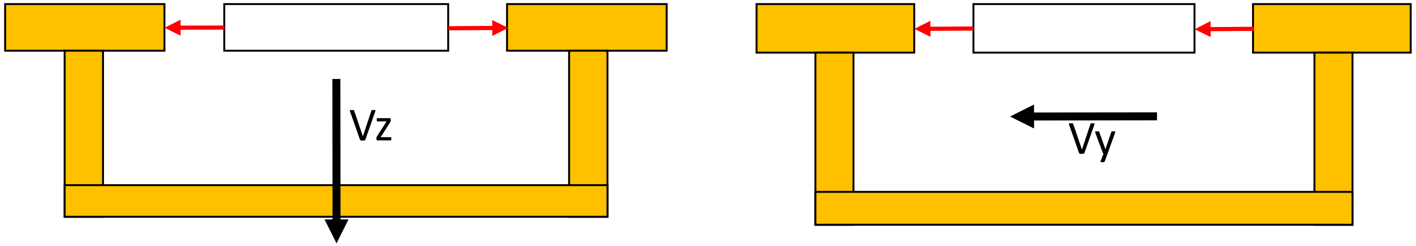
The cracked reinforced concrete section

- Basic rule: change of the shear stress depending on the normal force
- So we have a constant shear stress in the cracked zone
- Elastic concrete would yield a quadratic shear stress τ
- non linear plastified concrete ?



How to distribute shear for hollow sections

- *Use the general method with a deformation closure*
- *Select cuts to allow for symmetry conditions.*



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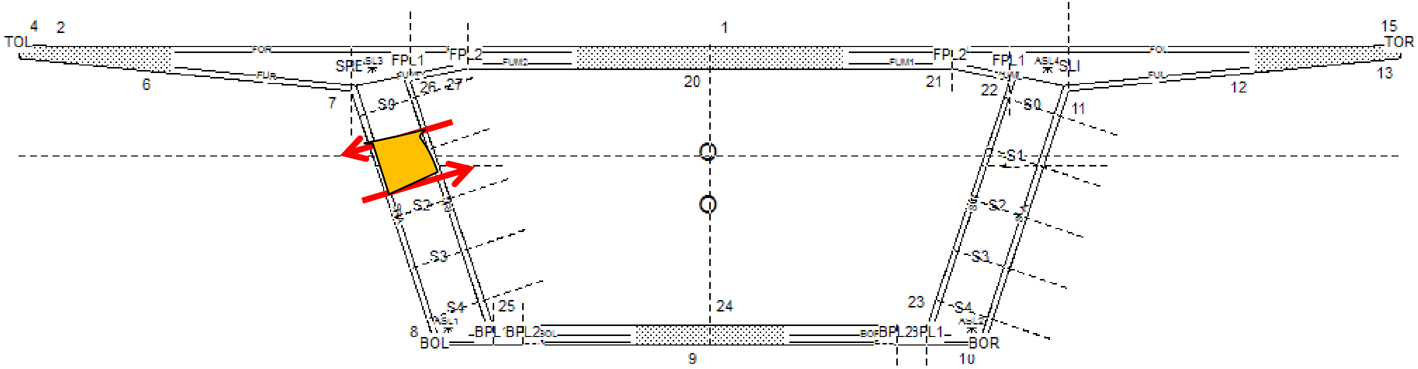
Attention!



- For small parts the shear may become zero
- Fall-back is the uncracked section

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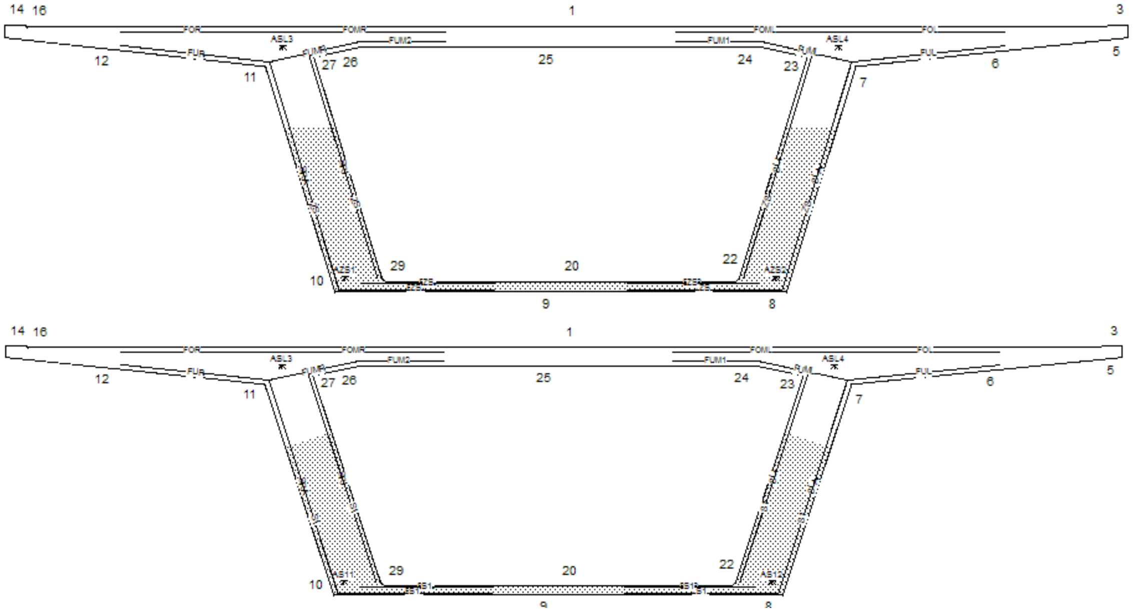
Cut through the web ?



- 5,5 % difference

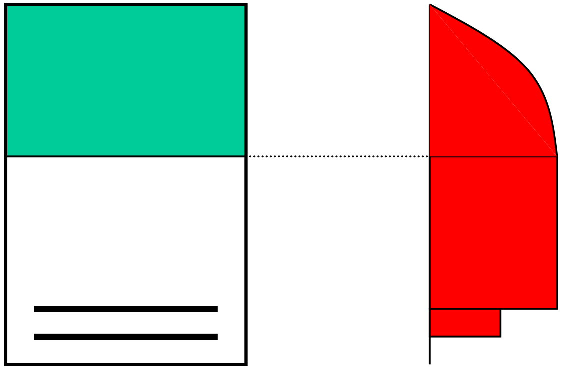
Cracked sections
Method of partial areas

- *Lever increases (e.g. 0.66h to 0.85d)*
- *Maximum shear flow V/z at neutral line*
- *Calculation of lever sometimes very difficult (e.g. for fully compressive or tensioned sections)*
- *Biaxial bending ?!*

$$T_{max} = \frac{\sqrt{V_y^2 + V_z^2}}{\sqrt{y^2 + z^2}}$$

„Katz Formula“

- Design code says:
  Ratio of separated tensile or compressive force.
- If the cut moves into the compressive zone the shear has to become less.
- Empirical rule



$$T = T_{\max} * ABS\left[\frac{Z}{Z_{ges}} - \frac{D}{D_{ges}}\right]$$

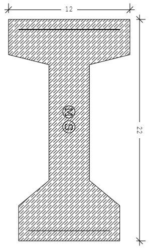
Concrete Design derived from uncracked section

- Stresses obtained from linear theory (as usual).
- Design is done in ultimalte limit state
- Shear flowT [kN/m]

$$T = \int \tau \approx \tau \cdot b$$

- Design on T instead of V/b with variable angle truss theory.
- Optional scaling with $z_{elastic}/z_{cracked}$

A small example



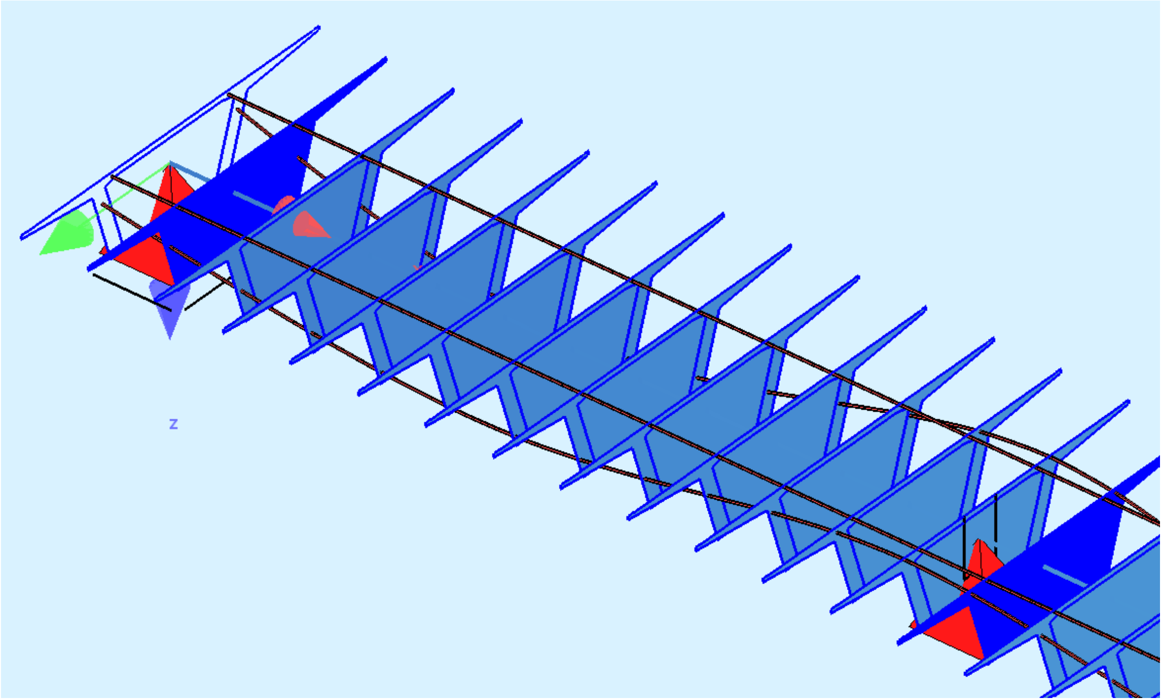
Required Reinforcements

x[m]	Ni [kN]	Myi/Mzi [kNm]	e1/yn [o/oo	e2/zn / mm]	nue C/S	rel tra	As [cm2]	L	Vz[kN]	z[m]
0.000	0.0	0.00	0.00	0.00	1.00	not calculated			23.45	0.210
0.135	0.0	6.13	-0.86	5.00	1.00	2.01	1.60	Z	21.82	0.192
0.400	0.0	13.35	-1.45	5.00	1.00	1.59	1.60	Z	18.62	0.192
0.913	0.0	16.37	-1.66	5.00	1.00	1.00	0.40	1	12.42	0.185
							1.60	Z		
1.427	0.0	21.16	-2.05	5.00	1.00	1.00	1.04	1	6.20	0.183
							1.60	Z		
1.940	0.0	22.75	-2.21	5.00	1.00	1.00	1.26	1	0.00	0.182
							1.60	Z		
2.453	0.0	21.16	-2.05	5.00	1.00	1.00	1.04	1	-6.20	0.183
							1.60	Z		
2.967	0.0	16.37	-1.66	5.00	1.00	1.00	0.40	1	-12.42	0.185
							1.60	Z		
3.480	0.0	13.35	-1.45	5.00	1.00	1.59	1.60	Z	-18.62	0.192
3.745	0.0	6.13	-0.86	5.00	1.00	2.01	1.60	Z	-21.82	0.192
3.880	0.0	0.00	0.00	0.00	1.00	not calculated			-23.45	0.210

Comparison with bond

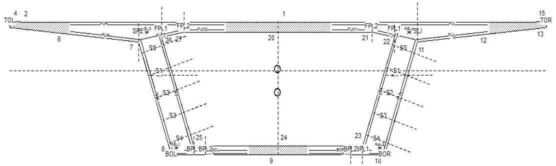
$x[m]$	Z_0 [kN]	Z_z [kN]	Z_s [kN]	reltra η	F $(Z_z+Z_s)/\eta$	$\Delta F_-/a$ [kN/m]	$\Delta F_+/a$ [kN/m]	$\Delta F/a$ [kN/m]	V/z [kN/m]	T_{elast} [kN/m]
0.135	31.8	95.9	-	5.78	16.6	123.0	111.3	118.8	113.6	133.1
0.400	72.0	106.5	-	2.31	46.1	111.3	85.0	98.1	97.0	113.6
0.913	72.0	106.5	-	1.19	89.7	85.0	51.6	68.3	66.4	75.8
1.427	72.0	106.5	9.7	1.00	116.2	51.6	17.5	34.6	34.0	37.8

An Example of a Bridge



Hogging Moment

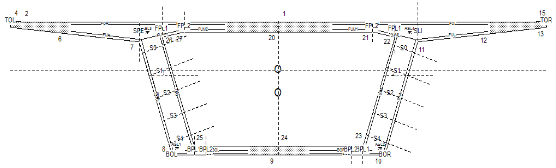
My= -44247 kNm, Vz= 8999 kN
z=3.15 m, T= 2856 kN/m.



Span 2 X=3.0	*S0*	*S1*	*S2*	*S3*	*S4*	*BPL*	*FPL1*	*FPL2*
width[m]	*0.70*	*0.70*	*0.70*	*0.70*	*0.70*	*0.260*	*0.397*	*0.260*
Shear (elastic)	*1596*	*1643*	*1534*	*1270*	*849*	*758*	*598*	*463*
Partial Shear	*1425*	*1428*	*1428*	*1292*	*928*	*574*	*400*	*380*
Differential shear 3m	*2094*	*1688*	*1688*	*1493*	*1062*	*656*	*343*	*17*
Differential shear 1m	*1660*	*1664*	*1659*	*1437*	*1017*	*628*	*329*	*16*

End support

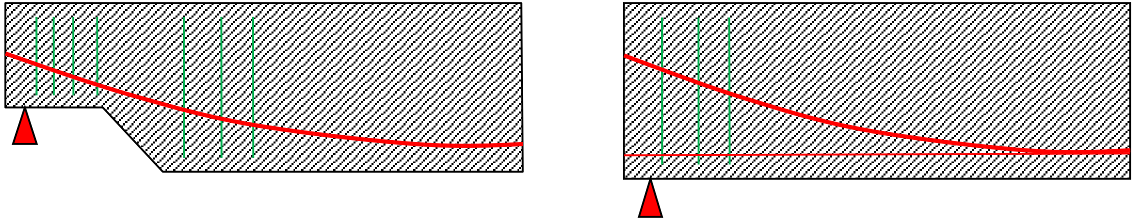
My=21751 kNm, Vz= 2974 kN
z=1.68 m, T= 1770 kN/m.



Span 1 X=3.0	*S0*	*S1*	*S2*	*S3*	*S4*	*BPL*	*FPL1*	*FPL2*
Width [m]	*0.70*	*0.70*	*0.70*	*0.70*	*0.70*	*0.260*	*0.397*	*0.260*
Shear (elastic)	*527*	*543*	*507*	*419*	*281*	*250*	*198*	*153*
Partial shear	*902*	*897*	*67*	*55*	*44*	*0*	*407*	*310*
Differential shear 3m	*1354*	*1346*	*1340*	*42*	*33*	*0*	*742*	*592*
Differential shear 1m	*1112*	*1105*	***	*34*	*27*	*0*	*587*	*447*

Assessment

- Lever is very small!
- => Shear becomes large
- => The construction is not properly selected
- Program should react on improper problem description!



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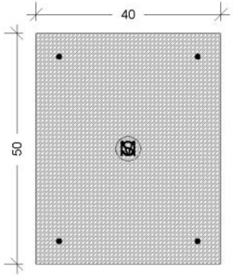
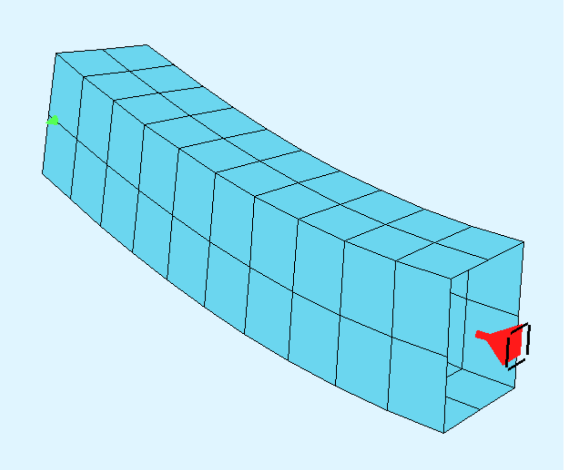
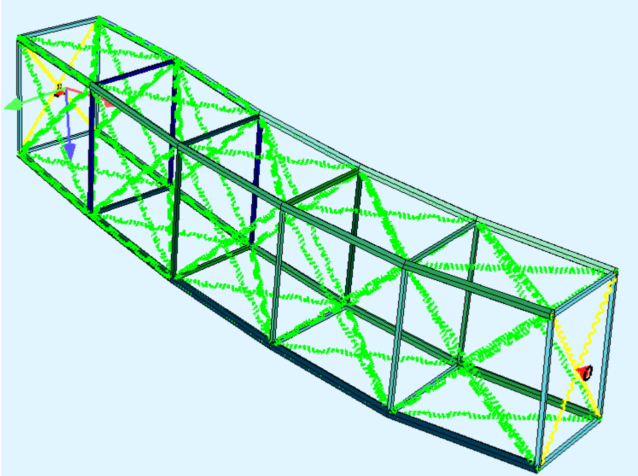
Spatial frame

- Rectangular section b/h = 40/50 cm
- Span width 4.0 m
- Loading 140 kN/m & 100 kN/m transverse







Classical Analysis (x=0.40 m)

- Forces and moments

  V_y = 160.0 kN M_z = -72.00 kNm

  V_z = 224.0 kN M_y = 100.80 kNm

- Shear flux per side (h‘=5 cm, z=0.9 d)

  T_y = ½ ·160.0/0.315 = 253.97 kN/m

  T_z = ½ ·224.0/0.405 = 276.54 kN/m

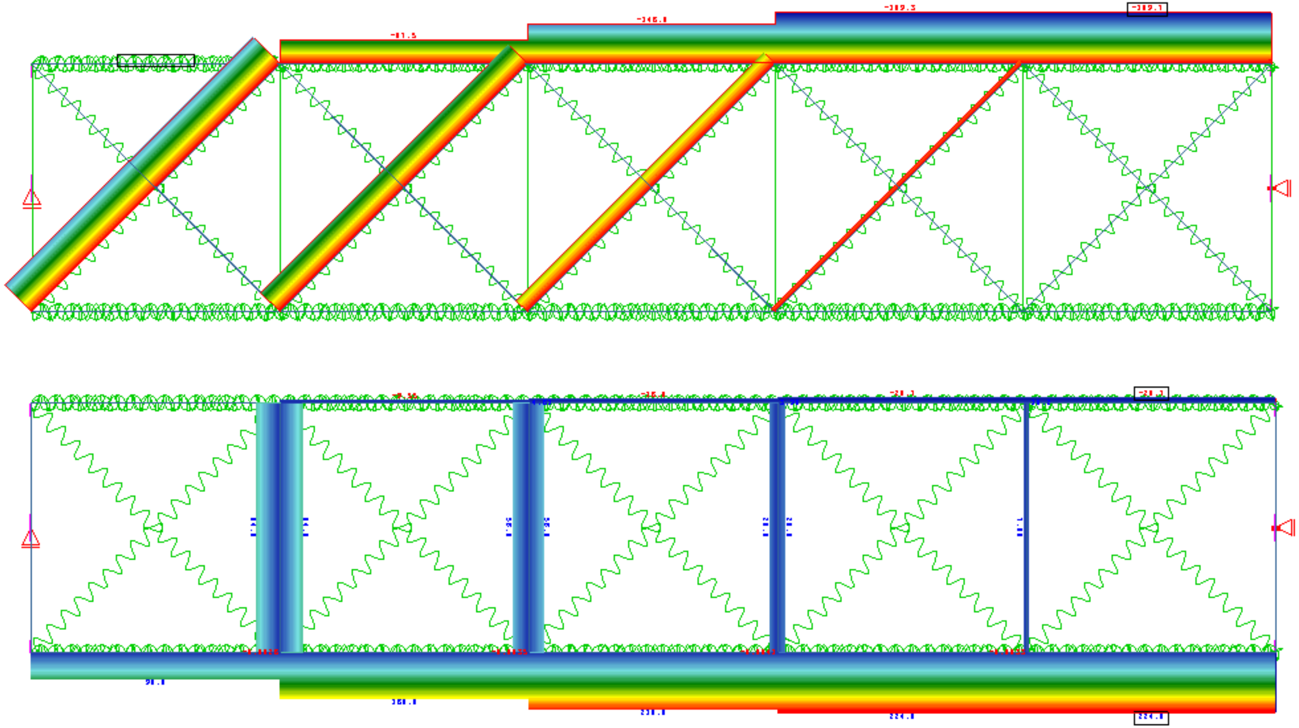
- Force in the shear links (θ = 45° , s = 40 cm)

  vertical 101.6 kN

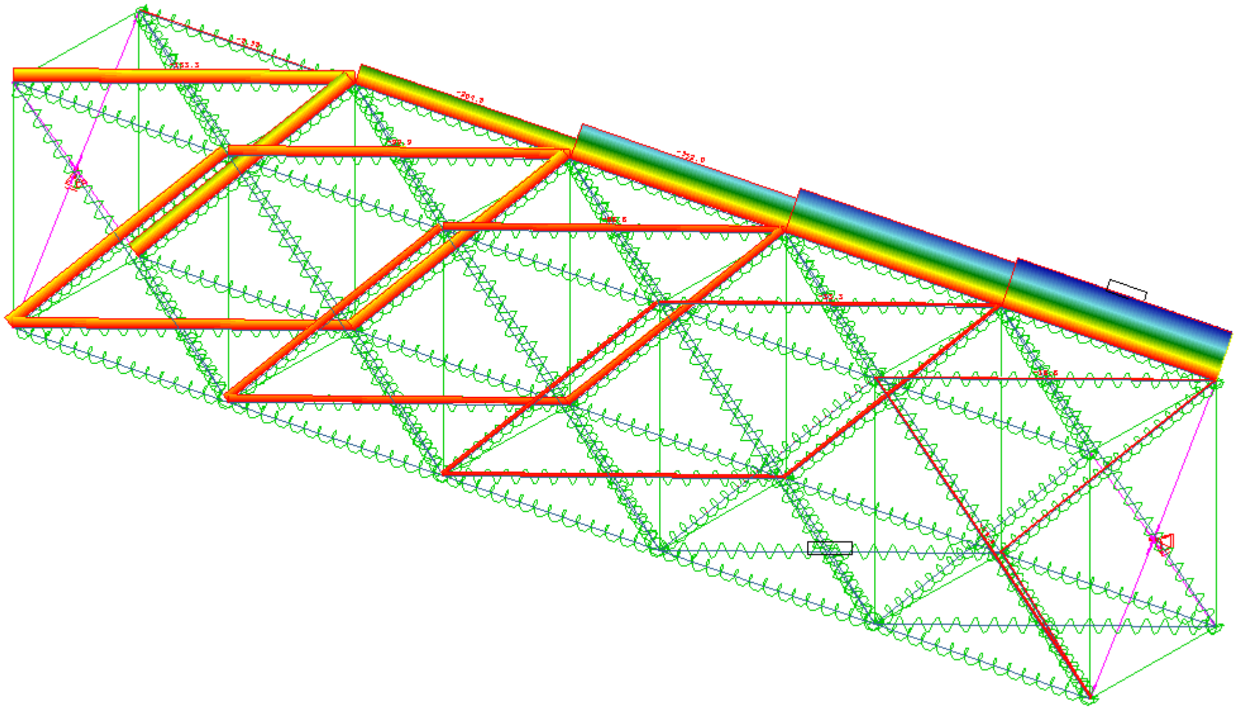
  horizontal 110.6 kN

- Forces are nearly independant from acting moments.

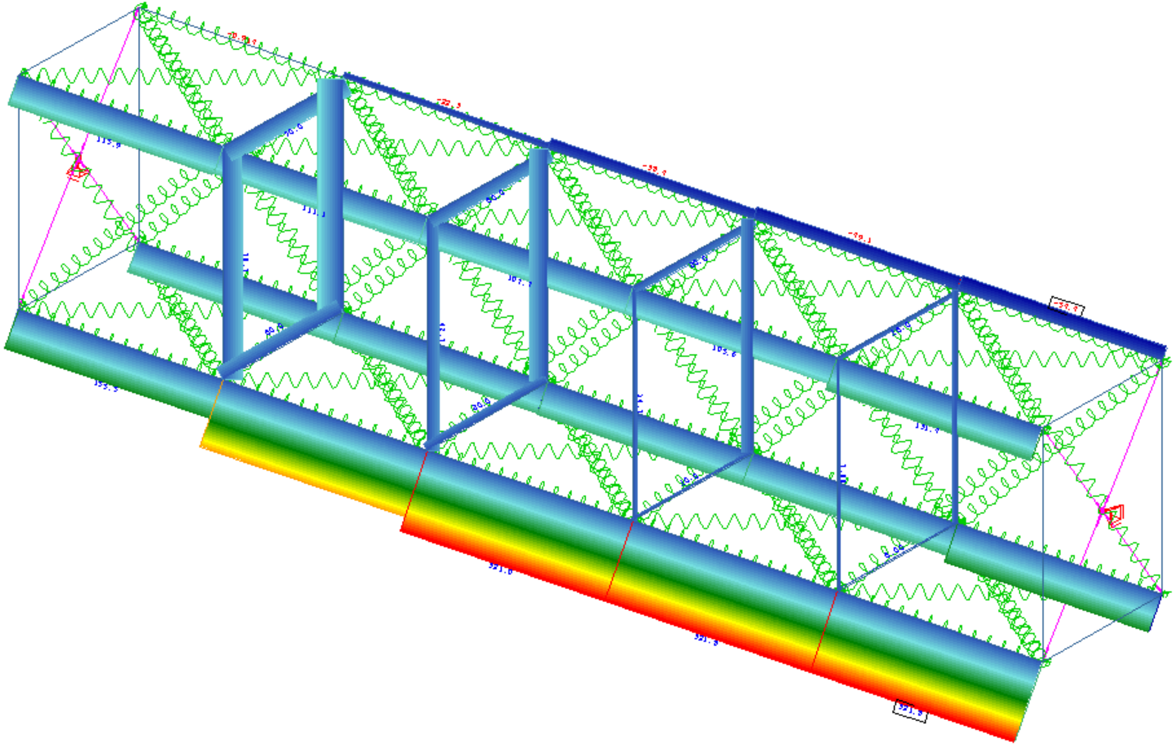
Truss forces uniaxial bending



Truss forces biaxial bending



Truss forces biaxial bending



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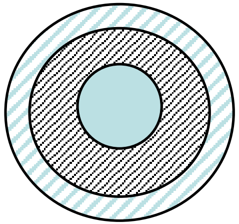
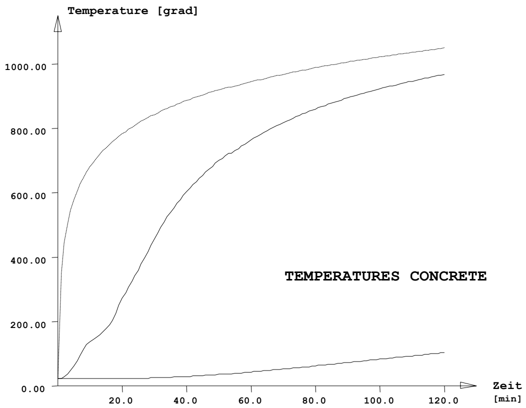
Comparisson of link forces

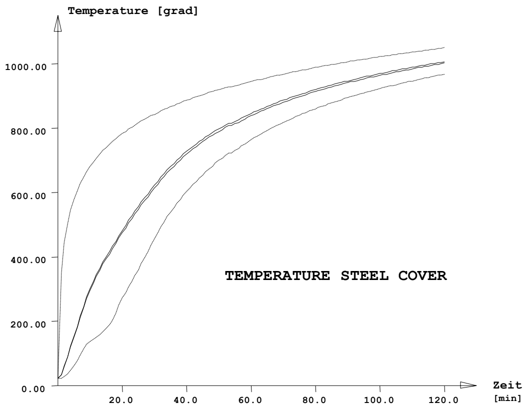
Loading	left	right	upper	lower
Classical	111	111	102	102
vertical	84	84	0	0.7
horizontal	0.4	0	60	60
combined	97.3	70.7	70	50
combined + M_z	97.7	70.3	70.3	49.7
combined - M_z	89.6	78.4	64.2	55.8

Hot Design (Fire)



- Transient Analysis of Temperatures with HYDRA
  - Temperature dependant properties
  - Radiation Boundary Conditions
  - Fire models





Effective Width I

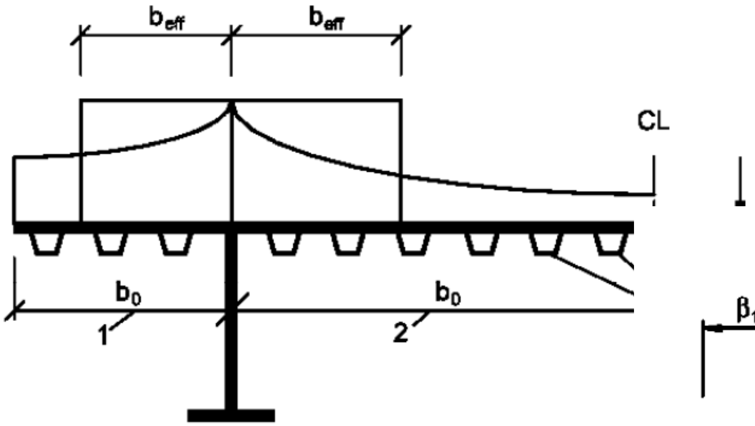
- For holes no normal stress can be taken at all (EN 1993-1-1 6.2.2.2)

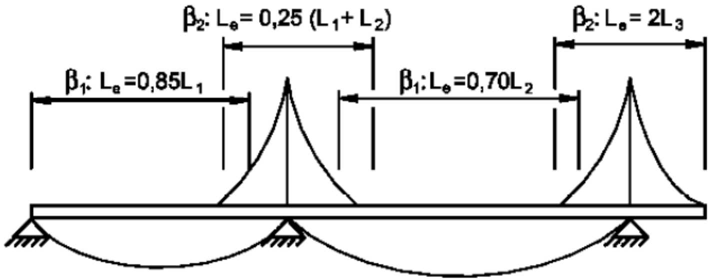
  (1) The net area of a cross-section should be taken as its gross area less appropriate deductions for all holes and other openings.

- There are no normal stresses at all
- The area and the inertia are missing completely
- The sectional description remains consistent
- But holes are no trenches, shear is still active!

Effective Width II

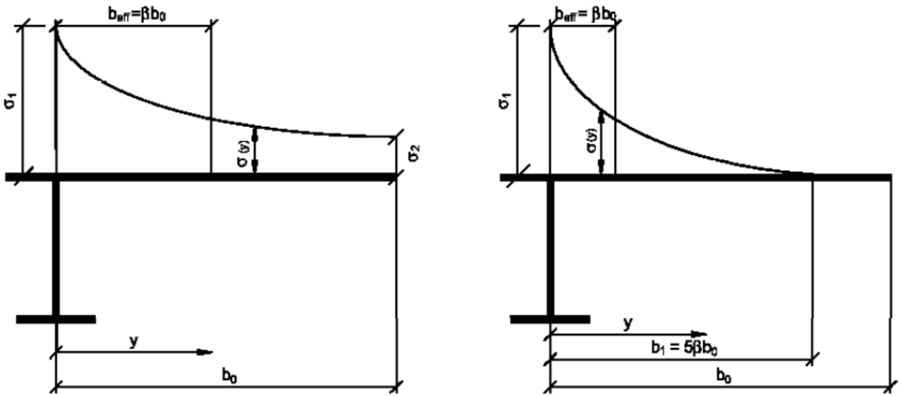
- The bending stress is not according to the Bernoulli-Hypothesis (EN 1993-1-5 3.2.1)





Effective Width III

- The bending stress is not according to the Bernoulli-Hypothesis (EN 1993-1-5 3.2.2)



$\beta > 0{,}20:$

$\sigma_2 = 1{,}25\,(\beta - 0{,}20)\,\sigma_1$

$\sigma(y) = \sigma_2 + (\sigma_1 - \sigma_2)\,(1 - y/b_0)^4$

$\beta \le 0{,}20:$

$\sigma_2 = 0$

$\sigma(y) = \sigma_1 (1 - y/b_1)^4$

Consequences – Normal force

- Normal forces are still considered on the total area
- A uniform strain creates no bending resultant with respect to the elastic centre of the total section
- Thus all external normal forces are referenced to the elastic centre of the total section.
- The eccentricity of a beam with a reference axis is the centre of the effective section for the normal force!

Consequences – Bending Moment

- The bending moment is defined relative to the centre of the total section
- Then it is shifted to the elastic centre of the effective section
- All bending stresses are calculated based on the effective section
- Thus the integral of the bending stresses is zero!
  No normal force is created!
- There are two moments and two non effective directions
  (Swain’s formula is partly valid)

Consequences – Nonlinear Analysis

- It is not required to account for a shift of the reference point of forces, if the equilibrium is not violated.
- The non effective parts have only uniform strains, but this strain is not necessarily the strain at the elastic centre!
- Biaxial bending with rotation of neutral axis becomes even more complex.
- So we are restricted to the strain definition.
- But there is no better solution! (?)

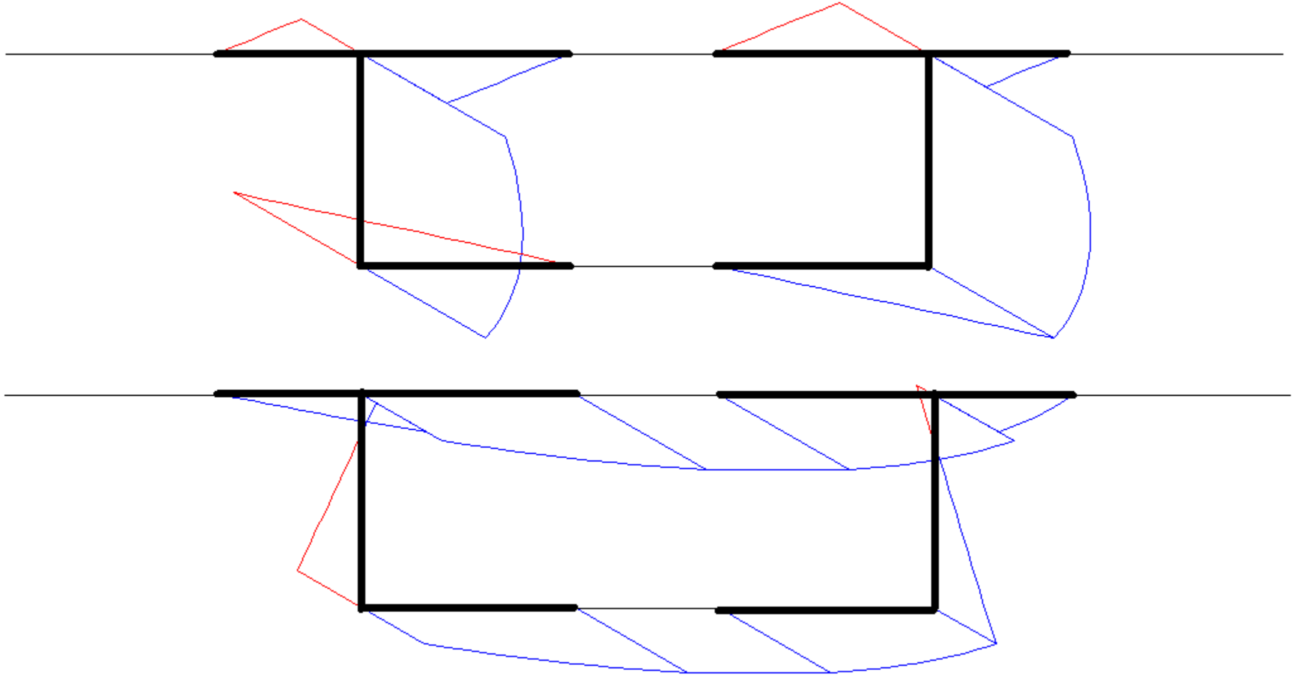
Consequences – Shear

- Area does not exist for longitudinal stress
- Area exists for shear stress
- Equilibrium:

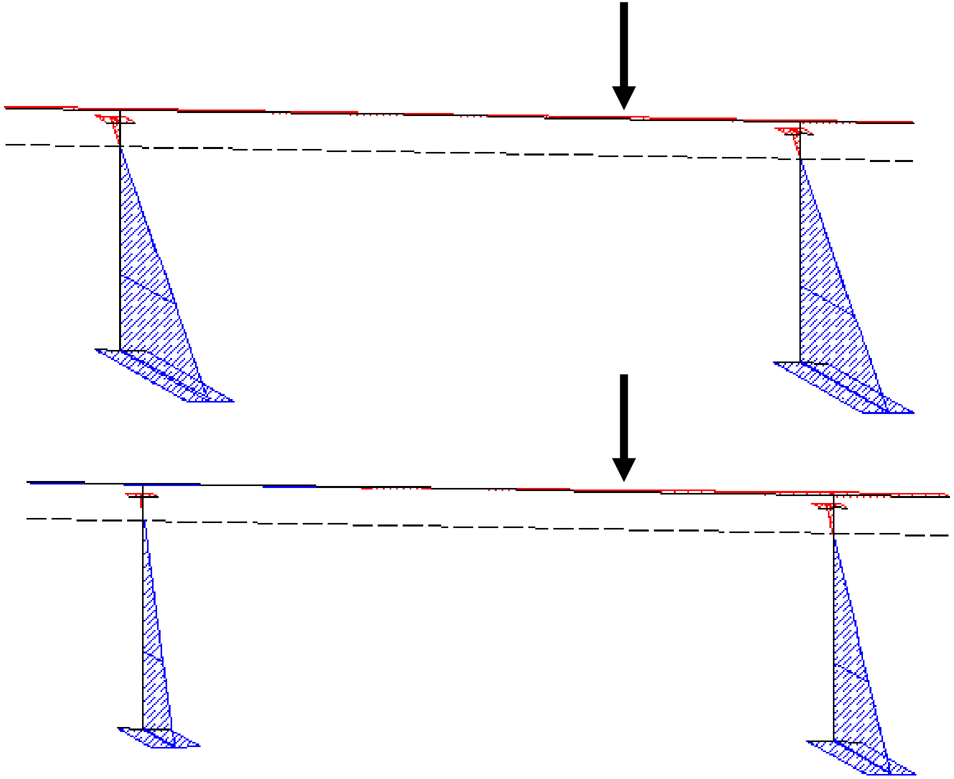
$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \sigma_x}{\partial x} = 0$$

- The longitudinal stress σ_x is zero
- The derivative of the longitudinal stress σ_x is zero
- The derivative of the shear stress τ_{xy} is zero
- The shear stress is constant

Shear: Non effective parts for bending



Linear: Warping Torsion = Bimoment



Projects where Design by Analysis has been performed

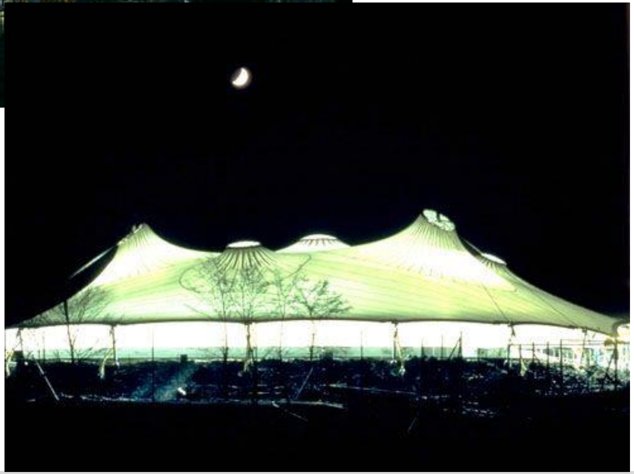
Hall 26, Fair Hannover

Munich Airport Hangar

New goverment district Berlin

Bridge „Wilde Gera“

Munich Airport Center



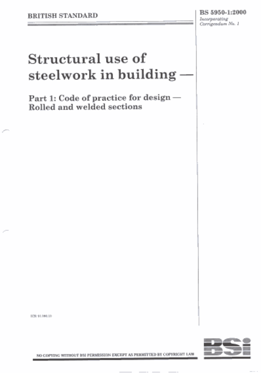
Membrane Roof, Hamburg

What is the purpose of a design code ?

- The customer wants a product which has assured properties even if he does not exactly know about them
- The safety requirements of the public has to be fulfilled
- Thus a design code may contain:
  - Clear technical definitions especially of required material properties and tests
  - General guidelines for the Design
  - Hints for comprehensive design methods
  - Hints for simplified design methods

Example: Design codes for structural steelwork

- D: Stahlbauten Bemessung und Konstruktion; Stabilitätsfälle, Knicken von Stäben und Stabwerken
  Steel structures, design and construction, Stability, Buckling of beams and frames (paperback, 93 pages)
- F: Règles de calcul des construction en acier
  Analysis rules for structural steel constructions (hardcover, 397p.)
- GB: Structural use of steelwork in building,
  Code of practice for design.(loose-leaf edition, 219 pages)
- I: Norme tecniche per il calcolo, l'esecuzione ed il collaudo delle strutture in cemento armato, normale e precompresso e per le strutture metalliche. (21 pages for structural steel)
- RU: SNIP 23.81 (260 pages)
- EC: Design of Steel Structures, 344 pages



2nd Order Theory

DIN 18800: General Method:

«Anstelle der in Tabelle 1 angegebenen Verfahren dürfen auch vereinfachte Nachweise geführt werden»

CM 66: Only a hint:

«Lorsque exceptionnellement on a besoin d'une meilleure approximation, il est nécessaire de tenir compte de la présence des irrégularités de forme et de structure qui influent sur la position des points d'inflexion dans un système hyperstatique de barres réelles, comme elles influent sur la charge d'affaissement d'une barre bi-articulée. »

BS 5950:

«Detailed recommendations for practical direct application of "second order" methods of global analysis (based on the final deformed geometry of the frame), including allowances for geometrical imperfections and residual stresses, strain hardening, the relationship between member stability and frame stability and appropriate failure criteria, are beyond the scope of this document. However, such use is not precluded provided that appropriate allowances are made for these considerations.»

Legal base (Germany)

- German BGB §§ 631 ff. (contracts of manufacture)
  - „The work has to be fulfilled in such a way, that it has the assured properties and has no faults reducing or annihilating the value or the fitness for the general purpose or that assured by the contract“
- VOB § 13. (Contracting in Civil Engineering)
  - „The contractor assures that his work has the assured properties and fits the generally accepted technical rules and has no faults, reducing or annihilating the value or the fitness for the general purpose or that assured by the contract“

generally accepted technical rules

- Not the state of art of science
- Not the contents of the design codes
- Not the way how it was since a long time before (Conserving the stone age)
- It is more than the absence of errors
- Thus
  - It has to be a technical rule for the design or build of a structure
  - The theoretical base has to be known and generally accepted.
  - The rule itself has to be broadly known and generally to be accepted.

Deviations

- Are not the only way to deal with a problem !
- Deviations should be possible !
  - e.g. Load values, safety factors
  - e.g. Design procedures
- Legal consequences:
  - Conforming to the codes ?
  - Reliability in case of damage or even before ?
- Esthetical and Technical consequences
  - Only square houses allowed ?
  - Technical progress is always something new !

Why should we deviate ?

- If a design code does not cover the problem itself
  - Modern architecture
  - Unusual load cases
- If we want to be more economical
- Implies that the provisions of the design codes are on the safe side, but quite often they are not:
  - Stability cases of DIN 18800
  - Plastic Resistance of sections $M_{plas} = 1.14\ M_{el}$
  - Shear design of concrete: Lever arm = 0.9 d
  - Crack width of concrete with tabulated values
  - Buckling length itself

Consistent Design Rules

- Design code is not consistent

  - Admissable shear stress DIN 1045 (1972)

    τ_v up to τ_{03} but τ_{v+t} only up to $1.3\tau_{02} < \tau_{03}$

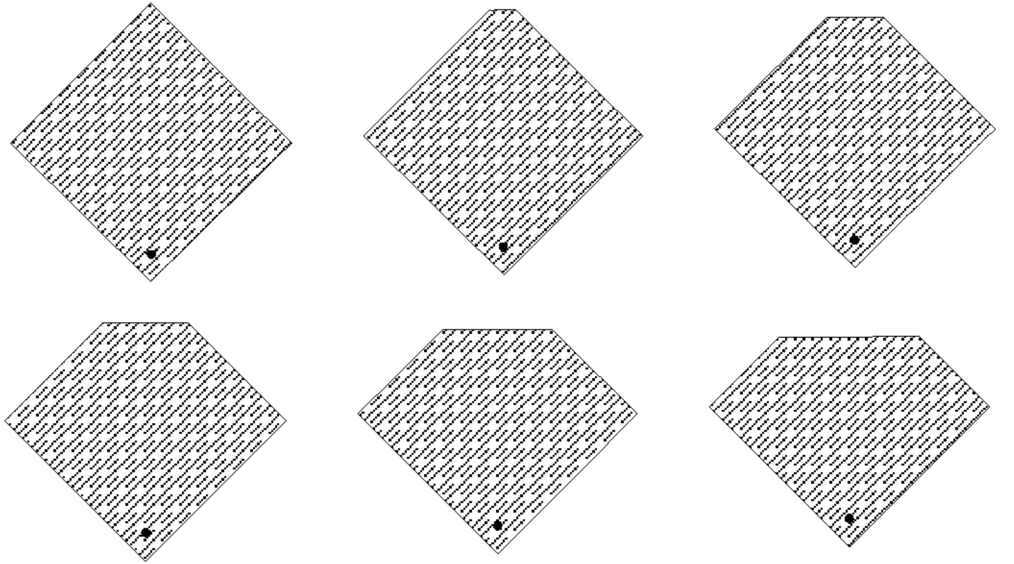
  - Admissable shear stress DIN 1045 (1988)

    $\tau_q/\tau_{03} + \tau_t/\tau_{02} < 1.3$

A newer example

- Strength of concrete for ultimate design
- Coefficient α = 0.85 to account for long term effects etc.
- “This coefficient is not valid if the width of the compressive zone is reducing in the direction to the extreme compression fibre (α = 0.80)”.
- Despite that this rule is wrong in general, for a biaxial bending of a rectangular section the strength will drop if a fly is producing a transverse bending.
- EC 2 – DIN 1045 (1996) – DIN 1045 (2001)

The removed corner



The removed corner

Required reinforcement:

Moment	Q 1	Q 2	Q 3	Q 4	Q 5	Q 6
800	42.5	42.8	44.1	46.3	49.2	52.9
1000	56.5	56.6	58.3	61.1	64.9	70.1
1200	96.6	80.4	77.6	81.4	99.5	238.2

The design codes give a higher strength if parts of the structure are removed!

Burried knowledge

- Plastic Moment Resistance of DIN 18800

  „If we have a form factor α_{pl} > 1.25 and it is not allowed to do the analysis according to 1st order theory, the plastic resistance of the bending moment has to be reduced by a factor of 1.25/ α_{pl} if normal or shear forces are also present."

- Why ?
  - A limit of plastic strains should be observed to assure sufficient rotational capacities
  - The redistribution of forces should be limited
  - The interaction formulas are not valid otherwise
  - The ultimate load is not obtained with plastic hinges
- How to convert that to software ?

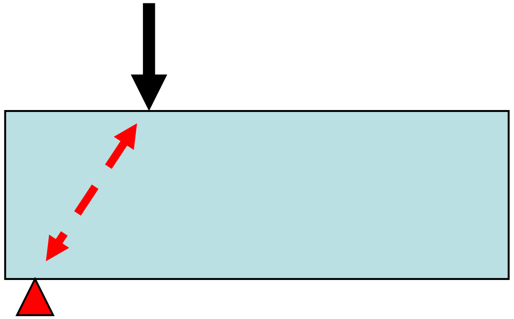
Misuse of Design Regulations

- Plastic Moment for Double-T-Shapes
  - An estimated factor 1.14 is found in DIN 18800 in paragraph (750) for local yielding in case of two axial bending!
  - Schneider Bautabellen:
    „Values in Table 8.23a are obtained as follows:
    max($\sigma_{R,d} \cdot 2 \cdot S_y$; $1.14 \cdot \sigma_{R,d} \cdot W_y$)
  - The cheated values are up to 7 % on the unsafe side

Forces and Moments

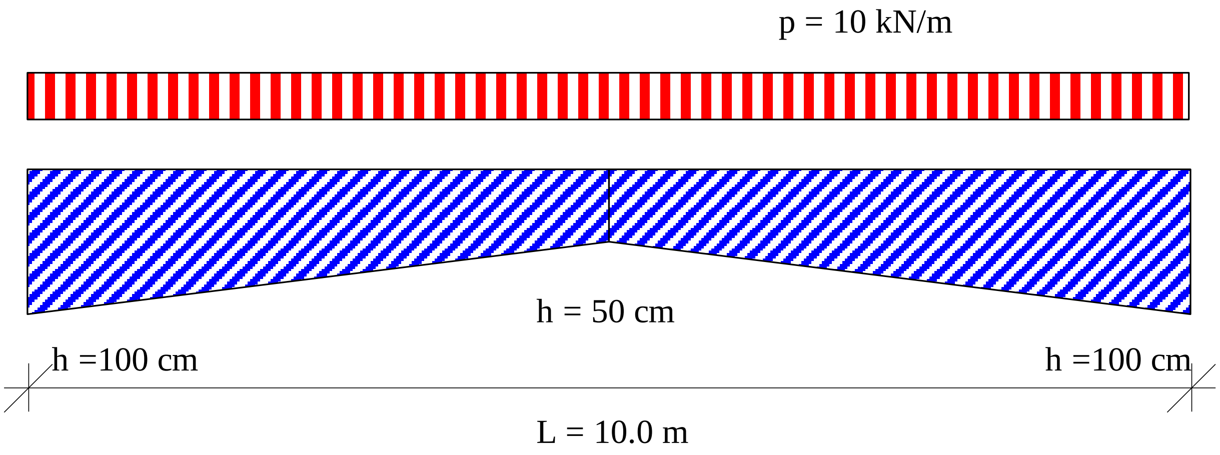
- Elasticity Theory:
  general rule, or only admissible rule ?
- 2nd Order Theory:
  only needed if results change with more than 10 % !
- Safety factors:
  are larger or lower stiffness values unfavourable?
- Negative sagging moments:
  „if despite a built in connection ...“
- Positive sagging moments:
  if not a more precise analysis is performed...
- Hogging moments:
  DIN 1045-1 minimum value 60 % of the fully built in values

Point Loads near the support

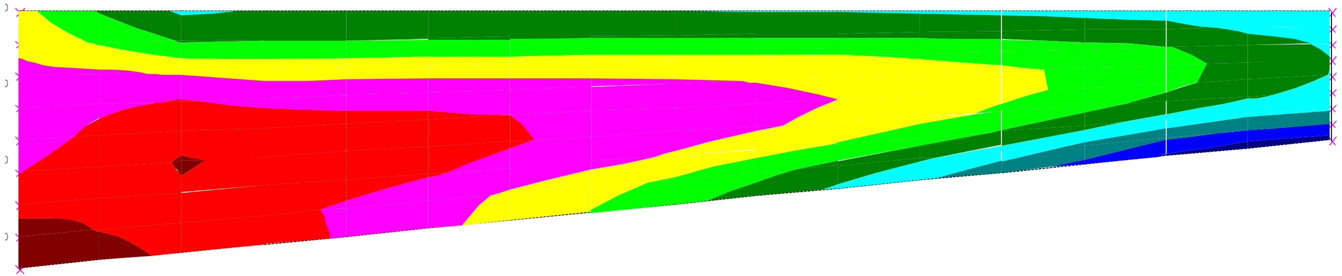


- Idea 100 % correct:
  reduce the partly shear force for the shear link evaluation,
  but not for the compressive stress.
- The part of the shear force for that load has to be evaluated on a single beam, a separate load case may be needed for that purpose
- Improper reduction of the total shear force in many design codes

Shear Stress in Haunches

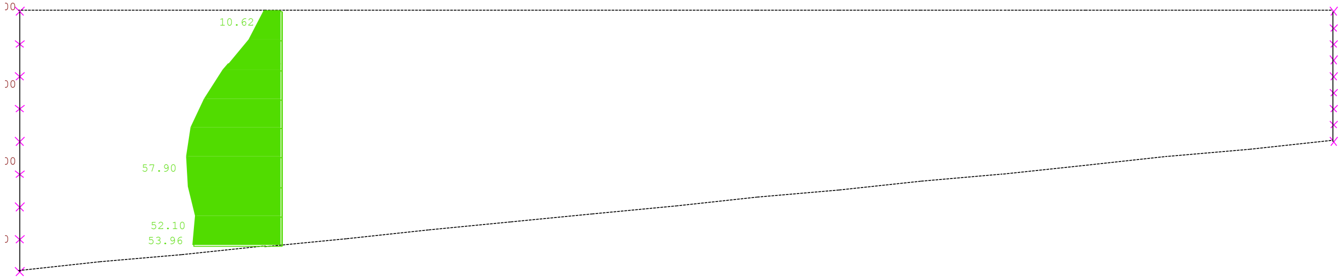


FE-Analysis



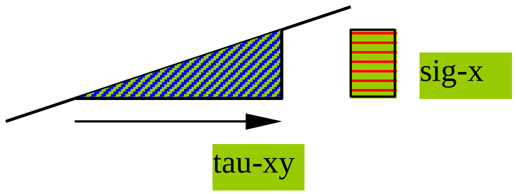
tau-xy

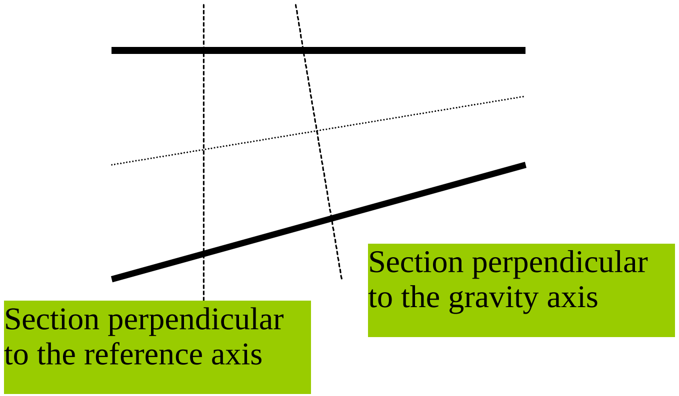
FE-Analysis



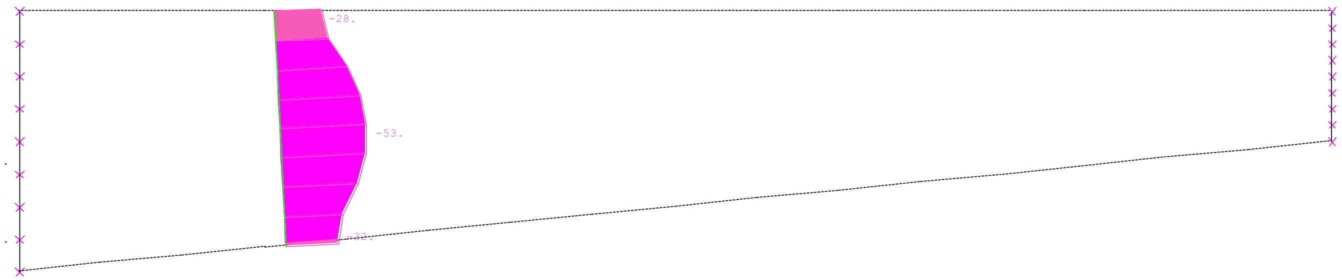
tau-xy

Shear Stress in Haunches





FE-Analysis



tau-nq

Shear Stress in Haunches

- Transverse force T 40.0 kN
- Transverse Shear V 39.9 kN
- Wrong shear stress 63.2 kN/m^2
- Reduction force M/d* tanα 5.6 kN
- Reduced shear stress 54.9 kN/m^2
- FE-shear stress 53.0 kN/m^2
- Conclusion
  - maximum is covered by design code
  - Distribution is not

Principal Compression

- Limit on compressive stress is natural
- If lower values are used, a check is made only if $\tau > 0.1\ \beta_{wn}$
- DIN 1045 old had no rules at all for that
- Increase the truss angle until the stress criteria is met
- Increase the truss angle if more shear links are provided than required.

Torsion

- Thickness of equivalent hollow section is variable

$$2\,d' < t < A/U$$

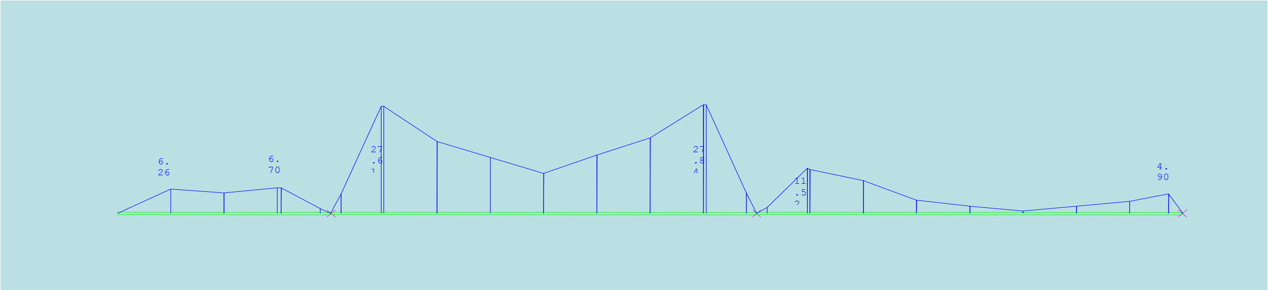
Norm	Width [m]	Ak [m2]	tan	tau-T	sig-II	Asw *fy
DIN 1045 alt	0.10	0.810	1.00	4.74*Mt	9.48*Mt	0.617*Mt
DIN 4227	0.15	0.723	1.00	4.61*Mt	9.22*Mt	0.617*Mt
EC2 / DIN 1045 new (min)	0.10	0.810	1.00	4.74*Mt	9.48*Mt	0.617*Mt
EC2 (max)	0.25	0.563	1.00	3.56*Mt	7.11*Mt	0.617*Mt
EC2 / DIN 1045 new (min2)	0.10	0.810	0.57	4.74*Mt	11.0*Mt	0.353*Mt
EC2 (max2)	0.25	0.563	1.75	3.56*Mt	8.26*Mt	1.558*Mt

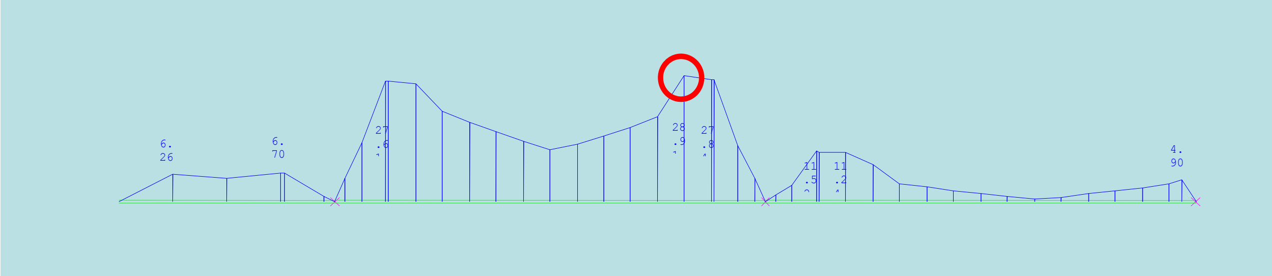
Lever of internal forces

$$\tau = \frac{V}{b \cdot z}$$

- z = 0.85 d or 0.90 d
- Uncracked section yields 0.66 h
- Distance of Tension and Compression
- If we have no / only small tension ?

Required shear links





Details of shear design

	x = 6.0	x = 6.5	x = 7.0
Shear Force	-628.8	-693.8	-758.8
Moment	+255.4	-75.25	-438.4
Lever	135	100	114
Shear stress II	1.33	1.98	1.90
Required shear links	19.37	28.91	27.74

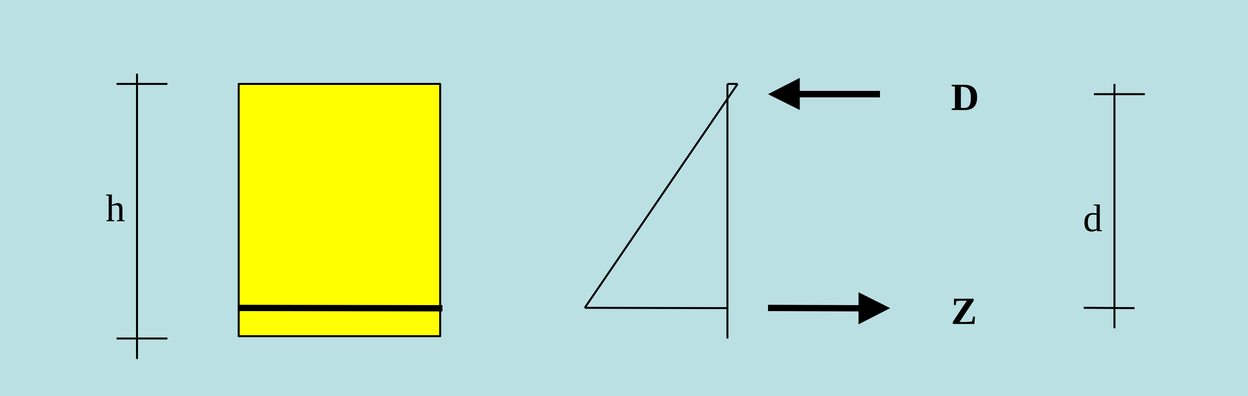
Diagnosis

- Lever is considerably to small (uncracked)
- It was allowed to use the principal tension instead of the shear stress

  σ_I = 3.69 MPa

  τ = 1.98 Mpa
- So what does that mean “it is allowed”

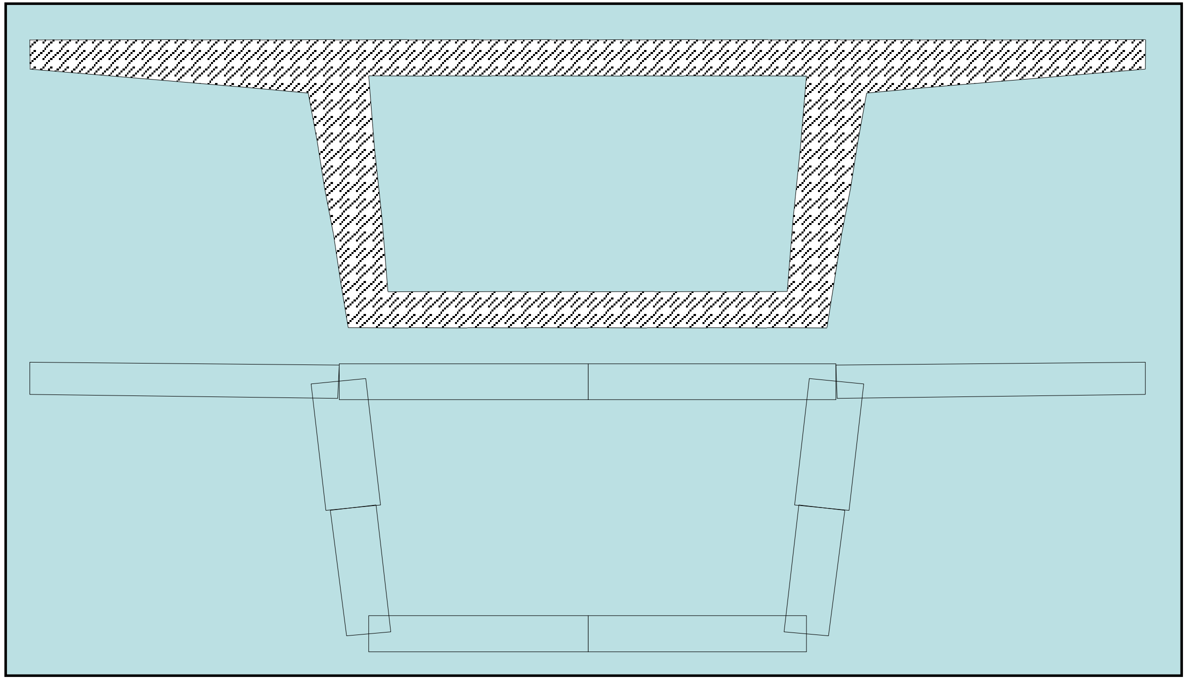
What are the limits



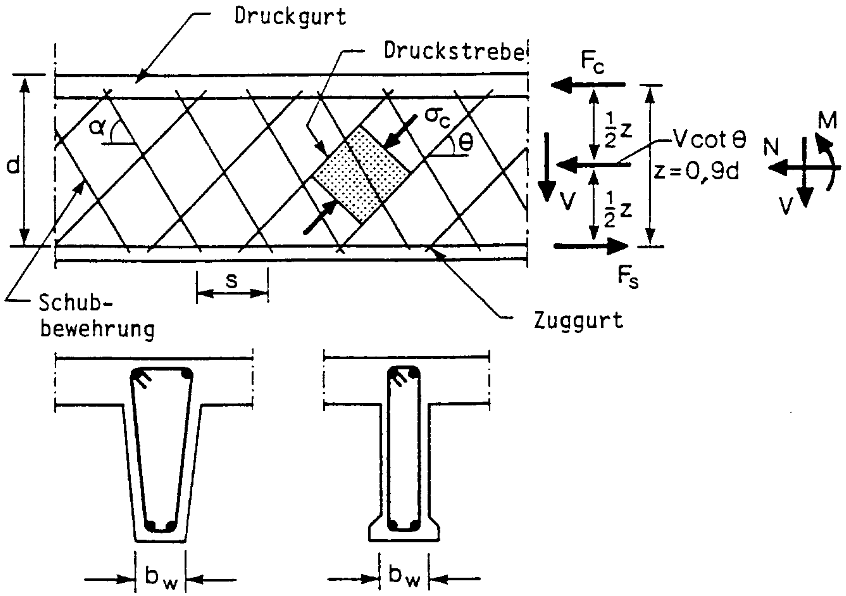
Minimum reinforcements

- Minimum reinforcements increase the longitudinal tensile force
- To get the equilibrium we need a larger compressive force
- So we get a smaller lever
- And a higher shear link area
- And a more even safety level ?

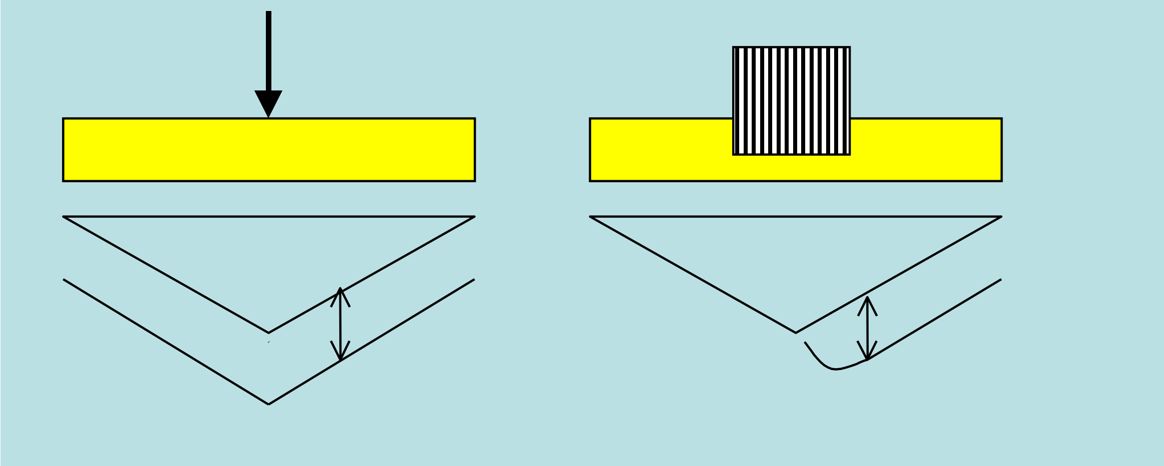
General Section model



Tensile force due to shear



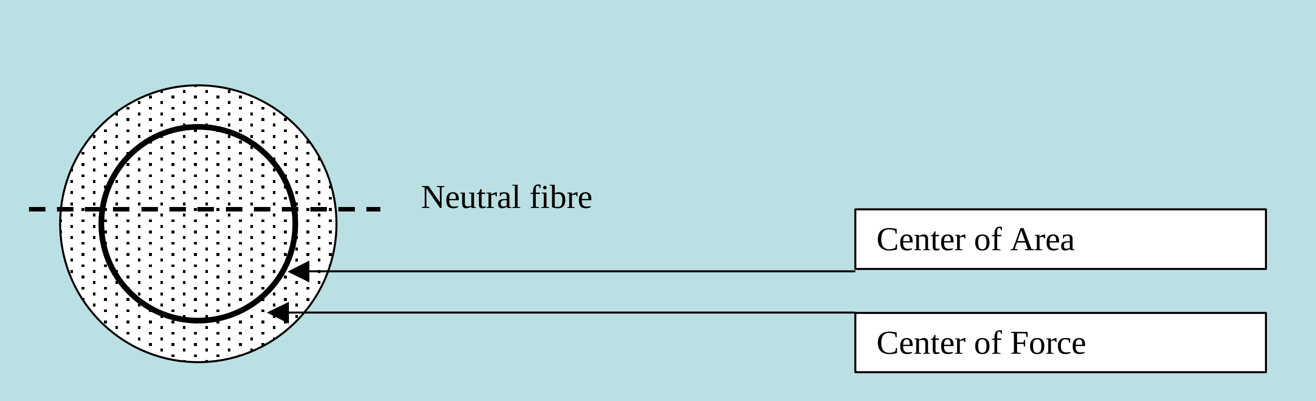
Shift Rule (Versatzmaß)



Height of effective tensile zone

- Old DIN 1045 / 4227
  - max 0.8 m
- Leonhardt / CEB / Heft 400
  EC 2 / DIN 1045-1 1996-1998
  - 2.5 times d’
  - max ((h-x),h/2)
  - max (h-x)/3
    may become difficult

Height of effective tensile zone



Height of effective tensile zone

$$k3 = \frac{\sigma_{b1} + \sigma_{b2}}{2\sigma_{b1}}$$

• Leonhardt / CEB	k3 on tensile zone
• Heft 400	k3 simplified
• EC 2	k3 on total section
• DIN 1045-1 1996	k3 on half section
• DIN 1045-1 1998	coefficient dropped

Variations

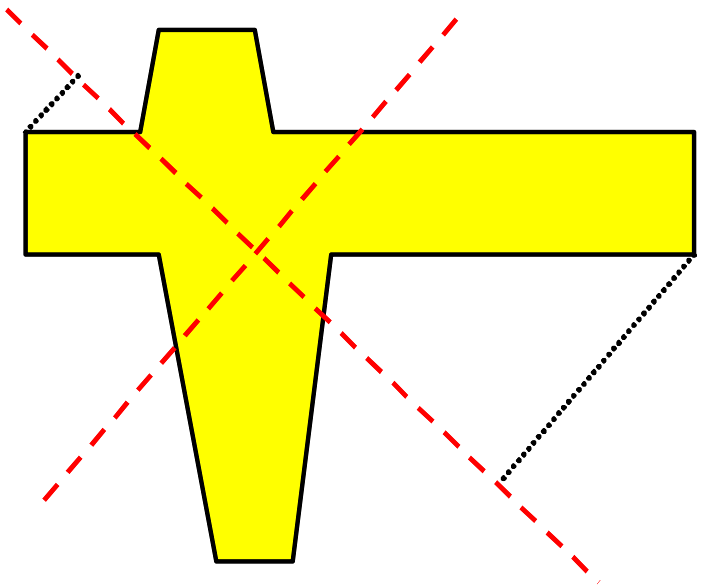
Biegebemessung	48.0 cm2
DIN 1045 (1972)	82.0 cm2
Heft 400 / EC2 (k3=0.915)	76.6 cm2
Heft 400 / EC2 (k3=0.500)	55.0 cm2
DIN 1045-1 1996 (k3=0.676)	62.4 cm2
DIN 1045-1 1998	57.0 cm2

B25 /BSt 500
b/h = 100/120
s = 6 cm
M = 1438.80 kNm
wk = 0.2 mm
D = 28 mm

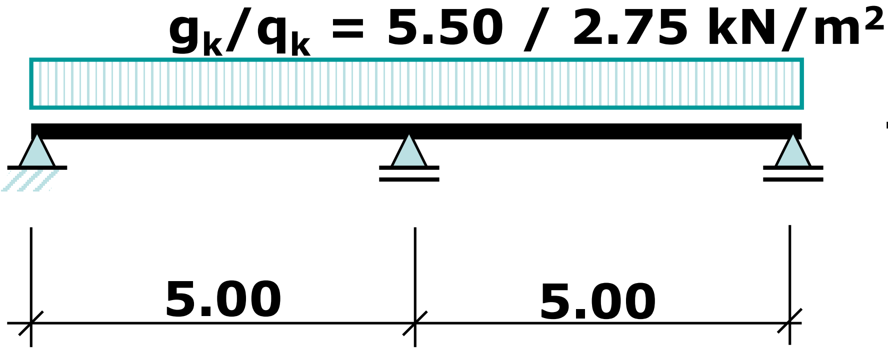
Initial cracking stress

- Formulas for pure tension
- Rectangular sections
- Scaling of cracked solution
- Initial cracking stress >> yield stress ?
  For sections with low reinforcements
  even negative Steel strain: $\varepsilon_{sm} \geq 0.4\ \sigma_s/E_s$

Edge and Corner Stresses

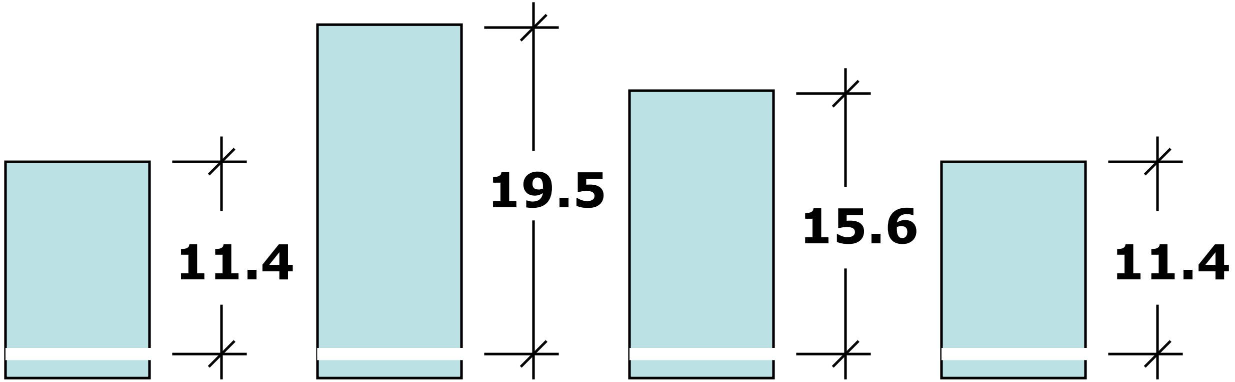


Reinforced Concrete Slab



Concrete C 20
Steel S 500
Thickness
h = 16 cm

Required static heights



DIN 1045
Nov. 1976

EC 2

E DIN 1045
Feb. 1997

E DIN 1045
Nov. 1998

Check for deflections

To allow for additional pay load it is recommended to limit the deflections for quasi permanent loadings to

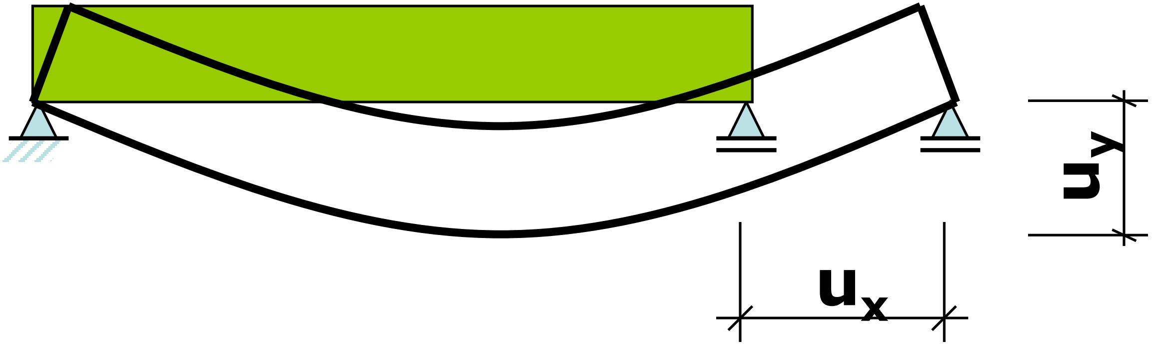
$$f \le \frac{l_{eff}}{500} = \frac{5.00}{500} = 1.0\ cm$$

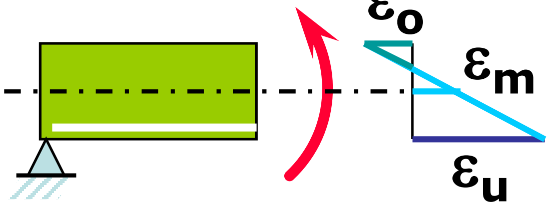
Combination coefficient ψ_2 = 0.3

Check for deflections

Stiffness according to	Deformation by analysis	allowed Deformation	
uncracked	0.24	1.0	cm
cracked	1.79		cm
Cracked with tension Stiffening	1.07		cm

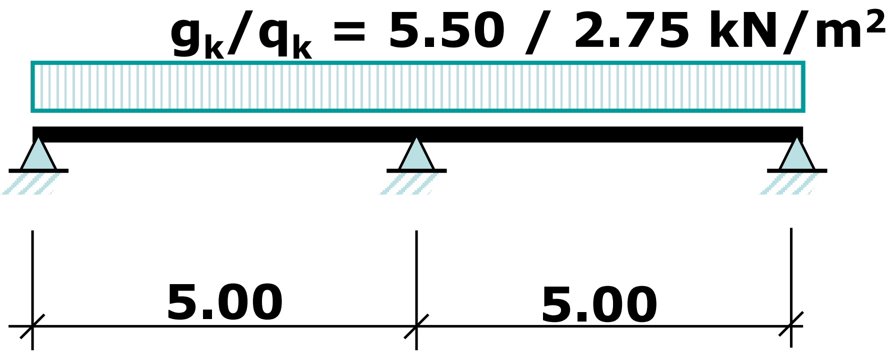
Deformed System





$$u_x = \int \varepsilon_m \cdot dx$$

Changed Boundary Condition



Concrete C 20
Steel S 500
thickness
h = 16 cm

quasi permanent loading = 5.50 + 0.3 x 2.75 kN/m²

calc f = 0.51 cm

Conclusion

- Requirements for a usable design code
  - General provisions for the materials, loadings and safety
  - General provisions for the tasks to do
  - Clearly stated sources and intensions
  - Clear logic = Complete Algorithms
  - „Limit states of assumptions“
  - Methods only in appendices / references = not mandatory

Conclusion

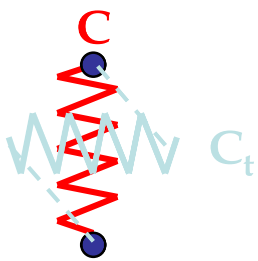
- The FE-Community
  - Implementation of codes may be wrong within software
  - The code may be wrong and the software correct.
  - There is hardly a benchmark for the code validation
  - Do not allow design codes to be written without people knowing about software.

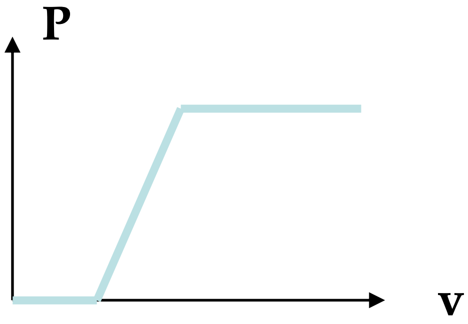
Composite sections

- Any combination of different materials
- Connection may be
  - rigid
  - flexible
- Design may be
  - Elastic (Stresses)
  - Plastic (Forces)

Composites within FE-System

- All kind of structures may be defined
- Any type of materials
- Link with spring elements





Composites within FE-System

- Special remarks for beams
  - Additional eccentricities
  - Stepped transverse shear
  - Normal force is constant,
    but longitudinal shear has parabolic distribution
- General Problems
  - Difficult to define within CAD
  - Choose spring stiffness appropriate
  - Parallel- and serial connection of springs

Composite sections

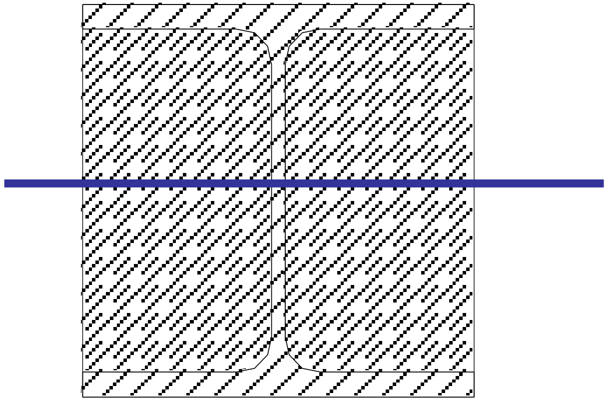
- Section values
  - Ideal section values
    partial section values
  - Shear cuts, stress points
  - Sectional values for shear and torsion
  - Fully plastic forces
  - Reinforcements

Shear in a composite section

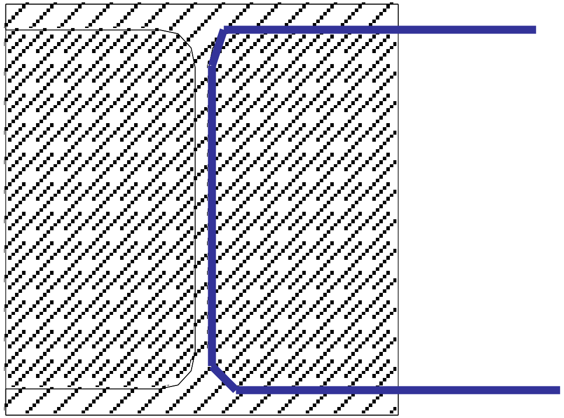
- EC4 chapter 4.4.2:
  „If the effectiveness of the concrete is not included in the ultimate limit state, the limit shear force should be taken from the structural steel area only.“
- Composite sections (EC4 4.8.2.6 bis 8.):
  For the bond stress between an uncracked concrete and the steel admissible stresses between 0.0 and 0.6 MPa are defined.

Shear cuts

- Chamber concrete



τ = constant ?



S_z=0.0

Shear cuts

- More Problems
  - Sequence and Uniqueness
  - Multiple connected regions
  - Discrete shear connections (dowels)

Shear deformations

- Theory Timoshenko/Marguerre

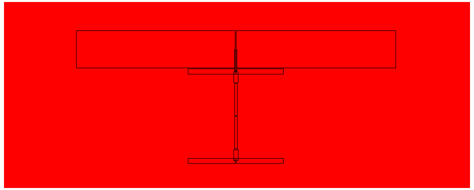
$$\Theta_y = \frac{Q_z}{GA_z} \qquad \Pi = V\theta = \frac{V^2}{GA_s} = \int_A \frac{\tau^2}{G}$$

$$\varphi_y = \frac{\partial w}{\partial x} + \Theta_y \qquad A_s = \frac{1}{G \bullet \Pi(V = 1)}$$

Not an exact Theory for shear deformations

Evaluation by an energetic equivalence

Shear deformation areas



Thickness [mm]	Az – Total [cm^2]	Az – Steel [cm^2]	Az – Concrete [cm^2]
Rigid	47.51	42.51	32.36
D = 100	41.40	35.46	38.50
D = 50	40.50	34.82	36.85
D = 10	34.51	26.76	34.93
D = 5	29.13	26.19	19.06
D = 2	19.85	18.48	8.85
D = 1	12.96	12.38	3.77
Steel only	30.91	30.91	0

Is that true ?

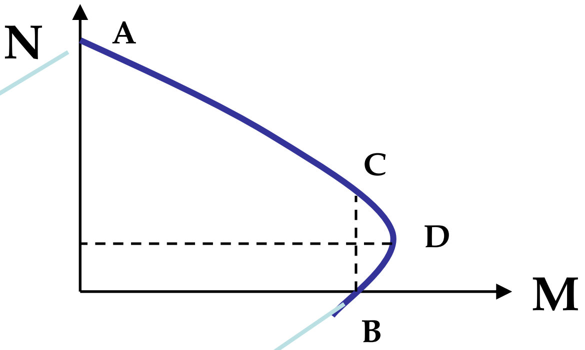
- Simply supported beam l=12 m with Point Load

$$w = \frac{Pl^3}{48EI} \qquad w_v = \frac{Pl}{GA_z}$$

- **Deflections in [mm] for P in [kN]**
  - **Rigid bond**
    w_b = 0.275 P, w_v = 0.031 P, w = 0.306 P
  - **Very flexible bond**
    w_b = 0.275 P, w_v = 0.114 P, w = 0.389 P
  - **No bond = two separate beams**
    w_b = 0.630 P, w_v = 0.003 P, w = 0.633 P
- **The inertia is calculated based on a fully rigid bond (=Bernoulli Hypothesis) !**

Plastic Forces and Moments

- Interaction curve
- characteristic/design



	N[kN]	Vy[kN]	Vz[kN]	Mt[kNm]	My[kNm]	Mz[kNm]	y[mm]	z[mm]	BUCK
K	-5677.9	1184.84	588.54	820.28	655.10	466.97	12.7	-140.	C B
K	3577.9	234.66	557.91	398.74	-448.55	-466.97	-12.7	-5.0	
K	-1500.3		573.22		686.91	.00	.0	-112.	COMB
K	-1050.0	1184.84			.00	471.36	.0	.0	COMB
K	-599.7		573.22		-481.79	.00	.0	-112.	COMB
K	-1050.0	234.66			.00	-471.36	.0	.0	COMB

Design requirements

- Construction stages
  - Stress limits dependant on materials
  - Bond force, longitudinal force
  - Interaction for usage of plastic forces
  - Creep and shrinkage
  - Ultimate limit state with real behaviour
  - Serviceability limit state with real behaviour

Bond

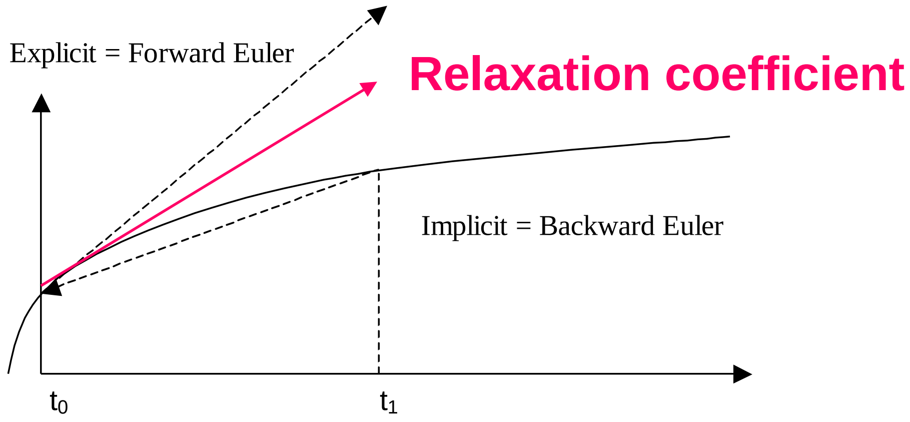
- Rigid bond / Flexible Bond
- Total longitudinal force in mid span
- Total bond force with sectional modulus
- Difference of forces for half span / quarter span
- More detailed analysis with cuts
- Design for number of dowels
- Inclusion of creep effects for longitudinal shear

Analysis of Creep and Shrinkage

- Methods to use
  - Summation approach
  - Product approach
  - Closed formulas or Diagrams / empirical values
- Enhanced Design codes
  - EN 1992-1-1
  - EN 1992-1-2
  - CEB Modell Code

System behaviour

- “Unconstrained” creep of the section itself
- Strain and curvatures acting on the system
- Subdivision in creep increments



Shrinkage

$$N = A_s E_s \varepsilon_s = \rho A_b E_s \varepsilon_s$$

$$\sigma_b = +\frac{N}{A_b + nA_s} = \frac{N}{A_b\left(1+n\rho\right)} = \frac{\rho E_s \varepsilon_s}{1+n\rho}$$

- Volume change of unloaded concrete
- Drying starts at the surface, creating tensions
- Shrinkage is uniform, stress free if plain concrete
- Tensile stresses if deformation is not free
  - Caused by outer constraints
  - Caused by provided reinforcements

Reinforcement for Shrinkage ?

- It is only possible to provide reinforcement for outer forces (e.g. constrained deformation of a foundation slab)
- For any freely moving element the reinforcement will increase the tensile stress of the concrete

ε_s	$-25\ 10^{-5}$
σ_s	-52.50 MPa
σ_b (ρ=2%)	0.92 MPa
σ_b (ρ=6%)	2.20 MPa

- Considerably effects for pre stressed concrete are possible

Shrinkage coefficient

- DIN 4227

  Table 7 defined for cases 1 to 4. Function of development in time given in Picture 3 dependant on the effective thickness.

- DIN 1045-1 / EC-2 / B 4750

  final end values supplied for 4 cases (dry and wet versus thick and thin)

- Appendix 1 / Heft 525

  detailed formulas in dependance of humidity, type of cement, temperature

- AASHTO

$$\Delta f_s = 17000 - 150 RH$$

Creep

- Basic Relation defined with constant stress

$$\varepsilon = \varphi_t \frac{\sigma_b}{E_b}$$

- E-Modulus has to be selected properly
- Creep for tensile stresses ?

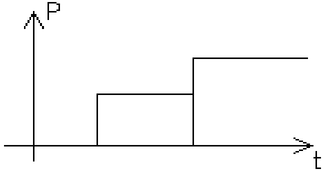
$$\varphi = \begin{Bmatrix} \varphi_0 & \sigma \leq 0 \\ \alpha \cdot \varphi_0 & 0 \geq \sigma \geq f_{ct} \\ 0 & \sigma \geq f_{ct} \end{Bmatrix}$$

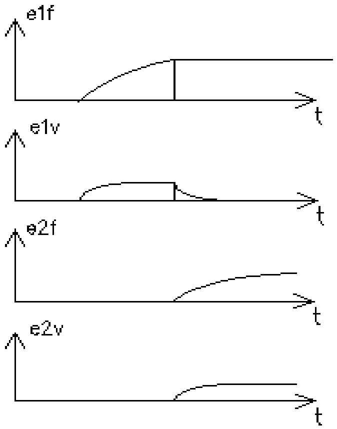
- Creep for which load cases ?

Summation approach

- Plastic Flow and delayed elasticity

$$\varphi_t = \varphi_{vo} k_{v(t-t2)} + \varphi_{fo} k_{v(t-t2)} - \varphi_{vo} k_{v(t2-t1)} k_{v(t-t2)}$$





Product approach

$$\varphi_t = \varphi_o \beta_c(t,t_0) = \varphi_{RH}\beta(f_{cm})\beta(t_0)\beta_c(t,t_0)$$

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.10\sqrt[3]{h_0}}\alpha_1\right]\alpha_2$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}(EC2) = \beta(f_{cm}) = \frac{5.3}{\sqrt{f_{cm}/f_{cm0}}}(DIN)$$

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.20}}$$

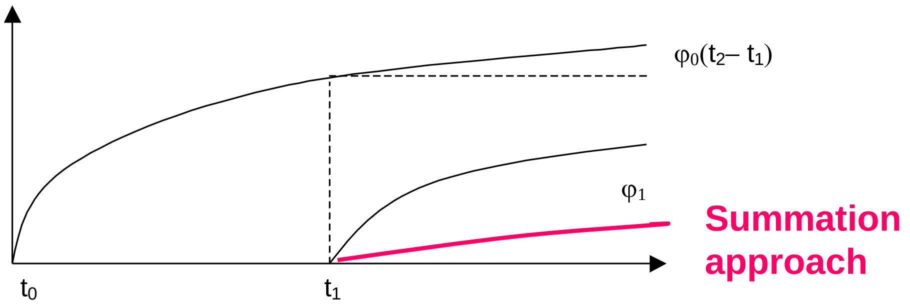
$$\beta_c(t,t_0) = \left(\frac{t - t_0}{\beta_H + t - t_0}\right)$$

$$\beta_H = 1.5\left[1 + (0.012RH)^{18}\right]h_0 + 250\alpha_3 \le 1500$$

Big Problem !?

- Summation approach of DIN 4227
  Creep value of a time slice is defined by the difference of the kf-values and allows a distinct analysis and superposition of results.
- Product approach does NOT allow the full superposition!
  We have different creep values for a load which is persistent since a longer time and a load which is applied now.
- Thus we have only the possibility to calculate the effect of stress changes

Problem of creep stages



$$\varepsilon = \varphi_0 \frac{\sigma_{b0}}{E_b} + \varphi_1 \frac{\Delta\sigma_{b1}}{E_b}$$

$$\varepsilon_0 = \varphi_0(t_0, t_1)\frac{\sigma_{b0}}{E_b}$$

$$\varepsilon_1 = \varphi_0(t_1, t_2)\frac{\sigma_{b0}}{E_b} + \varphi_1(t_1, t_2)\left[\frac{\sigma_{b1}}{E_b} - \frac{\sigma_{b0}}{E_b}\right] = \varphi_1(t_1, t_2)\frac{\sigma_{b1}}{E_b} + \left[\varphi_0(t_1, t_2) - \varphi_1(t_1, t_2)\right]\frac{\sigma_{b0}}{E_b}$$

Creep Stages

- „If the climate conditions change ... they have to be accounted for by changed creep and shrinkage coefficients for every time slice.“ (Leonhardt Vorlesungen Vol I)
- Effective age of concrete has to be accumulated for all creep stages.
- Creep coefficient is given by the difference of the residual creep of the loading compared to a new load applied at this stage.
- Negative creep coefficients have to be considered as negtaive loading multiplied with a positive creep factor!

Creep according to AASHTO

- There are easy rules given in the AASHTO:

$$\Delta f_p = 12\sigma_{c,z} - 7\sigma_{c,fds}$$

- The loss is given by the concrete stress at the tendon fibre due to dead weight and prestress

Tendon relaxation

- Product approach
  - Dependency on stress level defined as value for 1000 h
  - Function in time derived from time values or as prescribed factor.
- Values are defined in EN 1992
- An Iteration is needed for high stress levels
- Very high usage of stress will yield results quite close to those with smaller initial stresses
- Do not mix with the relaxation coefficient by Trost

Analysis Method

- Relaxations coefficient by Trost

$$\frac{\partial \varepsilon_b(t)}{\partial \tau} = \frac{\sigma}{E_b} \cdot \left(1+\varphi(t,\tau)\right)$$

$$\varepsilon_b(t) = \frac{\sigma_0}{E_b} \cdot \left(1+\varphi(t,\tau_0)\right) + \int_{\tau_0}^{t} \frac{\partial \sigma(\tau)}{\partial \tau} \frac{1}{E_b} \cdot \left(1+\varphi(t,\tau)\right) d\tau + \varepsilon_s(t)$$

$$\rho(t,\tau_0) = \frac{\int_{\tau_0}^{t} \frac{\partial \sigma(\tau)}{\partial \tau} \varphi(t,\tau) d\tau}{\left[\sigma(t) - \sigma(\tau_0)\right] \varphi(t,\tau_0)} \leq 1.0$$

$$\varepsilon_b(t) = \frac{\sigma_0}{E_b} \cdot \left(1+\varphi(t,\tau_0)\right) + \frac{\sigma(t)-\sigma_0}{E_b} \cdot \left[\left(1+\rho(t,\tau_0)\varphi(t,\tau)\right)\right] + \varepsilon_s(t)$$

Analysis Method

- Equilibrium of stresses in section
  = 3 equations for 3 unknown strain parameters

$$\sigma_{c,k+s} = \frac{E_b}{1+\rho\cdot\varphi}\cdot\varepsilon_{c,k+s} - \sigma_b \frac{\varphi}{1+\rho\cdot\varphi} - \frac{\varepsilon_S E_b}{1+\rho\cdot\varphi}$$

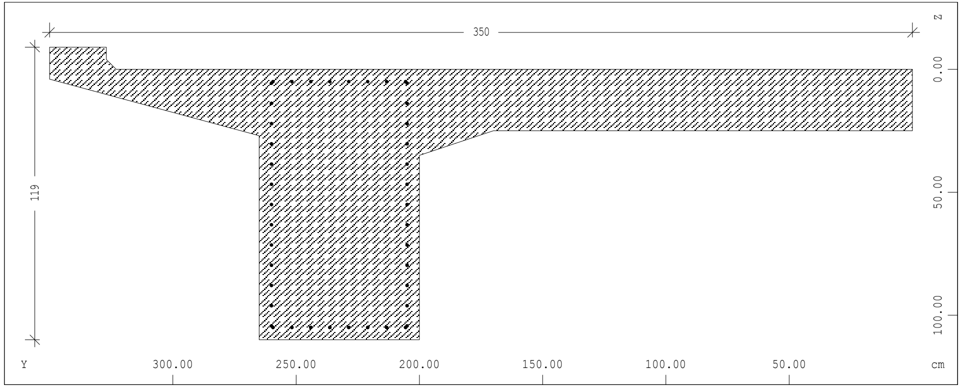
$$\varepsilon_{c,k+s} = \varepsilon_{0,k+s} + z\cdot\kappa_{y,k+s} - y\cdot\kappa_{z,k+s}$$

$$N = \int \sigma_{b,k+s} + \sigma_{St,k+s} = 0$$

$$M_y = \int z\cdot\left(\sigma_{b,k+s} + \sigma_{St,k+s}\right) = 0$$

$$M_z = \int y\cdot\left(\sigma_{b,k+s} + \sigma_{St,k+s}\right) = 0$$

Example


350
119
z
0.00
50.00
100.00
Y
300.00
250.00
200.00
150.00
100.00
50.00
cm

One step only

- EIGE 1 T 10000 T0 28

φ_{RH} = 1+(1-0.40)/(0.1*305**(1/3)) = 1.891
$\beta(f_{cm})$ = 16.8/38**(1/2) = 2.725
$\beta(t_0)$ = 1/(0.1+28**0.2) = 0.488

```
CREEP AND SHRINKAGE

MNr h[mm]     phi      eps   RH T[C]  dt[d]    t[d]  t0[d]  tw[d]  Relaxation
  1    305    2.52 -61.E-5   40    20  10000   10000      28  10029  0.80
            * 0.98   * 0.87
```

h=2A/U

$\beta_c(t-t_0)$ = (9972/(707+9972)**0.3

```
 Beam      x[m]   NQ   M   LK       N[kN]     My[kNm]     Mz[kNm]    phi  max-dsz -prz
    1    11.000    1   1   10     -5617.3     -980.15        0.00   2.47  -53.E-5
                                   1061.3      667.99        0.00         -170.50 19.9
                       N/My/Mz     1091.2      686.80        0.00
                      ex/ky/kz     -0.771      -0.096       0.000
```

Two steps

```
CREEP AND SHRINKAGE
MNr h[mm]     phi      eps   RH T[C]  dt[d]    t[d]  t0[d]  tw[d] Relaxation
  1    305    2.52 -61.E-5   40   20   100      100     28    128  0.80
             * 0.53   * 0.17
  Stab      x[m]  NQ   M  LK      N[kN]    My[kNm]    Mz[kNm]   phi  max-dsz -prz
     1   11.000    1   1  11    -5617.3    -980.15       0.00  1.35 -11.E-5
                                  454.1     285.80       0.00          -73.02   8.5
                    N/My/Mz       467.3     294.13       0.00
                   ex/ky/kz      -0.255     -0.161      0.000
--------------------------------------------------------------------------
MNr h[mm]     phi      eps   RH T[C]  dt[d]    t[d]  t0[d]  tw[d] Relaxation
  1    305    1.88 -61.E-5   40   20  10000   10100    128  10129  0.80
             * 0.98   * 0.70
  Stab      x[m]  NQ   M  LK      N[kN]    My[kNm]    Mz[kNm]   phi  max-dsz -prz
     1   11.000    1   1  11    -5617.3    -980.15       0.00 -0.72
                       1  12    -5163.3    -694.35       0.00  1.84 -42.E-5
                                  574.1     361.31       0.00          -92.56 10.8
                    N/My/Mz       592.4     372.83       0.00
                   ex/ky/kz      -0.512      0.096      0.000
```

Two steps + change of environment

```
KRIECHEN UND SCHWINDEN
MNr h[mm]    phi      eps  RH T[C]  dt[d]   t[d]  t0[d]  tw[d] Relaxation
  1   305   1.92 -43.E-5  70   20    100    100     28    128  0.80
          * 0.53  * 0.17
 Stab     x[m]  NQ   M  LK      N[kN]    My[kNm]    Mz[kNm]   phi max-dsz -prz
    1  11.000    1   1  11    -5617.3    -980.15       0.00  1.02 -7.4E-5
                               349.1     219.69       0.00        -56.02  6.5
                N/My/Mz        358.5     225.66       0.00
               ex/ky/kz       -0.189     -0.133      0.000
--------------------------------------------------------------------------
MNr h[mm]    phi      eps  RH T[C]  dt[d]   t[d]  t0[d]  tw[d] Relaxation
  1   305   1.88 -61.E-5  40   20  10000  10100    128  10129  0.80
          * 0.98  * 0.70
  Stab     x[m]  NQ   M  LK      N[kN]    My[kNm]    Mz[kNm]   phi max-dsz -prz
     1  11.000    1   1  11    -5617.3    -980.15       0.00 -0.99
                      1  12    -5268.3    -760.46       0.00  1.84 -42.E-5
                                 528.5     332.65       0.00        -85.13  9.9
                 N/My/Mz         544.8     342.91       0.00
                ex/ky/kz        -0.485      0.111      0.000
```

Relaxation
for 1.2 higher stresses

```
EIGE  1 T 10000 T0 28
EIGE 12 T 10000
```

```
CREEP AND SHRINKAGE
MNr h[mm]    phi      eps   RH T[C]  dt[d]   t[d]  t0[d]  tw[d] Relaxation
  1   305   2.52 -61.E-5   40   20  10000  10000     28  10029   0.80
           * 0.98  * 0.87
 12         3.00 * R1000            10000  10000                 0.00 -24.0
  Stab     x[m]  NQ  M  LK     N[kN]   My[kNm]    Mz[kNm]   phi  max-dsz -prz
     1   11.000   1  1  10   -6740.5  -1679.98       0.00  2.47 -53.E-5
                              2095.5   1318.90       0.00       -341.88 32.1
                   N/My/Mz    2115.9   1331.74       0.00
        ex/ky/kz     -0.765    -0.049     0.000
```

Influence of passive reinforcements

```
CTRL QWF 1.0

Section            A[m2]     ys[m]     zs[m]  IYZ[m4]     IY[m4]     IZ[m4]
  1  gros       1.383E+00    1.919     0.341 0.00E+00 1.412E-01 9.300E-01
     CS 0       1.402E+00    1.919     0.341 0.00E+00 1.433E-01 9.344E-01
     CS 1       1.443E+00    1.919     0.359 0.00E+00 1.591E-01 9.418E-01

CREEP AND SHRINKAGE
MNr h[mm]    phi      eps  RH T[C]  dt[d]   t[d]  t0[d]  tw[d] Relaxation
  1   305   2.52 -61.E-5  40   20  10000  10000     28  10029  0.80
          * 0.98  * 0.87
  Stab     x[m]   NQ  M  LK     N[kN]    My[kNm]    Mz[kNm]   phi max-dsz -prz
     1  11.000    1  1  10    -5455.4    -920.56       0.00  2.47 -53.E-5
                               1551.1     728.65       0.00      -152.53 17.8
                    N/My/Mz     976.2     614.40       0.00
                   ex/ky/kz    -0.729     -0.023      0.000
```

Conclusions

- Band width of entry parameters is quite large
- Band width of effects may be large too
- A concise analysis with fixed parameters may be misleading or wrong
- Safety concepts require upper and lower fractile values
- Program have to perform precise analysis based on estimated parameters.
- Live remains interesting !