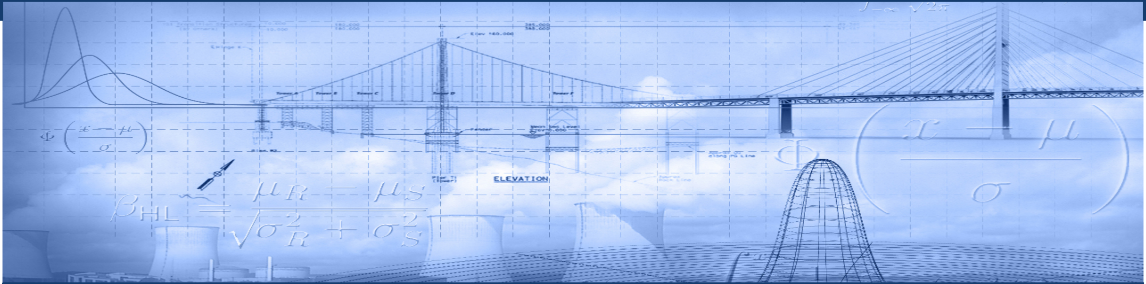
ETH zürich



Time-variant reliability using time-dependent surrogate models

JCSS Workshop on Time-Variant Reliability Analysis: Old Challenges and New Solutions

Styfen Schär, Stefano Marelli, Bruno Sudret

Chair of Risk, Safety and Uncertainty Quantification, ETH Zurich

December 02, 2024

How to cite?

This presentation is an invited lecture given at the JCSS Workshop on "Time-variant Reliability Analysis: Old Challenges and New solutions" on December 2nd, 2024, at the Technical University of Munich (Germany).

How to cite

Schär, S., Marelli, S. and Sudret, B. *Time-variant reliability using time-dependent surrogate models*, Joint Committee on Structural Safety (JCSS) Workshop on "Time-variant Reliability Analysis: Old Challenges and New solutions", Technical University of Munich (Germany), *Invited lecture*, December 2nd, 2024.

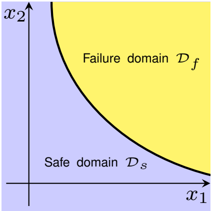
Blaues Pferd (Franz Marc, 1911) – Lenbachhaus, Munich

Time-invariant reliability problems

- The failure criterion is cast as a limit state function (performance function) $g : \boldsymbol{\xi} \in \mathcal{D}_{\boldsymbol{\Xi}} \mapsto \mathbb{R}$ such that:

$$\begin{array}{ll} g\left(\boldsymbol{\xi}, \mathcal{M}(\boldsymbol{\xi})\right) \leq 0 & \text{Failure domain } \mathcal{D}_f \\ g\left(\boldsymbol{\xi}, \mathcal{M}(\boldsymbol{\xi})\right) > 0 & \text{Safety domain } \mathcal{D}_s \\ g\left(\boldsymbol{\xi}, \mathcal{M}(\boldsymbol{\xi})\right) = 0 & \text{Limit state surface} \end{array}$$

e.g. $g(\boldsymbol{\xi}) = y_{adm} - \mathcal{M}(\boldsymbol{\xi})$ when Failure $\Leftrightarrow$ $QoI = \mathcal{M}(\boldsymbol{\xi}) \geq y_{adm}$



Probability of failure

$$P_f = \mathbb{P}\left(\left\{\boldsymbol{\Xi} \in D_f\right\}\right) = \mathbb{P}\left(g\left(\boldsymbol{\Xi}, \mathcal{M}(\boldsymbol{\Xi})\right) \leq 0\right) = \int_{\mathcal{D}_f = \{\boldsymbol{\xi} \in \mathcal{D}_{\boldsymbol{\Xi}} :\, g(\boldsymbol{\xi}, \mathcal{M}(\boldsymbol{\xi})) \leq 0\}} f_{\boldsymbol{\Xi}}(\boldsymbol{\xi})\, d\boldsymbol{\xi} \leq 0$$

- Multidimensional integral ($d = 10 - 100^+$), implicit domain of integration
- Failures are (usually) rare events: sought probability in the range 10^{-2} to 10^{-8}

Time-dependent computational models

Problem statement

- Consider a computational model of a dynamical system:

$$\mathcal{D}_{\boldsymbol{\Xi}} \times [0, T] : \ (\boldsymbol{\xi}, t) \mapsto \mathcal{M}(\boldsymbol{\xi}, t)$$

  where $\boldsymbol{\Xi}$ is a random vector of uncertain parameters with given PDF $f_{\boldsymbol{\Xi}}$

- Uncertainties may be in:
  - The excitation, denoted by $x(\boldsymbol{\xi}_x, t)$
  - And/or in the system's characteristics ($\boldsymbol{\xi}_s$):

  i.e.:

$$\mathcal{M}(\boldsymbol{\xi}, t) \equiv \mathcal{M}(x(\boldsymbol{\xi}_x, t), \ \boldsymbol{\xi}_s)$$

Time-variant reliability problems

Limit-state function

$$g(\boldsymbol{\xi}, t) = y_{adm} - \mathcal{M}(x(\boldsymbol{\xi}_x, t),\ \boldsymbol{\xi}_s)$$

Cumulative probability of failure

- Defined over a time interval $[0, T]$

$$P_{f,c} = \mathbb{P}\left(\exists\, t \in [0, T],\ g(\boldsymbol{\Xi}, t) \leq 0\right)$$

- After time discretization: $\mathcal{T} = \{0, \delta t, 2\delta t, \dots, (N-1)\delta t\}$:

$$\begin{aligned} P_{f,c} &\approx \mathbb{P}\left(\bigcup_{i=1,\dots,N} \{g(\boldsymbol{\Xi}, t_i) \leq 0\}\right), \qquad \text{with } t_i = (i-1)\delta t \\ &= \mathbb{P}\left(\left[\min_{i=1,\dots,N} g(\boldsymbol{\Xi}, t_i)\right] \leq 0\right) \\ &= \mathbb{P}\left(y_{adm} - \max_{i=1,\dots,N} \mathcal{M}(\boldsymbol{\Xi}, t_i) \leq 0\right) \end{aligned}$$

Monte Carlo estimation

Procedure

- Sample a large number of scenarios $\{\boldsymbol{\xi}_k,\ k = 1,\ \dots\ , n_{\text{MCS}}\}$
- Estimate $P_{f,c}$ as

$$\hat{P}_{f,c} = \frac{\#\left\{y_{adm} - \max\limits_{i=1,\dots,N} \mathcal{M}(\boldsymbol{\xi}_k, t_i) \leq 0\right\}}{n_{\text{MCS}}}$$

Challenges

- Feasible for extremely fast-to-evaluate models
- ... or using surrogate models

Outline

Introduction

Surrogate models for time-variant models

- mNARX models
- Application: quarter-car model

Feature-based NARX models

- From mNARX to $\mathcal{F}$-NARX
- Application: Bouc-Wen oscillator

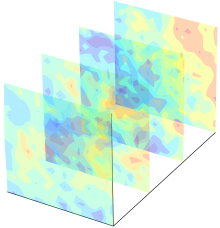
Surrogate modelling for dynamical systems

Setup

- Computational model $\mathcal{M}$ with time-dependent exogenous input $\boldsymbol{x}$ and output $\boldsymbol{y}$:

$$\boldsymbol{x} : \mathcal{T} \rightarrow \mathbb{R}^M, \boldsymbol{y} : \mathcal{T} \rightarrow \mathbb{R}$$

- Discrete time axis $\mathcal{T} = \{0, \delta t, 2\delta t, \dots, (N-1)\delta t\}$

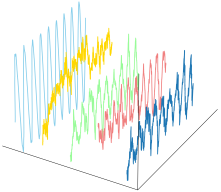


Objectives

- Replace the computational model with a fast-to-evaluate surrogate $\hat{\mathcal{M}}$

$$y(t) = \mathcal{M}(\boldsymbol{x}(\mathcal{T} \leq t),) \approx \hat{\mathcal{M}}(\boldsymbol{x}(\mathcal{T} \leq t))$$

- Surrogate is built on a limited number of model runs ($\approx \mathcal{O}(10^{1-2})$)



Modelling time-dependence

Nonlinear AutoRegressive with eXogenous inputs (NARX) model

- Autoregressive: uses its own past predictions to predict a future time step
- Exogenous input: excitation that governs the system response

$$\hat{y}(t) = \hat{\mathcal{M}}(\boldsymbol{\varphi}(t); \mathbf{c})$$

- $\boldsymbol{\varphi}(t) \in \mathbb{R}^{M_\varphi}$: gathers the exogenous inputs and system output at different time steps
- $\boldsymbol{c}$: finite set of NARX coefficients
- $\hat{\mathcal{M}}$ can be Gaussian process, polynomials, neural networks, etc.

NARX calibration

The lagged vector $\boldsymbol{\varphi}(t) \in \mathbb{R}^n$ for each trace reads:

$$\begin{aligned}\boldsymbol{\varphi}(t) = \{&y(t-\delta t), y(t-2\delta t), \ldots, y(t-n_y\delta t),\\ &x_1(t), x_1(t-\delta t), \ldots, x_1(t-n_{x_1}\delta t),\\ &\ldots,\\ &x_M(t), x_M(t-\delta t), \ldots, x_M(t-n_{x_M}\delta t)\}\end{aligned}$$

Calibrating a NARX model can be cast into a regression problem with regression matrix $\boldsymbol{\Phi} \in \mathbb{R}^{\tilde{N}\times n}$ and output vector $\boldsymbol{y} \in \mathbb{R}^{\tilde{N}}$:

$$\boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\varphi}(t_0) \\ \boldsymbol{\varphi}(t_0+\delta t) \\ \vdots \\ \boldsymbol{\varphi}(t_0+(N-1)\delta t) \end{pmatrix} \quad \boldsymbol{y} = \begin{pmatrix} y(t_0) \\ y(t_0+\delta t) \\ \vdots \\ y(t_0+(N-1)\delta t) \end{pmatrix}$$

where $t_0 = \max(n_y, n_{x_1}, \ldots, n_{x_M})\delta t$

NARX calibration (cont.)

Regression matrices and output vector from multiple traces can be concatenated:

$$\boldsymbol{\Phi}_{\text{ED}} = \begin{pmatrix} \boldsymbol{\Phi}^{(1)} \\ \vdots \\ \boldsymbol{\Phi}^{(N_{\text{ED}})} \end{pmatrix} \quad \boldsymbol{y}_{\text{ED}} = \begin{pmatrix} \boldsymbol{y}^{(1)} \\ \vdots \\ \boldsymbol{y}^{(N_{\text{ED}})} \end{pmatrix}$$

If the number of total time steps becomes too large, subsampling can be used:

$$\boldsymbol{\Phi}_{S} = \begin{pmatrix} \boldsymbol{\Phi}_{\text{ED},r_1} \\ \boldsymbol{\Phi}_{\text{ED},r_2} \\ \vdots \\ \boldsymbol{\Phi}_{\text{ED},r_k} \end{pmatrix} \quad \boldsymbol{y}_{S} = \begin{pmatrix} \boldsymbol{y}_{\text{ED},r_1} \\ \boldsymbol{y}_{\text{ED},r_2} \\ \vdots \\ \boldsymbol{y}_{\text{ED},r_k} \end{pmatrix}$$

where $r_i \in \{1, 2, \dots, |\boldsymbol{y}_{\text{ED}}|\}$ are randomly or deterministically drawn

NARX calibration (cont.)

Polynomial NARX model

$$\mathcal{P}_{\boldsymbol{\alpha}}(\boldsymbol{\varphi}(t)) = \prod_{i=1}^{n} \varphi_i(t)^{\alpha_i}$$

Computation of the coefficients

- The NARX coefficients $\boldsymbol{c}$ are estimated by minimizing the loss function $\mathcal{L}$:

$$\hat{\boldsymbol{c}} = \arg\min_{\boldsymbol{c}} \mathcal{L}\left(\boldsymbol{y}_{\text{ED}}, \hat{\mathcal{M}}(\boldsymbol{\Phi}_{\text{ED}}; \boldsymbol{c})\right)$$

- Ordinary least-squares minimization:

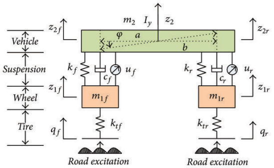
$$\hat{\boldsymbol{c}} = \arg\min_{\boldsymbol{c}} \left\|\boldsymbol{y}_{\text{ED}} - \mathcal{P}_{\boldsymbol{\alpha}}(\boldsymbol{\Phi}_{\text{ED}})\boldsymbol{c}\right\|^2$$

- Sparse solvers like least angle regression (LARS):

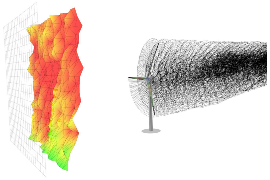
$$\hat{\boldsymbol{c}} = \arg\min_{\boldsymbol{c}} \left\|\boldsymbol{y}_{\text{ED}} - \mathcal{P}_{\boldsymbol{\alpha}}(\boldsymbol{\Phi}_{\text{ED}})\boldsymbol{c}\right\|_2^2 + \gamma||\boldsymbol{c}||_1$$

Challenges with dynamical systems

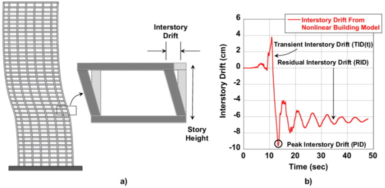
- Direct mapping from the exogenous input to the system response can be highly nonlinear
- Control systems can cause the response to be non-smooth
- Exogenous input can be high-dimensional (*e.g.* spatial field at each time instant)



Pang et al. (2017). Shock and Vibration 2017:6573567



Perez-Becker et al. (2021). Energies 14(3):783.



Rivard et al. (2022). Earthquake Spectra 38(2):875529302110533

Classical NARX modelling often fails under these conditions

Multistep surrogate modelling

Rationale

- Using the original input can result in a complex nonlinear mapping
- Constructing the surrogate on a more informative manifold $\zeta \in \mathbb{R}^{N \times M_\zeta}$ can simplify the mapping:

$$\hat{\mathcal{M}} : \zeta(\mathcal{T} \leq t) \rightarrow y(t) \text{ where } \zeta = \mathcal{F}(\boldsymbol{x})$$

Our approach

Manifold Nonlinear AutoRegressive with eXogenous input (mNARX) modelling - A multistep surrogate modelling approach

1) Input preprocessing	2) Manifold construction	3) Surrogate training
• Dealing with high dimensionality in $\boldsymbol{x}$ • Upsampling, scaling, etc.	• Incremental process • Incorporate prior knowledge of the system	• Built on the manifold • Use of autoregressive surrogate

Schär et al. (2024). Emulating the dynamics of complex systems using autoregressive models on manifolds (mNARX), MSSP.

Input preprocessing (optional)

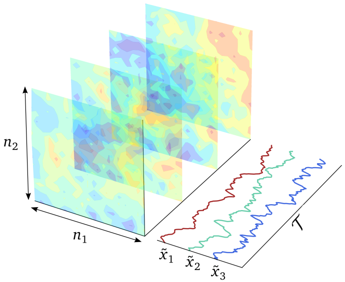
Goal

Reduce dimensionality of the system excitation $\boldsymbol{x}$ along non-temporal coordinates:

$$\tilde{\boldsymbol{x}} = \mathcal{G}(\boldsymbol{x})$$

where $\boldsymbol{x} \in \mathbb{R}^{N \times M}$ and $\tilde{\boldsymbol{x}} \in \mathbb{R}^{N \times m}$ such that $m \ll M$

- Data compression using principal component analysis, N-dimensional discrete cosine transform (DCT), etc.
- Original time scale $\mathcal{T}$ preserved



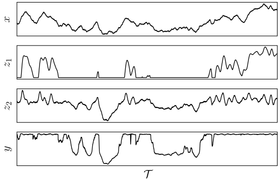
Manifold construction

- The manifold ζ includes a set of features $\boldsymbol{z}_i$ called auxiliary quantities:

$$\zeta = \{\boldsymbol{x}, \boldsymbol{z}_1, \ldots, \boldsymbol{z}_n\}$$

- Auxiliary quantities are constructed incrementally as a function of the information already available:

$$\begin{aligned}
\boldsymbol{z}_1(t) &= \mathcal{F}_1(\boldsymbol{x}(\mathcal{T} \leq t), \boldsymbol{z}_1(\mathcal{T} < t)) \\
\boldsymbol{z}_2(t) &= \mathcal{F}_2(\boldsymbol{z}_1(\mathcal{T} \leq t), \boldsymbol{x}(\mathcal{T} \leq t), \boldsymbol{z}_2(\mathcal{T} < t)) \\
&\vdots \\
\boldsymbol{z}_n(t) &= \mathcal{F}_n(\boldsymbol{z}_1(\mathcal{T} \leq t), \ldots, \boldsymbol{z}_{n-1}(\mathcal{T} \leq t), \boldsymbol{x}(\mathcal{T} \leq t), \boldsymbol{z}_n(\mathcal{T} < t))
\end{aligned}$$



Example (wind turbine)

- $\boldsymbol{x}$: Mean wind speed
- $\boldsymbol{z}_1$: Blade pitch
- $\boldsymbol{z}_2$: Rotor speed
- $\boldsymbol{y}$: Generator power

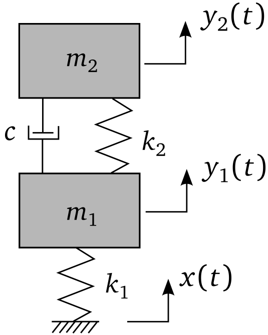
All details in Schär et al. (2024). Emulating the dynamics of complex systems using autoregressive models on manifolds (mNARX), MSSP.

Outline

Quarter-car model

Governing equations

$$\begin{cases} m_2\, \ddot{y}_2(t) = -k_2\,(y_2(t) - y_1(t))^3 - c\,(\dot{y}_2(t) - \dot{y}_1(t)) \\ m_1\, \ddot{y}_1(t) = k_2\,(y_2(t) - y_1(t))^3 + c\,(\dot{y}_2(t) - \dot{y}_1(t)) + k_1\,(x(t) - y_1(t)) \end{cases}$$



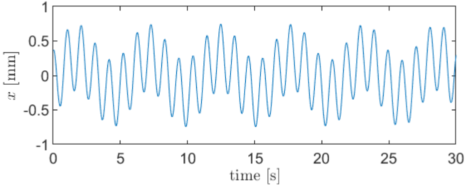
Parameter	Unit	Value
Spring stiffness k_1	N/mm	5,000
Spring stiffness k_2	N/mm	1,000
Mass m_1	kg	50
Mass m_2	kg	10
Damping ratio c	Ns/mm	50

Quarter-car model (cont.)

Random excitation

$$x(t) = \frac{1}{N_\omega} \sum_{i=1}^{N_\omega} A_i \sin\left(2\pi B_i t + C_i\right)$$

- $N_\omega \sim \mathcal{U}$ with $\mathbb{P}(N_\omega = i)_{i=1,\dots,5} = \frac{1}{5}$
- $A_i \sim \mathcal{U}(-1, 1)$
- $B_i \sim \mathcal{U}(-1, 1)$
- $C_i \sim \mathcal{U}(-\pi, \pi)$



Quarter-car model – Experimental designs

Problem setup

- Exogenous input $x(t)$
- Auxiliary quantity $y_1(t)$
- Quantity of interest $y_2(t)$

Random experimental design

- $N_{\text{ED}} = \{10, 50, 100\}$
- Selected randomly from dataset

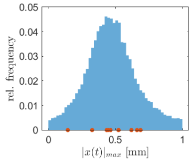
Biased experimental design

- $N_{\text{ED}} = \{10, 50, 100\}$
- Sampled uniformly between $|x(t)|_{\text{min}}$ and $|x(t)|_{\text{max}}$

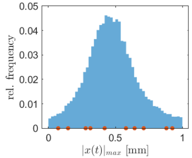
Validation dataset

- $N_{\text{val}} = 100{,}000$

Random experimental design



Biased experimental design



Selection of the best mNARX model

Mai, C. V., Polynomial chaos expansions for uncertain dynamical systems - Applications in earthquake engineering

- Select a reference training trace, *e.g.* the one with the maximal $|y_{\max}|$ over the whole experimental design
- For each candidate polynomial mNARX model (lags, max. degree, max . interaction, etc.):
  1. Compute the best sparse mNARX basis and related coefficients using LARS
  2. Recompute the coefficients of the sparse mNARX model using the whole experimental design by OLS
  3. Compute the mean forecast error over the whole experimental design:

$$\bar{\varepsilon} = \frac{1}{N_{\text{ED}}} \sum_{i=1}^{N_{\text{ED}}} \varepsilon^{(i)} \quad \text{with} \quad \varepsilon^{(i)} = \frac{1}{N} \frac{\sum_{j=0}^{N-1} \left(y^{(i)}(j\delta t) - \hat{y}^{(i)}(j\delta t) \right)^2}{\text{Var}(\boldsymbol{y}^{(i)}) + \gamma},$$

  where γ is a regularization term to avoid division by zero

- Select the mNARX model that minimizes the mean forecast error

Quarter-car model – mNARX configuration

NARX model for $y_1(t)$

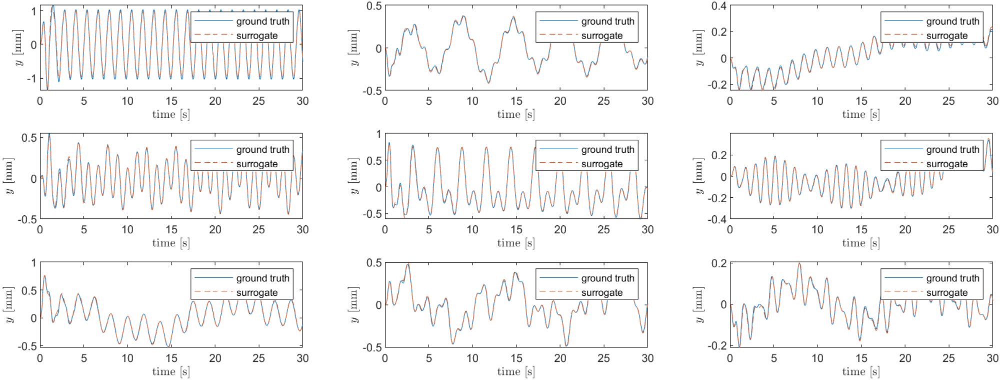
- Exogenous inputs: $x(t)$
- Exogenous input lags: $\{0\}$
- Autoregressive lags: $\{\delta t, 2\delta t\}$
- Maximum polynomial degree: 1
- Interaction order: 1

NARX model for $y_2(t)$

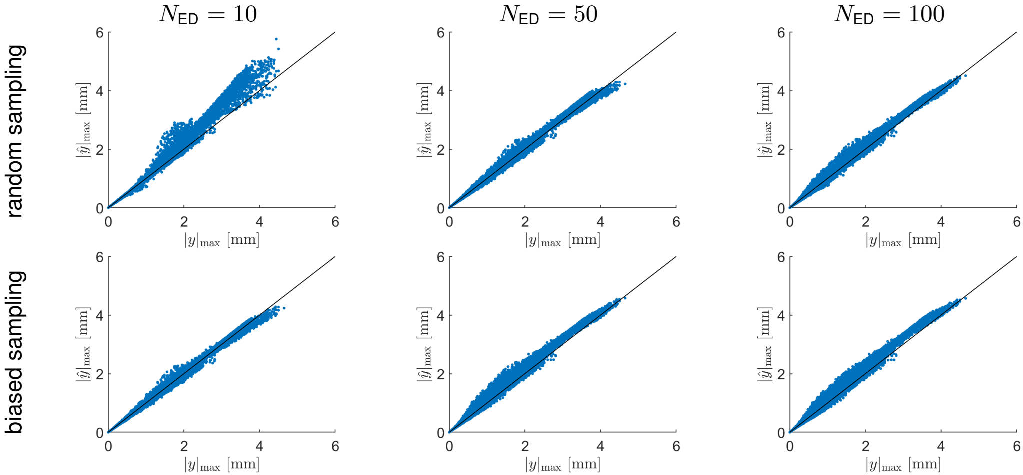
- Exogenous inputs: $x(t), \hat{y}_1(t)$
- Exogenous input lags: $\{0\}, \{0\}$
- Autoregressive lags: $\{\delta t, 2\delta t\}$
- Maximum polynomial degree: 3
- Interaction order: 2

Quarter-Car model – Predicted traces

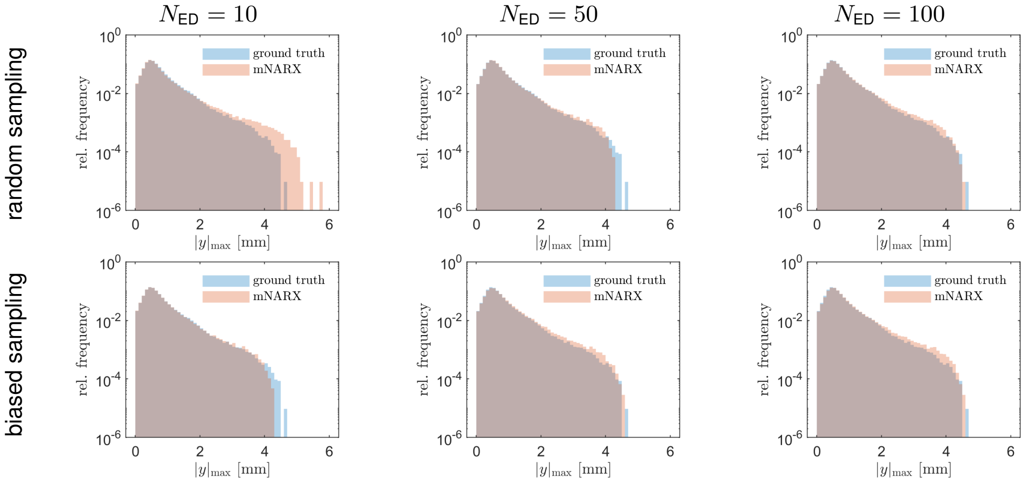
Surrogate built from random experimental design and $N_{\text{ED}} = 50$



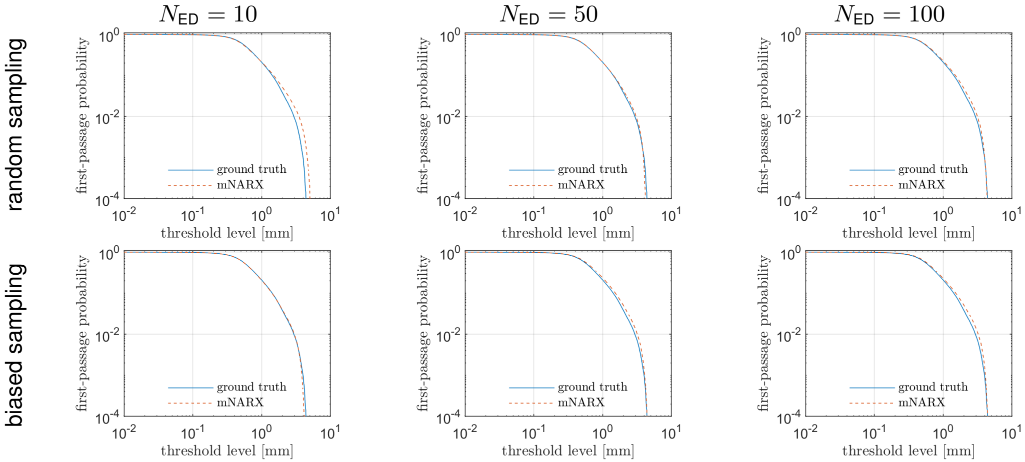
Quarter-Car model – Validation plots for $|y_{\max}|$



Quarter-Car model – Histograms of $|y_{\max}|$



Quarter-Car model – First passage probability



Outline

Introduction

Surrogate models for time-variant models

Feature-based NARX models

- From mNARX to $\mathcal{F}$-NARX
- Application: Bouc-Wen oscillator

Challenges with NARX Models

Lag selection

- Difficulty in selecting relevant time lags, especially with high sampling rate or long memory
- Having many lags increases model complexity and computational demands

Discrete-time-centric limitations

- Over-reliance on recent lags at high sampling rates
- Numerical instability due to high correlation between adjacent time steps
- Decimating the data is not always possible

Missed opportunities

- Do not fully utilize the continuous nature of real-world processes
- Fail to exploit temporal smoothness and compressibility

Our new proposal: Feature-centric autoregressive modelling

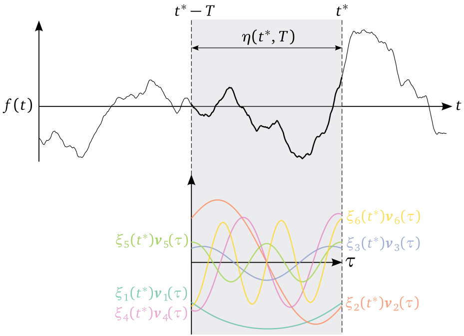
Rationale

- Mitigate challenges of lag selection and long system memories
- Exploit temporal smoothness and regularity

Our approach

- Functional Nonlinear AutoRegressive with eXogenous input ($\mathcal{F}$-NARX) modelling
- Extend classical NARX by leveraging features instead of time lags
- Captures the dynamics of a system using a time-feature representation:

$$\hat{y}(t) = \hat{\mathcal{M}}(\boldsymbol{\xi}(t); \boldsymbol{c})$$



Schär et al. (2024). Feature-centric nonlinear autoregressive models, arXiv.

Principal component analysis over the memory window

Starting from the lagged matrix $\boldsymbol{\Phi}_i$ of the i^{th} variable $\in \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M, \boldsymbol{y}\}$):

1. Standardize $\boldsymbol{\Phi}_i$:
$$\boldsymbol{Z}_i = \frac{\boldsymbol{\Phi}_i - \mu_i}{\sigma_i}$$
2. Compute covariance matrix:
$$\boldsymbol{C}_i = \frac{1}{\tilde{N} - 1} \boldsymbol{Z}_i^\top \boldsymbol{Z}_i$$
3. Compute eigenvalue decomposition:
$$\boldsymbol{C}_i \boldsymbol{v}_{ij} = \lambda_{ij} \boldsymbol{v}_{ij}$$
4. The first $\tilde{n}_i$ eigenvectors are retained (*e.g.* through cumulated explained variance):
$$\boldsymbol{\Lambda}_i = \{\boldsymbol{v}_{i1}, \boldsymbol{v}_{i2}, \ldots, \boldsymbol{v}_{i\tilde{n}_i}\}$$
5. The new features $\boldsymbol{\Xi}_i$ are obtained by projection:
$$\boldsymbol{\Xi}_i = \mathcal{K}_i^{\text{PCA}}(\boldsymbol{\Phi}_i) = \boldsymbol{\Phi}_i \boldsymbol{\Lambda}_i$$

$\mathcal{F}$-NARX calibration

- The obtained matrix of the temporal features for the i^{th} variable reads:

$$\boldsymbol{\Xi}_i = \mathcal{K}_i^{\text{PCA}}(\boldsymbol{\Phi}_i) = \begin{pmatrix} \boldsymbol{\xi}_i(t_0) \\ \boldsymbol{\xi}_i(t_0 + \delta t) \\ \vdots \\ \boldsymbol{\xi}_i((N-1)\delta t) \end{pmatrix}$$

- By concatenating horizontally, the full feature matrix $\boldsymbol{\Xi}$ reads:

$$\boldsymbol{\Xi} = \{\boldsymbol{\Xi}_{x_1}, \dots, \boldsymbol{\Xi}_{x_M}, \boldsymbol{\Xi}_y\}$$

- Multiple traces are handle similarly to the classical NARX case for (sparse) regression:

$$\boldsymbol{\Xi}_{\text{ED}} = \begin{pmatrix} \boldsymbol{\Xi}^{(1)} \\ \vdots \\ \boldsymbol{\Xi}^{(N_{\text{ED}})} \end{pmatrix}, \quad \boldsymbol{y}_{\text{ED}} = \begin{pmatrix} \boldsymbol{y}^{(1)} \\ \vdots \\ \boldsymbol{y}^{(N_{\text{ED}})} \end{pmatrix}$$

Number of features for each input / output

Truncation of the PCA

- Select a fixed number $\tilde{n}_i$ of eigenvectors corresponding to the largest eigenvalues, or
- Select $\tilde{n}_i$ out of n_i eigenvectors according to explained variance (*e.g.* 99%)

$$\nu_i = \frac{\sum_{k=1}^{\tilde{n}_i} \lambda_{ik}}{\sum_{\ell=1}^{n_i} \lambda_{i\ell}}.$$

Advantages of using PCA features

- Features are related to the memory time window and the smoothness of trajectories, not to time lags

  (For classical NARX models, the shortest lags are not always the most important ones)
- Even long memories can be represented by only a few features instead of many time lags

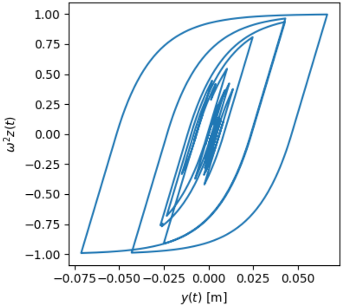
Outline

Bouc-Wen oscillator case study

Governing equation

$$\begin{cases} \ddot{y}(t) + 2\zeta\omega\dot{y}(t) + \omega^2(\rho y(t) + (1-\rho)z(t)) = -x(t) \\ \dot{z}(t) = \gamma\dot{y}(t) - \alpha|\dot{y}(t)||z(t)|^{n-1}z(t) - \beta\dot{y}(t)|z(t)|^n \end{cases}$$

Nonlinear behavior



Parameter	Unit	Value
ζ	-	0.02
ω	rad/s	10
ρ	-	0.2
γ	-	0.5
α	1/m	25
β	-	25
n	-	1

Bouc-Wen oscillator case study (cont.)

Stochastic excitation

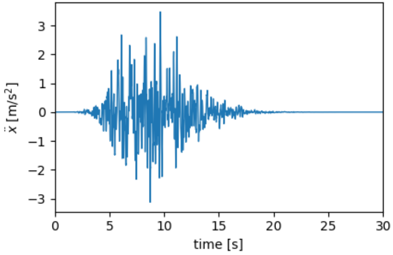
- Ground-motion acceleration $\ddot{x}(t)$
- Der Kiureghian ground motion model

Rezaeian, S. and Der Kiureghian, A. Simulation of synthetic ground motions for specified earthquake and site characteristics.

- Estimated from component 090 of the Northridge earthquake recorded at the LA 00 station

Mai, C. V., Polynomial chaos expansions for uncertain dynamical systems - Applications in earthquake engineering.

Parameter	Unit	Value
I_a	s.g	0.109
D_{5-95}	s	7.96
t_{mid}	s	7.78
$\omega_{mid}/2\pi$	Hz	$4.66 \times 2\pi$
$\omega'/2\pi$	Hz	$-0.09 \times 2\pi$
ζ_f	-	0.24



Bouc-Wen oscillator – Experimental designs

Problem setup

- Exogenous inputs $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$
- Auxiliary quantity $z(t)$
- Quantity of interest $y(t)$

Random experimental design

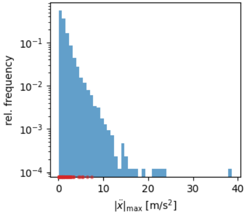
- $N_{\text{ED}} = 100$ realizations
- Selected randomly from dataset

Biased experimental design

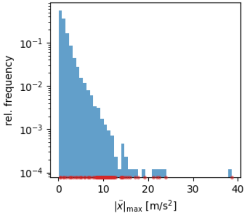
- $N_{\text{ED}} = 100$ realizations
- Sampled uniformly between $|\ddot{x}(t)|_{\min}$ and $|\ddot{x}(t)|_{\max}$

Validation dataset: $N_{\text{val}} = 10{,}000$

Random experimental design



Biased experimental design



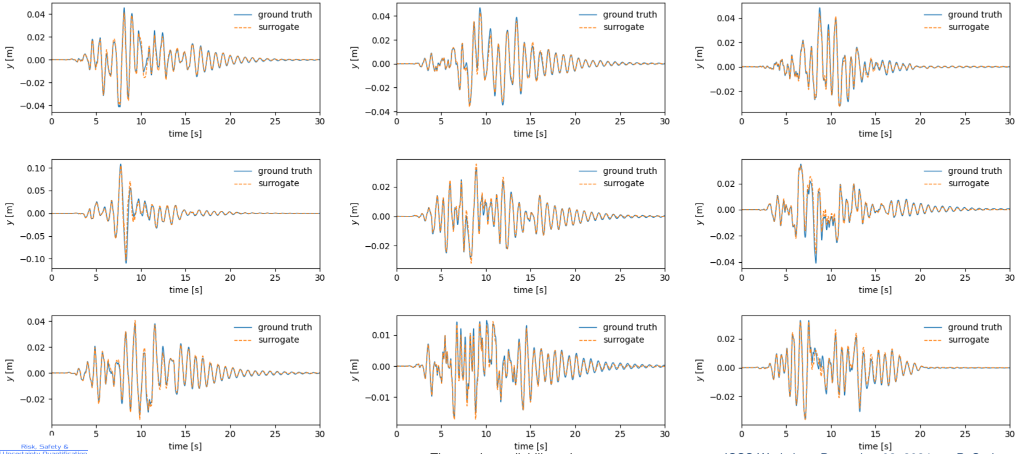
Bouc-Wen oscillator – NARX configuration

$$\begin{cases} \ddot{y}(t) + 2\zeta\omega\dot{y}(t) + \omega^2(\rho y(t) + (1-\rho)z(t)) = -x(t) \\ \dot{z}(t) = \gamma\dot{y}(t) - \alpha|\dot{y}(t)||z(t)|^{n-1}z(t) - \beta\dot{y}(t)|z(t)|^n \end{cases}$$

	Biased sampling		**Random sampling**	
Output quantity	$z(t)$	$y(t)$	$z(t)$	$y(t)$
Exogenous inputs	$\ddot{x}(t), \dot{x}(t), x(t)$	$\ddot{x}(t), \dot{x}(t), x(t), \hat{z}(t)$	$\ddot{x}(t), \dot{x}(t), x(t)$	$\ddot{x}(t), \dot{x}(t), x(t), \hat{z}(t)$
Exogenous memories	1 sec. / 100 lags	1 sec. / 100 lags	1 sec. / 100 lags	1 sec. / 100 lags
Autoregressive memory	1 sec. / 100 lags	1 sec. / 100 lags	1 sec. / 100 lags	1 sec. / 100 lags
Explained variance	85 %	85 %	90 %	85 %
Maximum polynomial degree	3	3	3	3
Interaction order	2	2	2	2

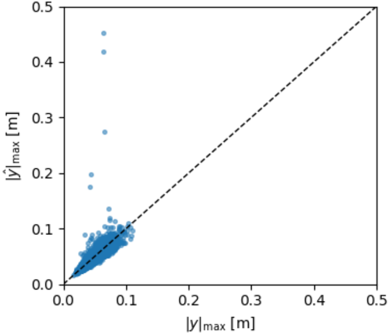
Bouc-Wen results – Predicted traces

Surrogate built from biased experimental design

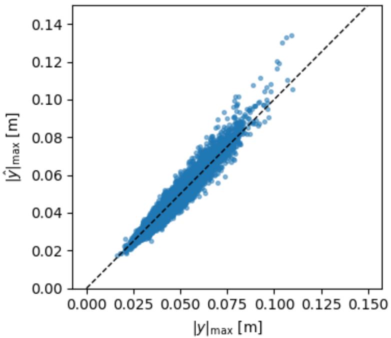


Bouc-Wen oscillator – Validation plots for $|y_{\max}|$

Random sampling ($N_{\text{ED}} = 100$)

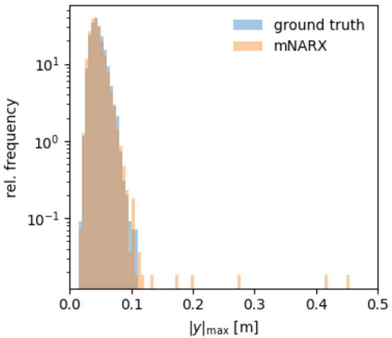


Biased sampling ($N_{\text{ED}} = 100$)

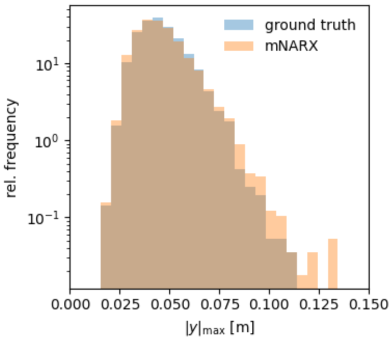


Bouc-Wen oscillator – Histograms of $|y_{\max}|$

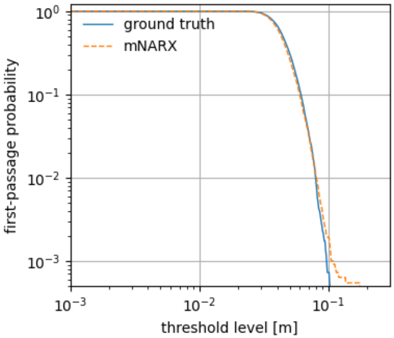
Random sampling ($N_{\text{ED}} = 100$)



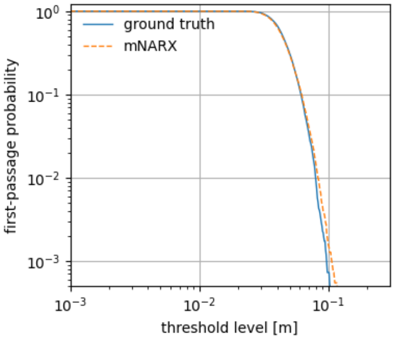
Biased sampling ($N_{\text{ED}} = 100$)



Bouc-Wen oscillator results – First-passage probabilities



Random sampling ($N_{\text{ED}} = 100$)



Biased sampling ($N_{\text{ED}} = 100$)

Conclusions

- Time-variant reliability for dynamical systems requires numerous, possibly costly, transient simulations
- Surrogate models for dynamical systems based on nonlinear auto-regressive models with exogenous inputs (NARX) are proposed
- For better accuracy, NARX models can be build on time-variant features which are extracted on-the-fly from the data (so-called mNARX, manifold-based NARX)
- A functional version of NARX allows us to consider a time-window instead of number of time steps, and use principal component analysis (or other extraction techniques, *e.g.* moving averages, Fourier coefficients, etc.) to define the features
- The obtained surrogates can be post-process at low cost for reliability analysis ... but also for other purposes

Questions ?

Chair of Risk, Safety & Uncertainty Quantification

www.rsuq.ethz.ch

The Uncertainty Quantification Software

www.uqlab.com

www.uqpylab.uq-cloud.io

UQ[py]Lab

The Uncertainty Quantification Community

www.uqworld.org

Thank you very much for your attention !