UNIVERSITY OF SÃO PAULO
SÃO CARLOS SCHOOL OF ENGINEERING
DEPARTMENT OF STRUCTURAL ENGINEERING

The Ergodicity Assumption in Performance-Based Engineering

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Dr. Rubia M. Bose, M. Isabela D. Rodrigues

Motivation

Study of Time-variant Reliability.

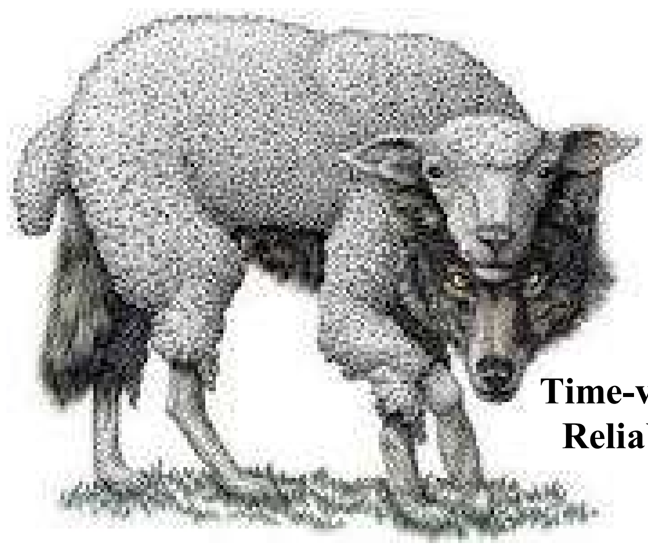
Motivation

Time-variant Reliability is **dead**!

Motivation

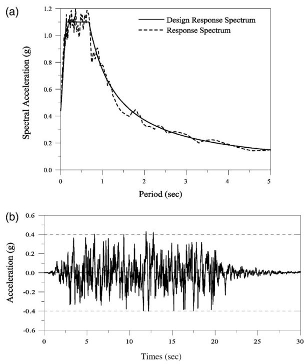
Time-variant Reliability analysis is **dead**!

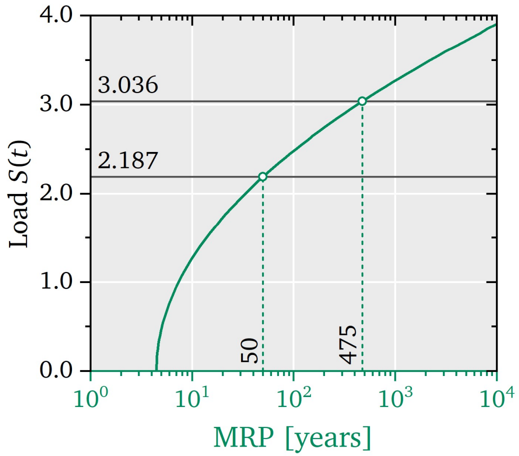
It has been updated to:
Performance Based Engineering.



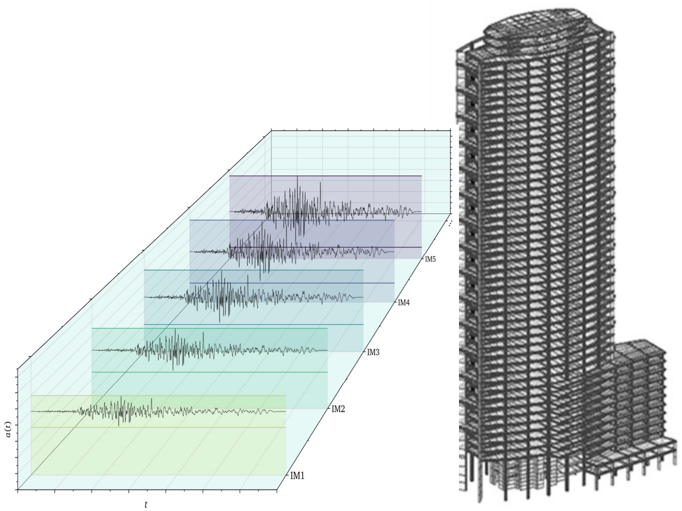
Performance Based Engineering

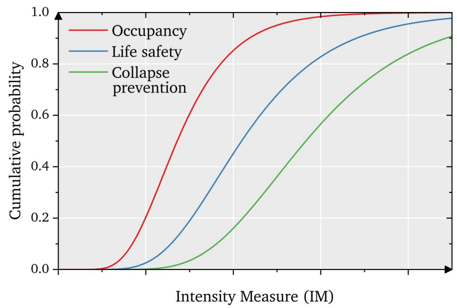
Hazard analysis (**IM**):





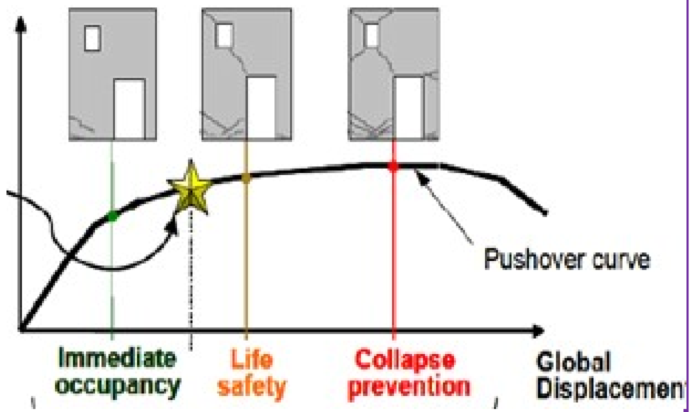
Demand analysis (**EDP**):





Damage & **loss** analysis (**DM**):

Static *pushover* analysis:



Performance Based Engineering

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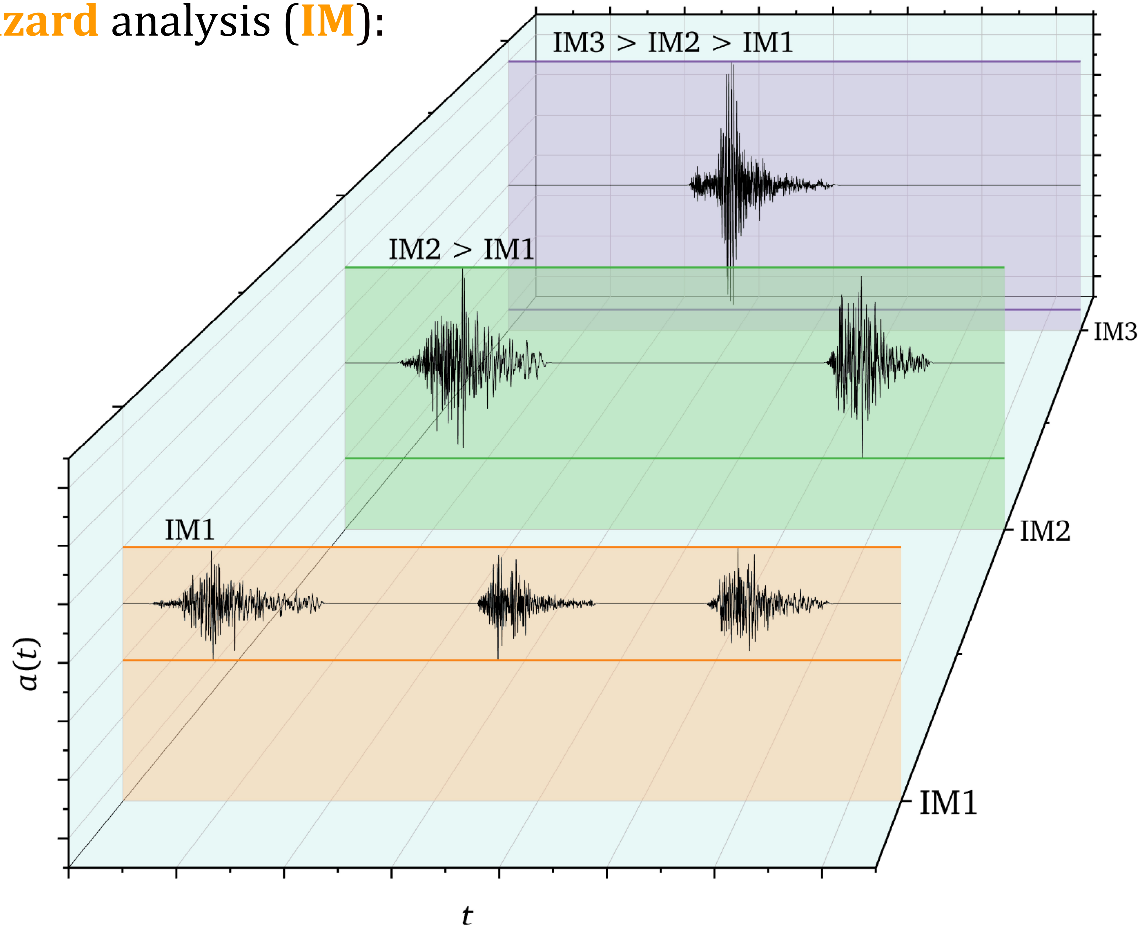
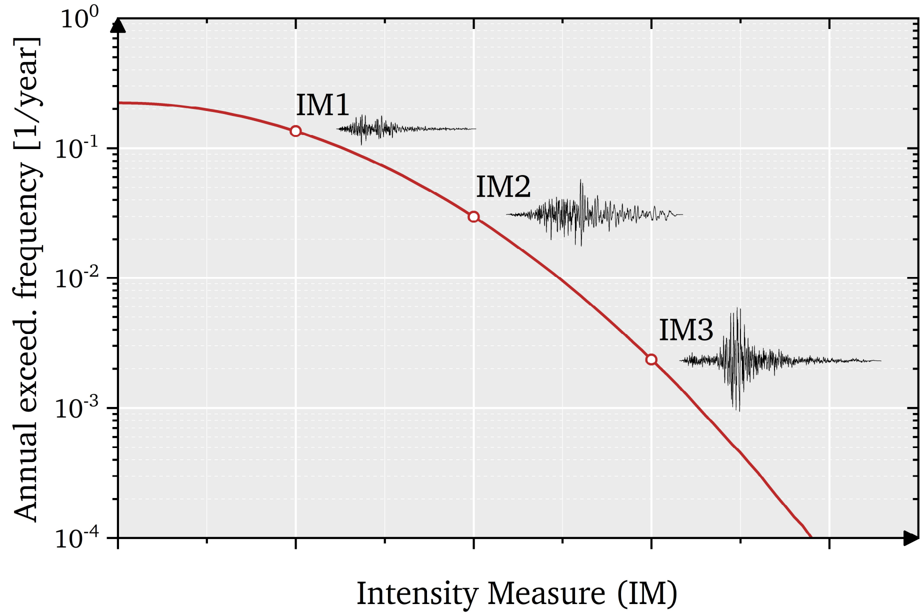




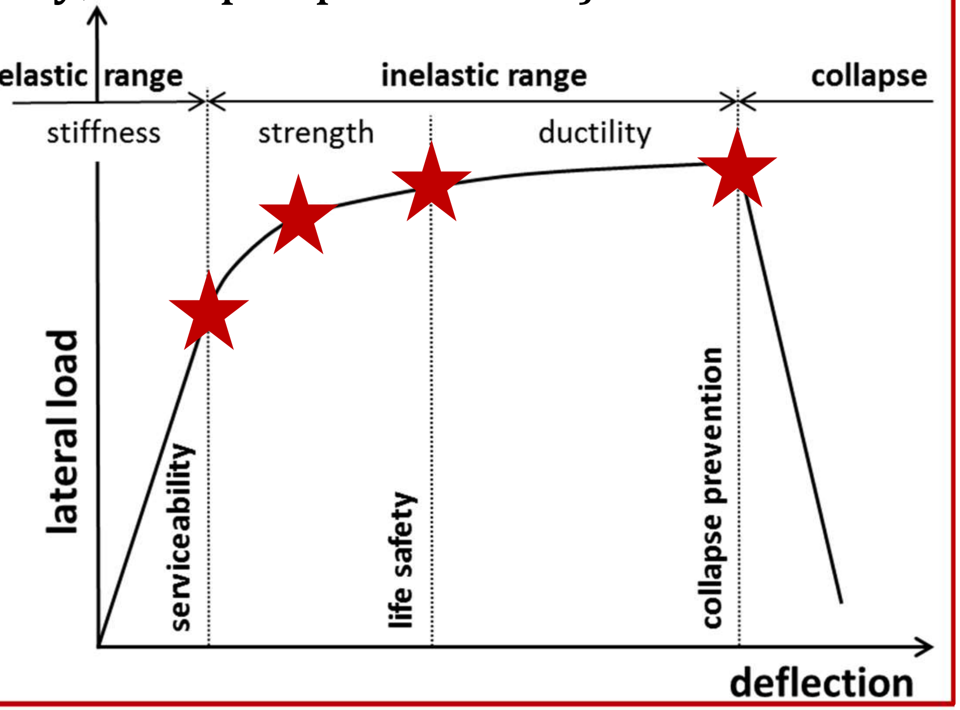
Performance Based Engineering

Demand analysis (**EDP**):

Limit states:

$\boldsymbol{LS}(\textbf{EDP}) = \{$Operational, damage control, life safety, collapse prevention$\}$

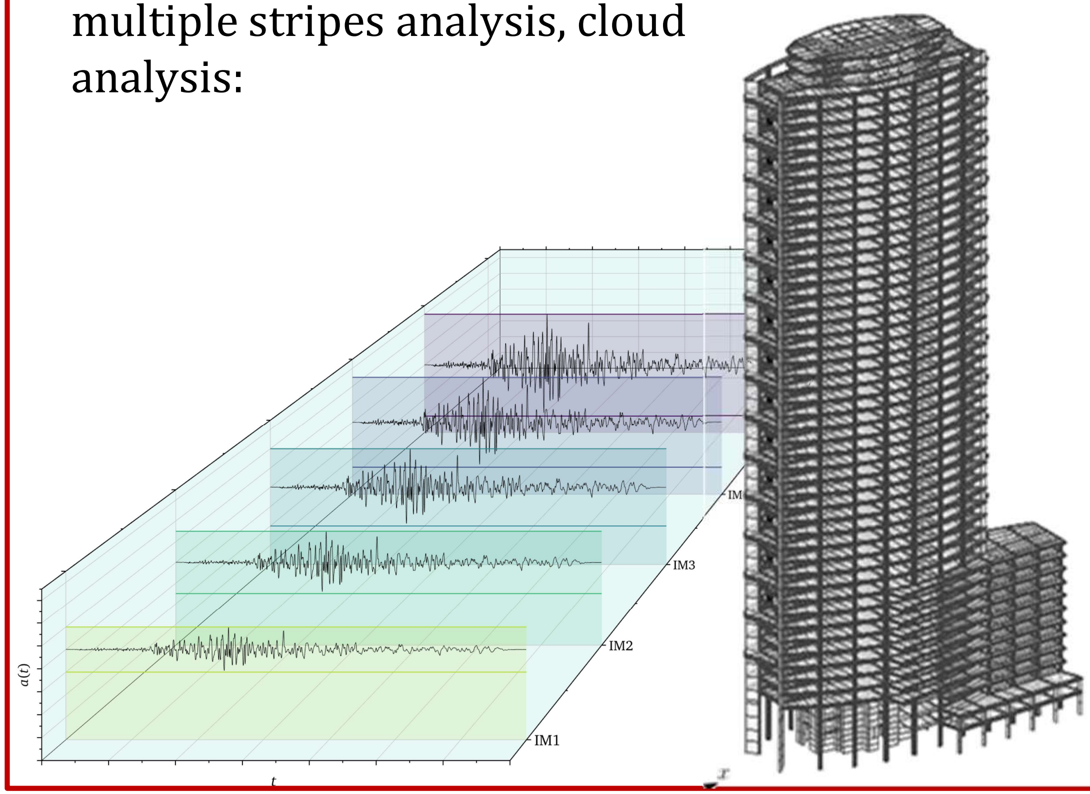
EDP $=$ *Engineering demand parameter*
Displacements;
acccelerations;
interstory drift ratios, etc.

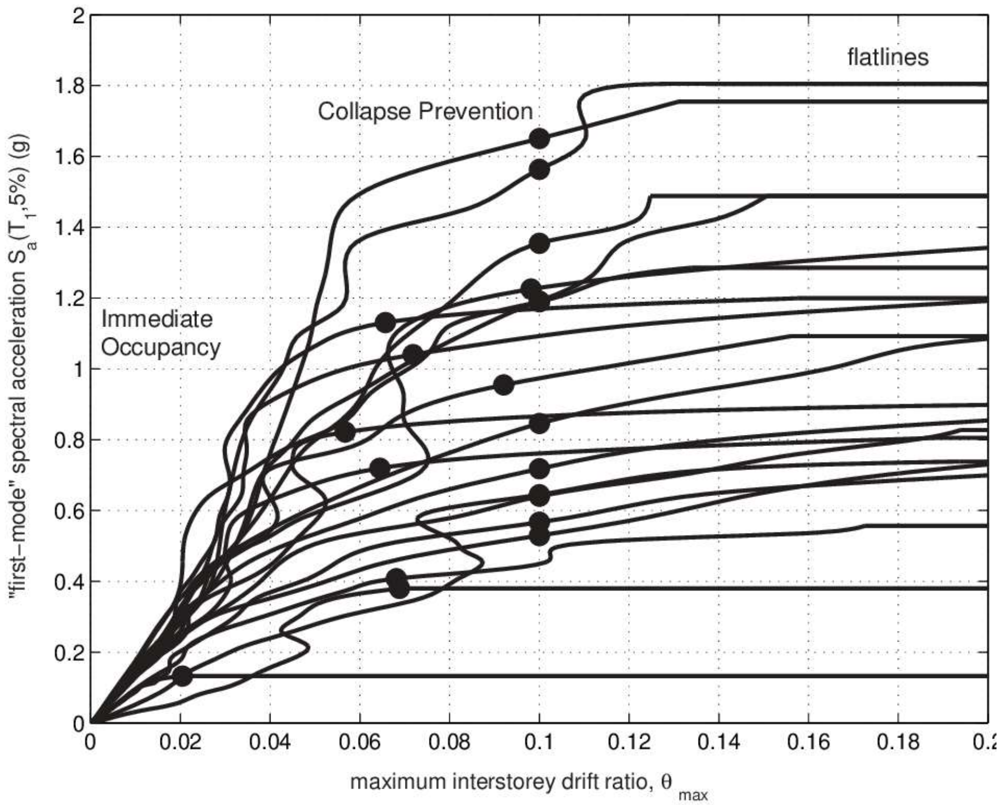


Performance Based Engineering

Demand analysis (**EDP**):

Incremental dynamic analysis (IDA), multiple stripes analysis, cloud analysis:

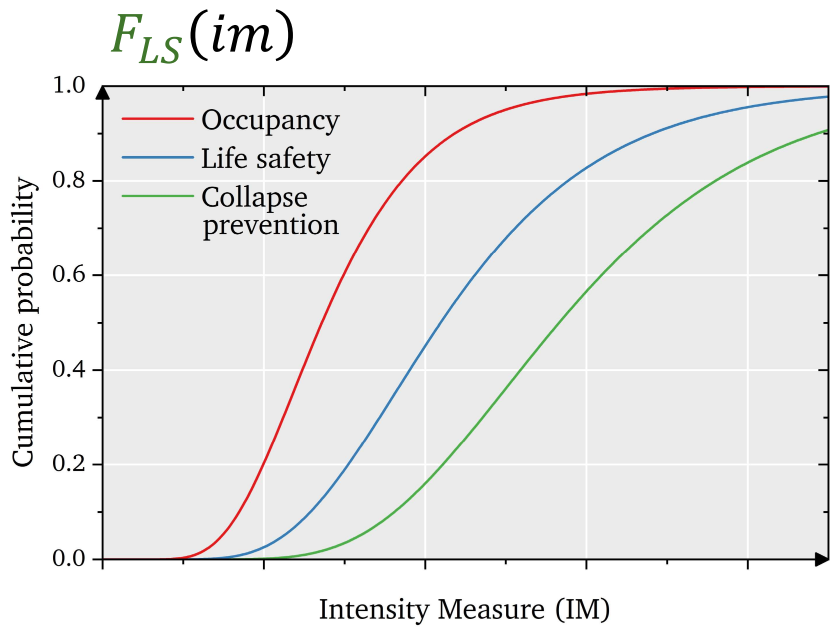




Performance Based Engineering

Fragility curves: $F_{LS}(im) = P[g_{LS}(\boldsymbol{x}|IM = im) \leq 0]$

$$= \int \int F(LS|\mathbf{EDP}) f(\mathbf{EDP}|\mathbf{SP}, \text{IM})\ f(\mathbf{SP}, \text{IM})\ d\mathbf{EDP}\ d\mathbf{SP}$$



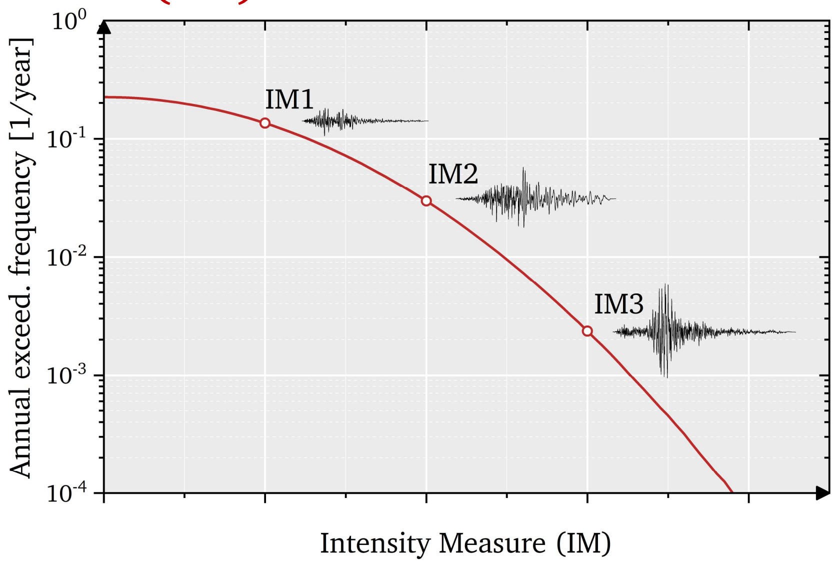
Performance Based Engineering

Fragility curves: $F_{LS}(im) = P[g_{LS}(\boldsymbol{x}|IM = im) \leq 0]$
$= \int\int F(LS|\mathbf{EDP}) f(\mathbf{EDP}|\mathbf{SP}, \mathrm{IM})\ f(\mathbf{SP}, \mathrm{IM})\ d\mathbf{EDP}\ d\mathbf{SP}$

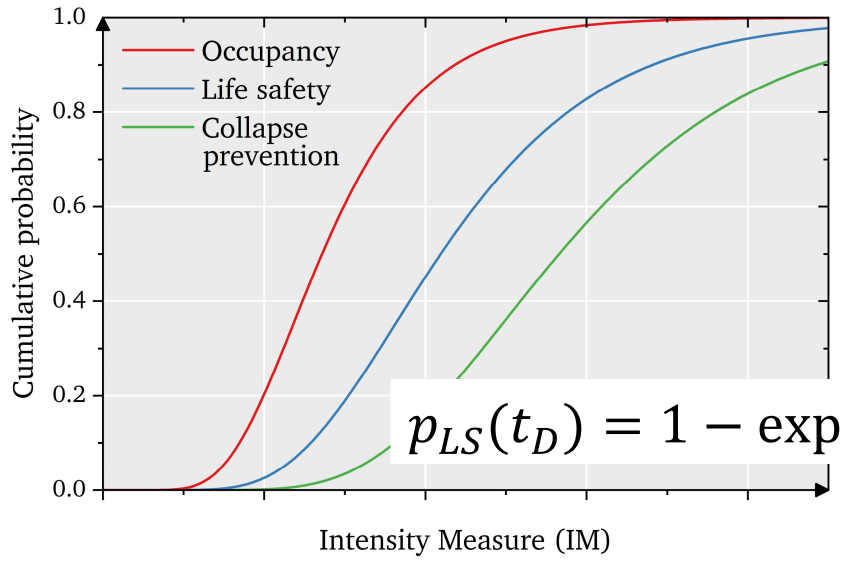
Annual failure rate: $\lambda_{LS} = \int_0^\infty |h(im)| F_{LS}(im) \mathrm{d}im = \int_0^\infty f_{LS}(im) H(im) \mathrm{d}im$ $\qquad h(im) = \frac{\mathrm{d}H(im)}{\mathrm{d}im}$

$$\left(p_f = \int_0^\infty f_S(s) F_R(s) ds\right)$$

$H(im)$



$F_{LS}(im)$



$$p_{LS}(t_D) = 1 - \exp[-\lambda_{LS} \cdot t_D]$$

Time-variant Reliability

$$p_f(\boldsymbol{r}, t_D) = p_{f0} + (1 - p_{f0})\left(1 - exp\left[-\int_0^{t_D} \eta(\boldsymbol{r}, t)\mathrm{d}t\right]\right)$$
$$\geq 1 - exp\left[-\int_0^{t_D} \eta(\boldsymbol{r}, t)\mathrm{d}t\right]$$

$\eta(\boldsymbol{r}, t)$ Rate of a non-homogeneous Poisson process;
$\boldsymbol{r}$ Vector of system parameters;
$E_{\boldsymbol{R}}[\,.\,]$ Expectation over $\boldsymbol{R}$.

$$p_f(t_D) = E_{\boldsymbol{R}}[\, p_f(\boldsymbol{r}, t_D)] = \int_{\boldsymbol{R}} \left(1 - exp\left[-\int_0^{t_D} \eta(\boldsymbol{r}, t)\mathrm{d}t\right]\right) f_{\boldsymbol{R}}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$$

Time-variant Reliability in PBE

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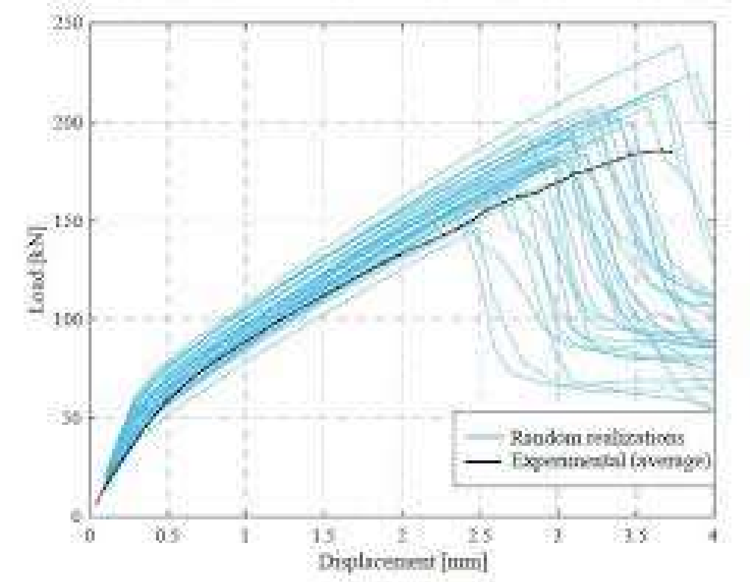
$$p_f(t_D) = E_{\boldsymbol{R}}\left[1 - exp\left(-\int_0^{t_D} E_{\boldsymbol{RTR}}[\eta(\boldsymbol{r}, im, t)]\mathrm{d}t\right)\right]$$

$E_{\boldsymbol{RTR}}[.]$ Expectation over variables with Record-to-Record variability: intensity, time-history, spectral shape;

$$p_f(t_D) = \int_{\boldsymbol{R}} \left[1 - exp\left(-\int_0^{t_D}\int_{IM} \lambda(\boldsymbol{r}, im, t)h(im)\mathrm{d}im\,\mathrm{d}t\right)\right] f_{\boldsymbol{R}}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$$

Ergodic variables: value changes from one load application to the next!
Non-ergodic variables: value is the same for all load applications!

Time-variant Reliability in PBE



Integration over im leads to the annual failure rate:

$$\lambda(\boldsymbol{r},t) = \int_{IM} \lambda(\boldsymbol{r},im,t)h(im)\mathrm{d}im$$

$$p_f(t_D) = \int_{\boldsymbol{R}} \left[1 - exp\left(-\int_0^{t_D} \lambda(\boldsymbol{r},t)\mathrm{d}t\right)\right] f_{\boldsymbol{R}}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}$$

Each sample of random system parameters should be exposed to the whole suit of ground motions: $N \times N_m$ non-linear structural dynamics time-history computations;

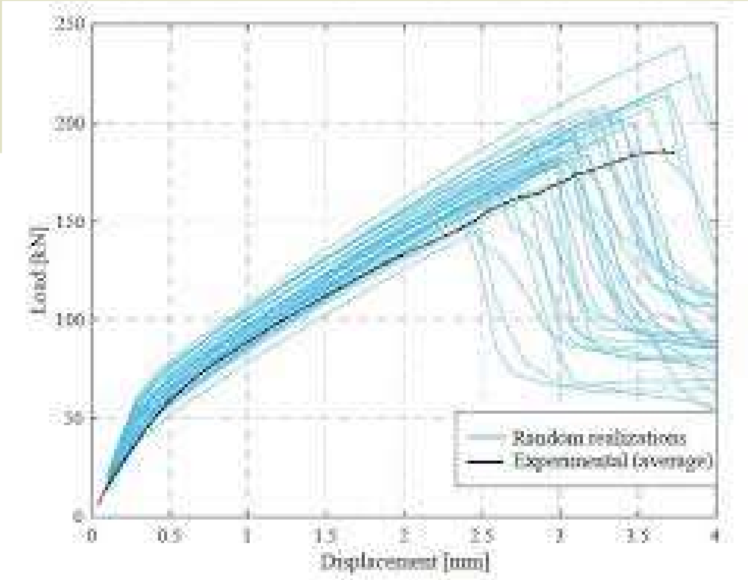
N is the number of ground motion records;

N_m is the number of system parameter samples.

Instead:

$$\lambda_E(t) = \int_{IM} \int_{\boldsymbol{R}} \lambda(\boldsymbol{r},im,t) f_{\boldsymbol{R}}(\boldsymbol{r})\mathrm{d}\boldsymbol{r}\, h(im)\mathrm{d}im$$

Time-variant Reliability in PBE



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Offends the Poisson assumption of independent crossings!

Der Kiureghian, 2005: Non-ergodicity and PEER's framework formula, Earthq. Eng. Struct. Dyn. 34.
Jalayer & Ebrahimian, 2020: Seismic reliability assessment and the nonergodicity in the modelling parameter uncertainties," *Earthq. Eng. Struct. Dyn.* 49.

Fragility function as a product of lognormals

$$\lambda_{LS} = \int_0^\infty F_Z(IM_C|im)h(im)\mathrm{d}im$$

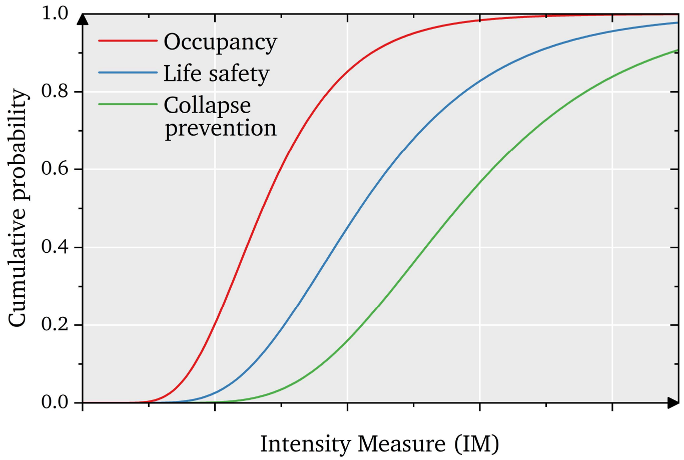
$$Z = g(X,Y) = X \cdot Y$$

Beck et al. 2022: On the ergodicity assumption in Performance-Based engineering, Structural Safety 97.

X effect of all ergodic variables, but the hazard intensity (RTR variability)

Y is the effect of random system parameters: $Y = w(\boldsymbol{R})$.

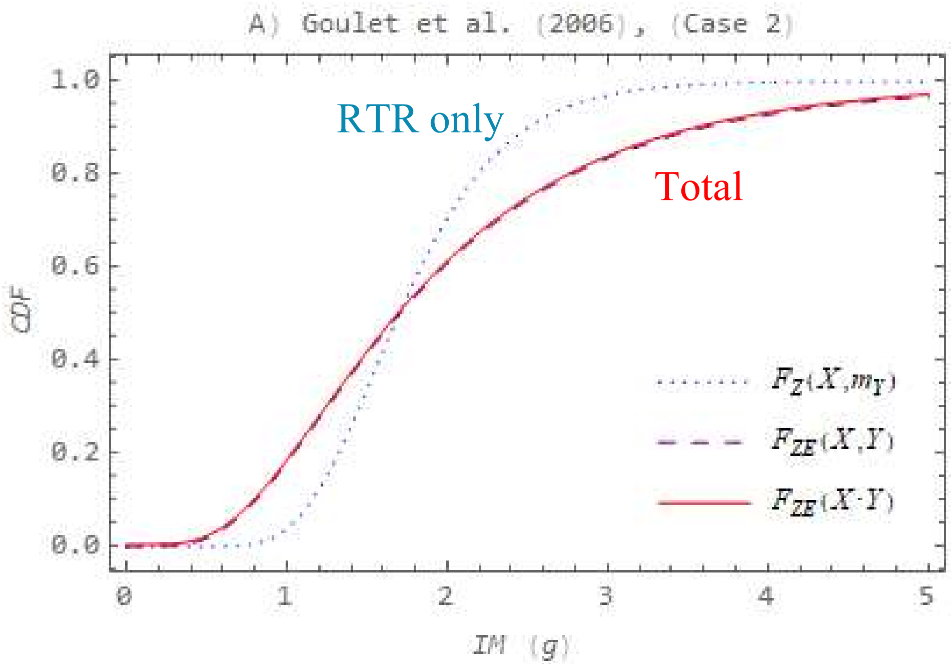
$F_Z(IM_C|im)$

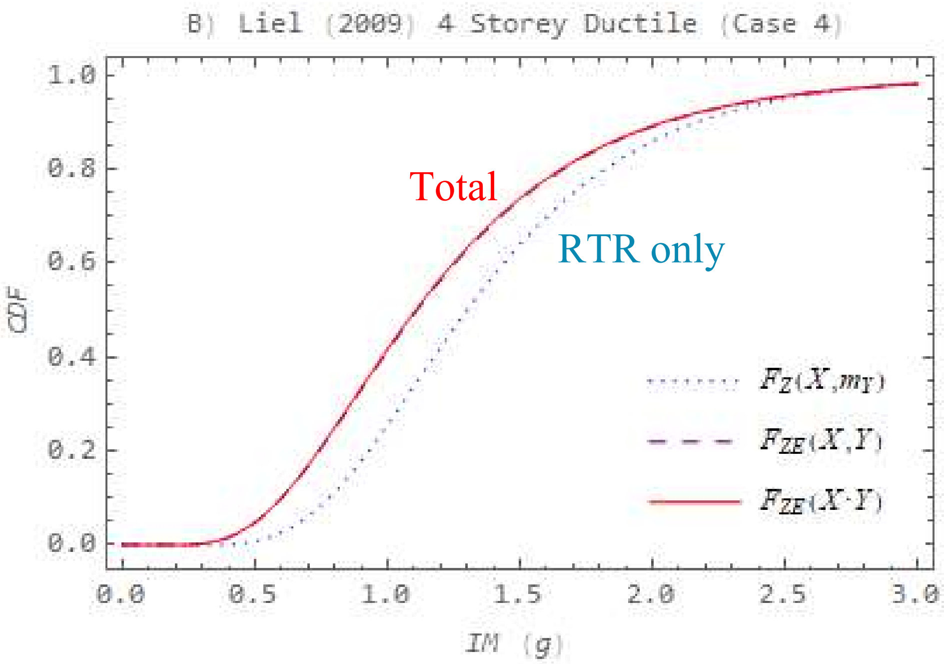


$$E[\ln(Z)] = E[\ln(X)] + E[\ln(Y)] = \widehat{m}_X + \widehat{m}_Y$$

$$Var[\ln(Z)] = Var[\ln(X)] + Var[\ln(Y)] = \sigma_{lnX}^2 + \sigma_{lnY}^2$$

Realistic fragility curves





4-story frame from:

Goulet et al. 2006: Evaluation of the seismic performance of a code-conforming reinforced-concrete frame building - Part I: Ground motion selection and structural collapse simulation.

4-story frame from:

Liel et al. 2009: Incorporating modeling uncertainties in the assessment of seismic collapse risk of buildings, Structural Safety 31.

Evaluation of the "ensemble crossing rate" error:

$$Z = g(X,Y) = X \cdot Y \qquad x = g^{-1}(z,y) = z/y$$
$$\partial g^{-1}/\partial z = 1/y$$

$$F_Z(z) = \int_0^{\infty}\int_0^{g^{-1}} f_{XY}(x,y)\mathrm{d}x\,\mathrm{d}y = \int_0^{\infty}\int_0^{z} \frac{1}{y} f_{XY}\left(\frac{z}{y},y\right)\mathrm{d}z\,\mathrm{d}y$$

$$F_Z(\mathrm{z}) = \int_0^{\infty}\int_0^{z} \frac{1}{y} f_X\left(\frac{z}{y}\right) f_Y(y)\mathrm{d}z\,\mathrm{d}y$$

Beck & Melchers, 2004: On the ensemble crossing rate approach to time variant reliability analysis of uncertain structures, *Probabilistic Eng. Mech.* 19.
Beck & Melchers, 2005: Barrier failure dominance in time variant reliability analysis, Prob. Eng. Mech. 20.
Beck, 2008: The random barrier-crossing problem, Probabilistic Eng. Mech., vol. 23

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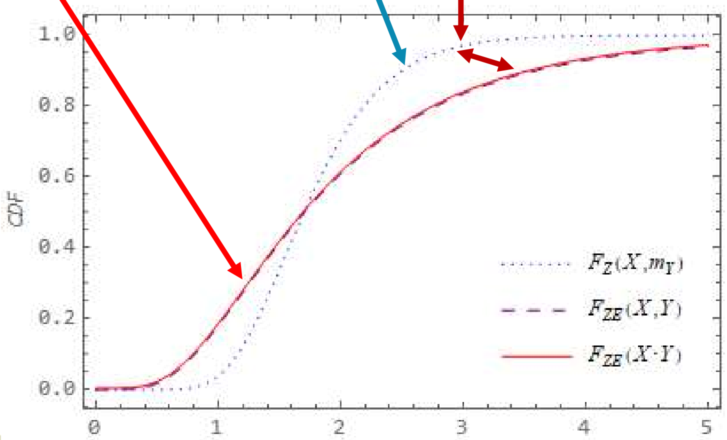
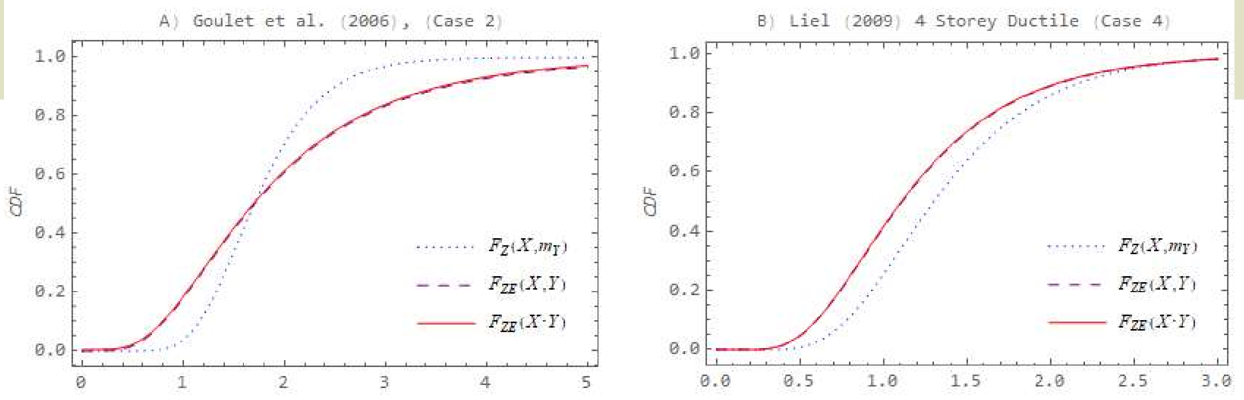


$Z(y) = X \cdot y$ **Record-to-Record** variability only!

$$\lambda_{LS}(y) = \int_0^\infty F_X(IM_C|im, y) h(im)\mathrm{d}im$$

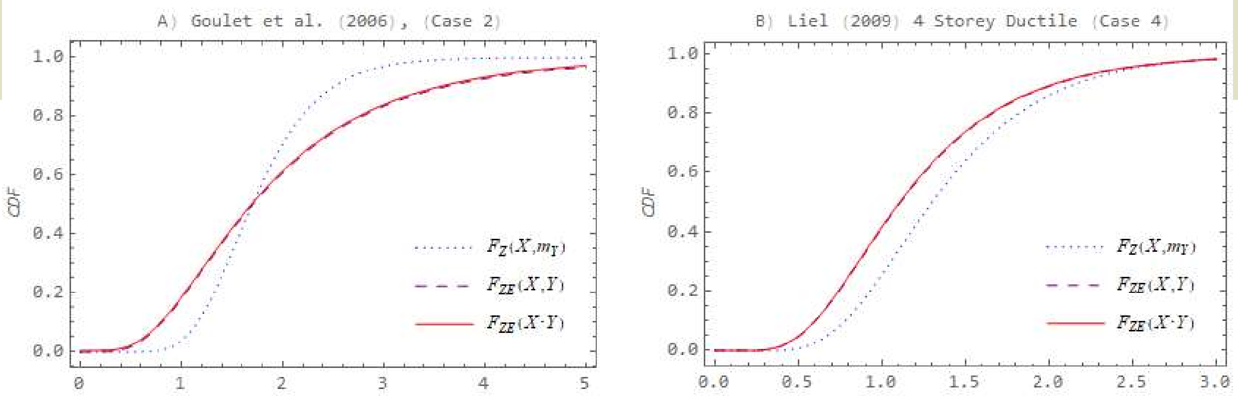
$$p_f(t_D) = \int_0^\infty \left[1 - exp\left(-\int_0^{t_D} \lambda_{LS}(y)\mathrm{d}t\right)\right] f_y(y)\mathrm{d}y$$

Realistic fragility curves:



Ref.	Case	Stories	T_1 (s)	Margin w.r.t. $S_a(T_1)_{2/50}$	RTR (X)		Total (Z)		System only (Y)	
					$\hat{m}$	σ_{ln}	$\hat{m}$	σ_{ln}	$\hat{m}$	σ_{ln}
[8]	1	4	1.00	2.11	1.7	0.30	1.7	0.58	1	0.5
	2		1.00	2.61	2.1	0.29	2.1	0.578	1	0.5
	3		1.00	3.48	2.8	0.34	2.8	0.605	1	0.5
[9]	4	4	1.12	1.52	1.3	0.40	1.10	0.48	0.85	0.26533
	5	12	2.01	1.23	0.61	0.473	0.56	0.52	0.918	0.21603
	6	12	1.98	0.63	0.3	0.45	0.28	0.50	0.933	0.21795
	7	12	2.26	0.73	0.35	0.415	0.38	0.49	1.086	0.26053

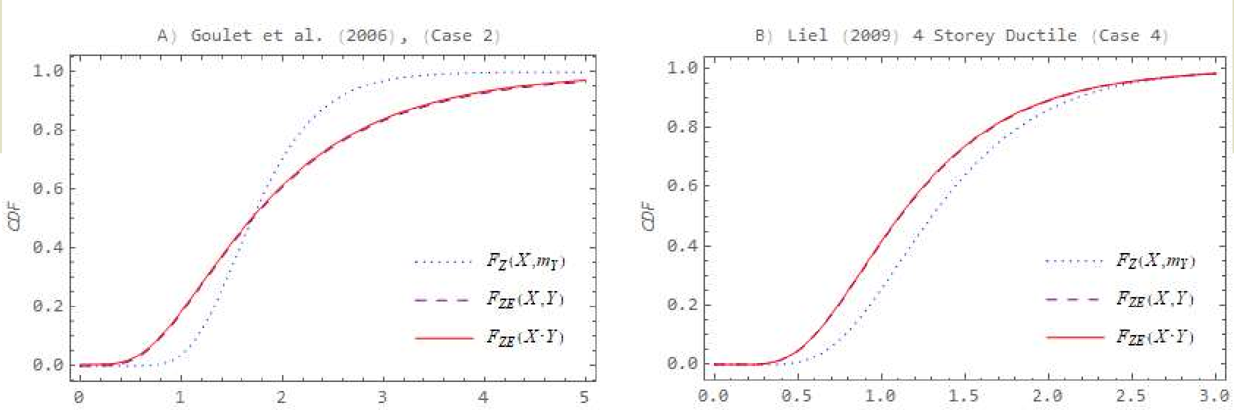
Error evaluation



Annual collapse rates $\lambda_{LS} \times 10^4$:

Frame	a) RTR only	b) Total, (exact)	c) Total, with ER error	Ratio b/a (effect of system uncertainty)	% **Error** λ100(c-b)/b
1	0.215	2.564	2.569	11.92	0.17
2	0.049	1.087	1.088	22.20	0.11
3	0.013	0.422	0.422	31.55	0.07
4	2.413	6.743	6.746	2.80	0.04
5	6.790	11.086	11.089	1.63	0.03
6	51.168	67.903	67.969	1.33	0.10
7	30.937	30.565	30.592	0.99	0.09

Error evaluation



Fifty-year $p_f(t_D = 50) \times 10^4$:

Frame	a) RTR only	b) Total, (exact)	c) Total, with ER error	Ratio b/a (effect of system uncertainty)	% **Error** λ100(c-b)/b
1	0.0011	0.0119	0.0128	11.03	7.62
2	0.0002	0.0052	0.0054	21.05	5.27
3	0.0001	0.0020	0.0021	30.60	3.05
4	0.0120	0.0326	0.0332	2.72	1.79
5	0.0334	0.0532	0.0539	1.59	1.40
6	0.2257	0.2772	0.2881	1.23	3.94
7	0.1433	0.1363	0.1418	0.95	4.07

Concluding remarks

Offending the Poisson assumption of independent crossings can be avoided in PBE by a *product-of-lognormals* scheme;

Results are particularly relevant for service limit states in *Performance Based Wind Engineering*, since the average number of load cycles is much larger (to be investigated);

Details in Beck et al. 2022: On the ergodicity assumption in Performance-Based engineering, Structural Safety 97.

Acknowledgements:

Thanks to:

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CAPES

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UNIVERSIDADE DE SÃO PAULO
SCIENTIA VINCES

The colleagues for your attention.

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