technische universität dortmund

Chair for Reliability Engineering

Simulation methods for analysis and design in stochastic linear dynamics

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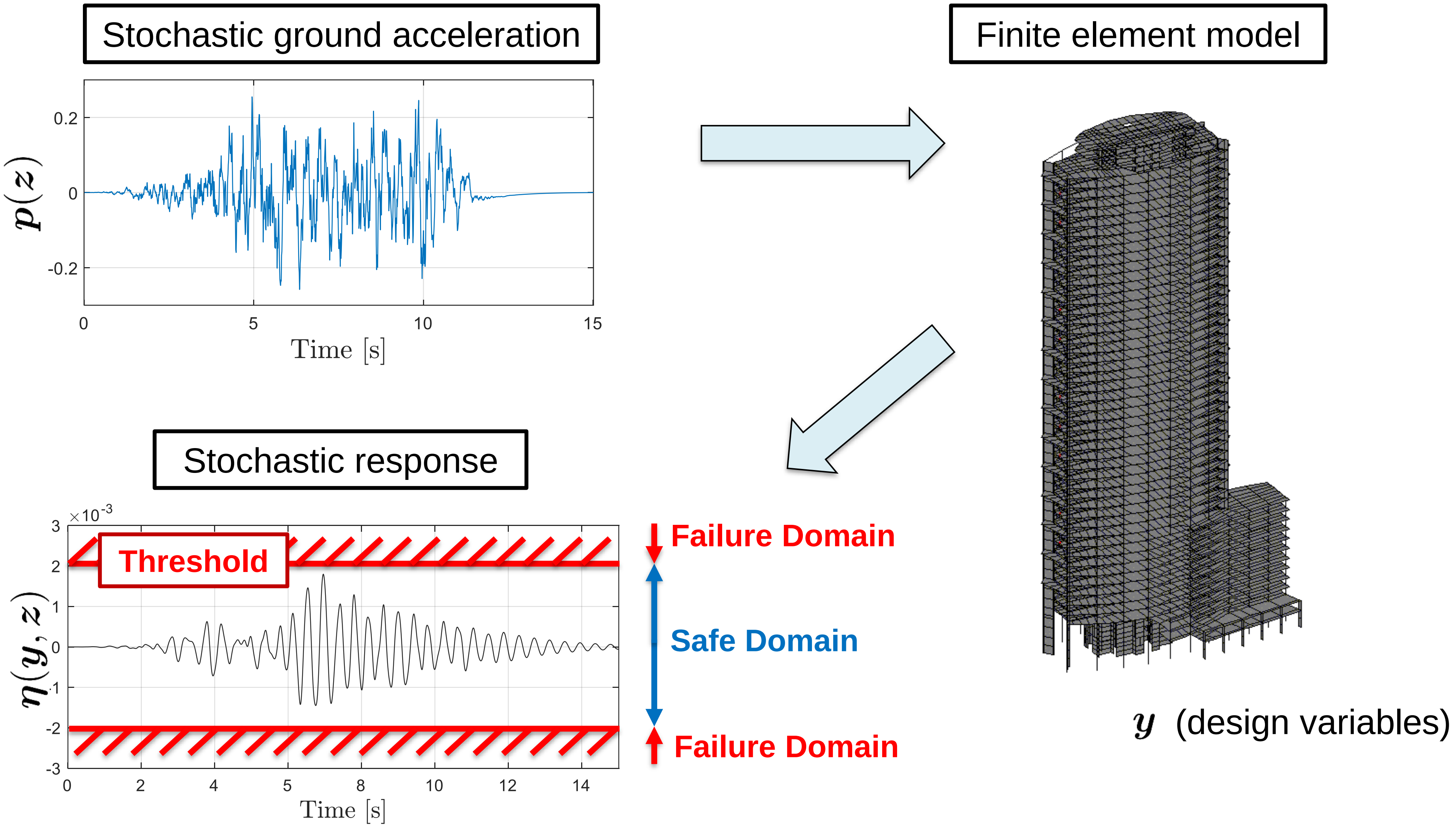
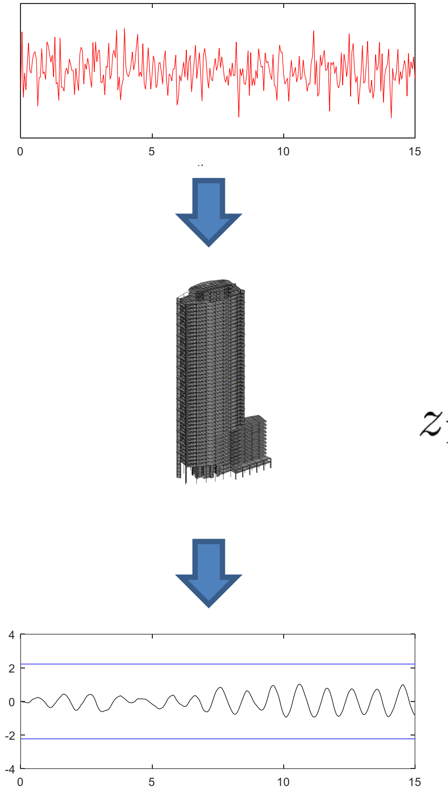
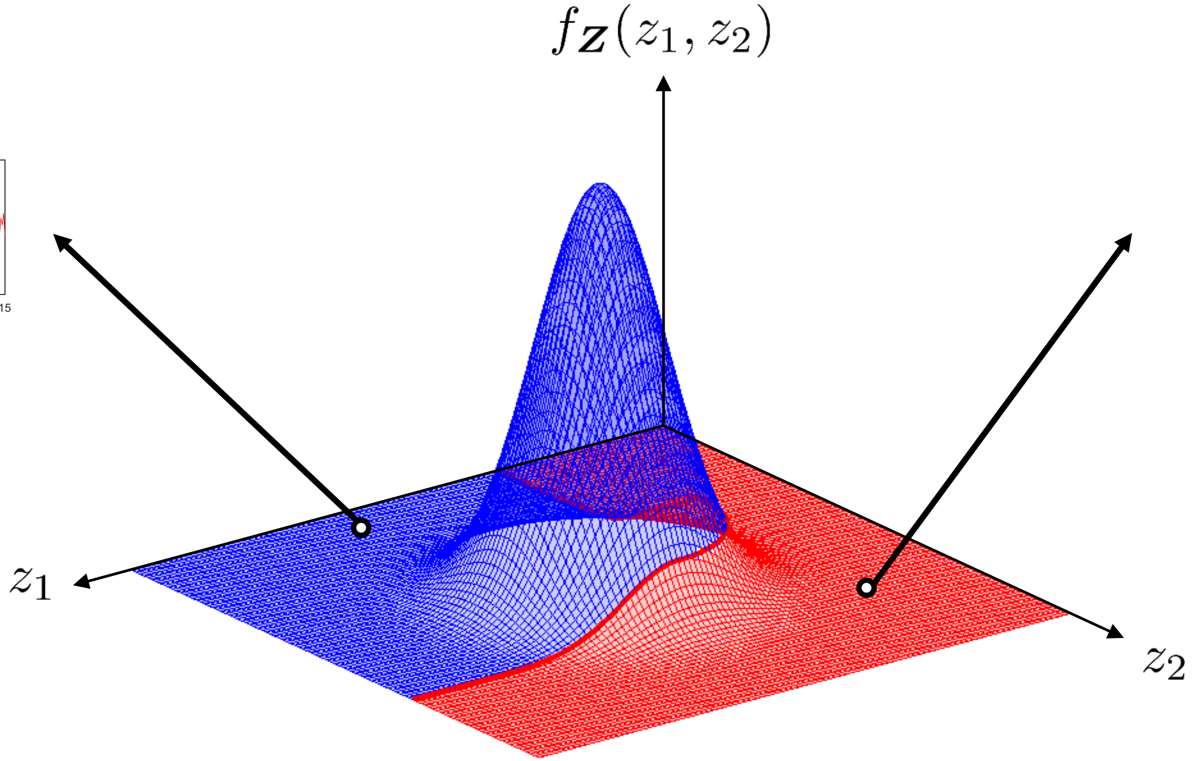
Motivation

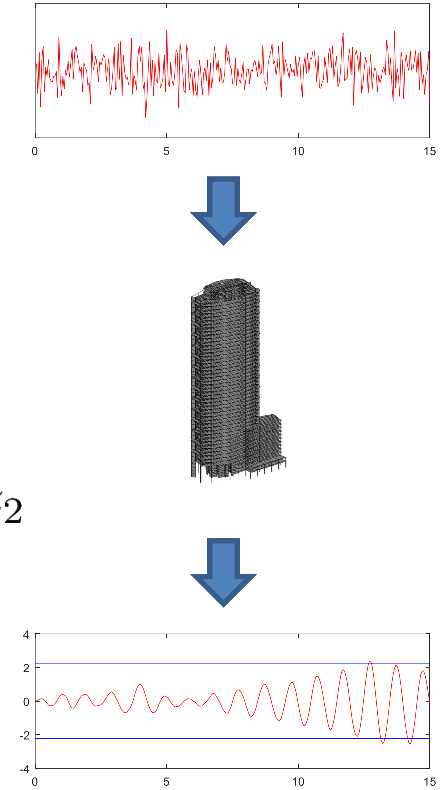


Stochastic ground acceleration
0.2
0
-0.2
p(z)
0
5
10
15
Time [s]
Finite element model
Stochastic response
×10^-3
3
2
1
0
-1
-2
-3
η(y, z)
Threshold
0
2
4
5
8
10
12
14
Time [s]
Failure Domain
Safe Domain
Failure Domain
y (design variables)

Motivation



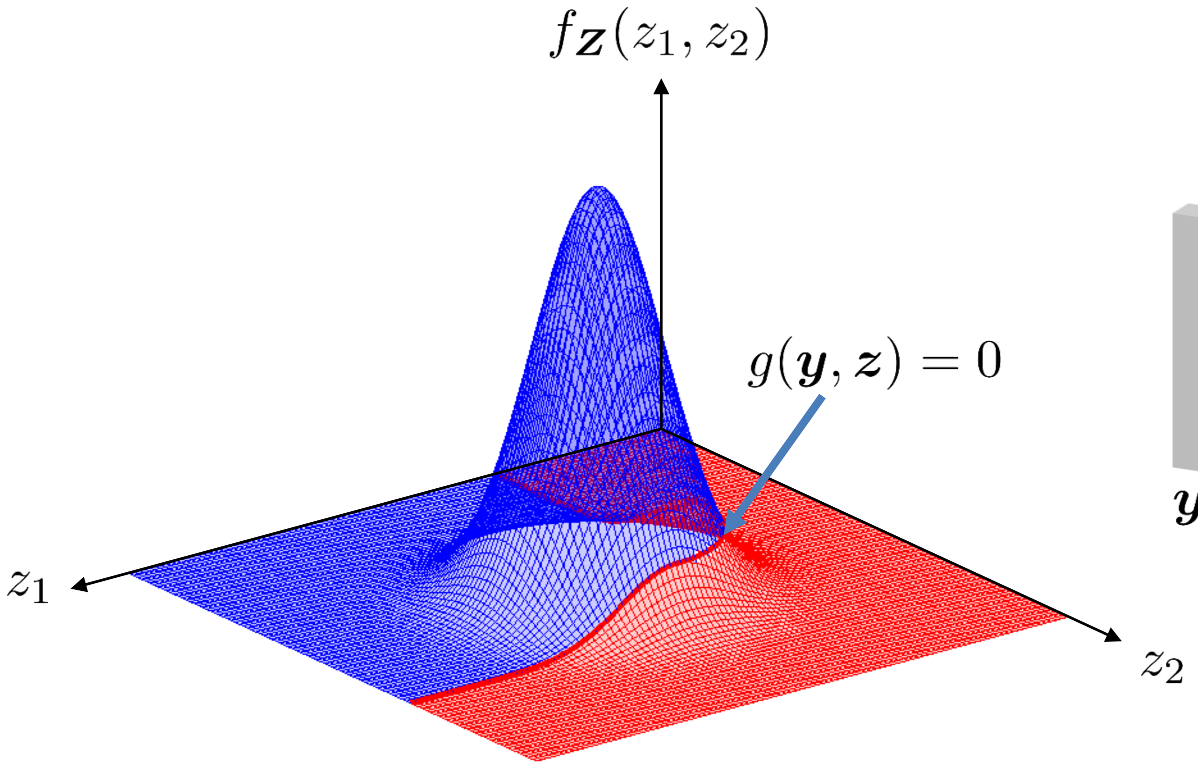


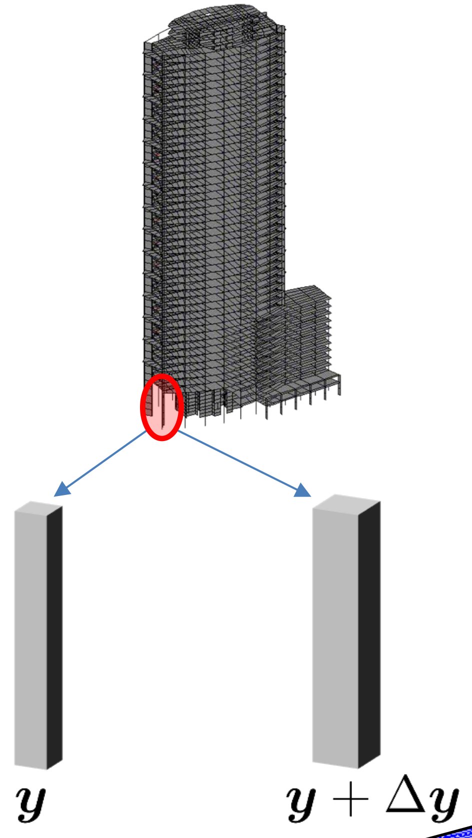


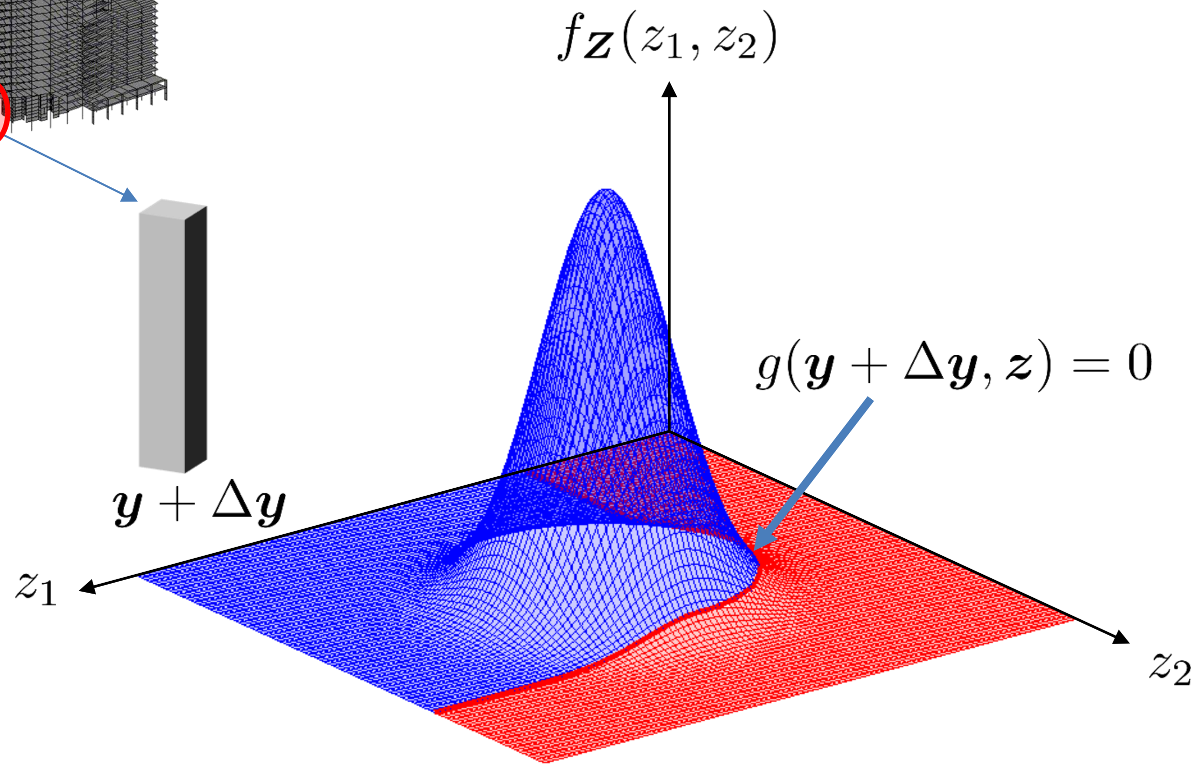
$$p_F(\boldsymbol{y}) = \int_{\boldsymbol{z} \in \Omega_z} I_F(\boldsymbol{y}, \boldsymbol{z}) f_{\boldsymbol{Z}}(\boldsymbol{z}) d\boldsymbol{z}$$

Challenge 1: probability estimation (dimensionality, non-Gaussianity, multiple failure domains, nonlinearities)

Motivation



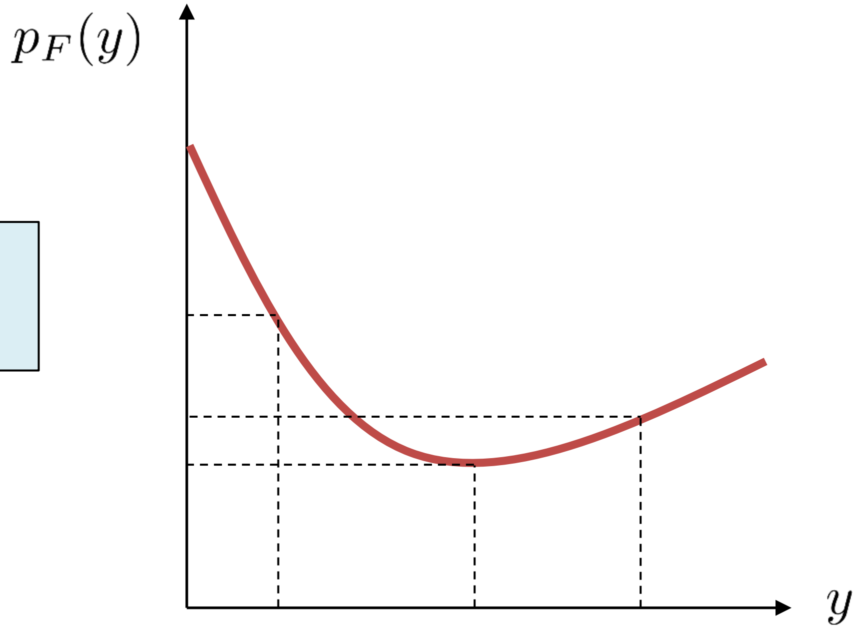




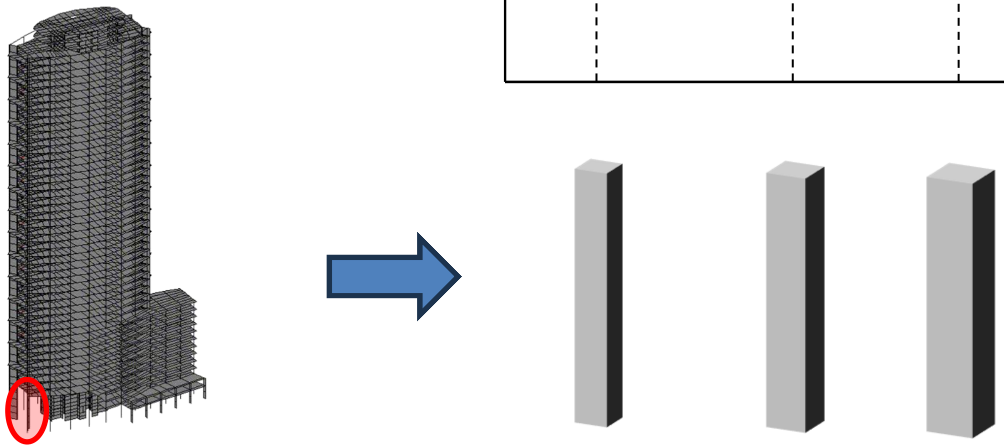
$$\frac{\partial p_F(\boldsymbol{y})}{\partial y_j} = \frac{\partial}{\partial y_j}\left(\int_{\boldsymbol{z}\in\Omega_z} I_F(\boldsymbol{y},\boldsymbol{z}) f_{\boldsymbol{Z}}(\boldsymbol{z}) d\boldsymbol{z}\right)$$

Challenge 2: sensitivity

Motivation

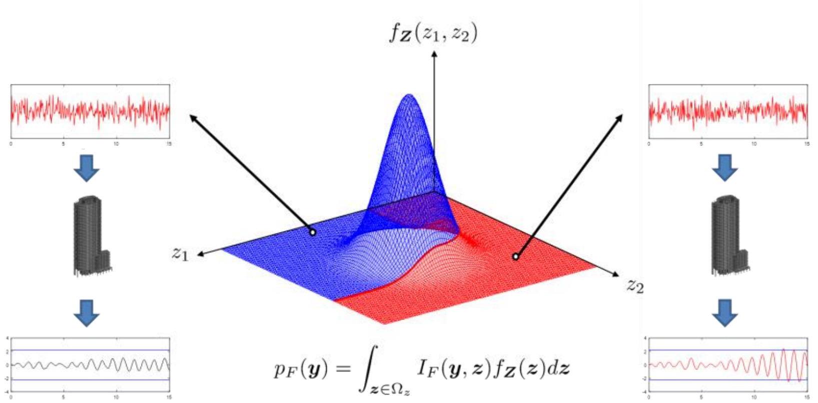


Challenge 3: optimal design

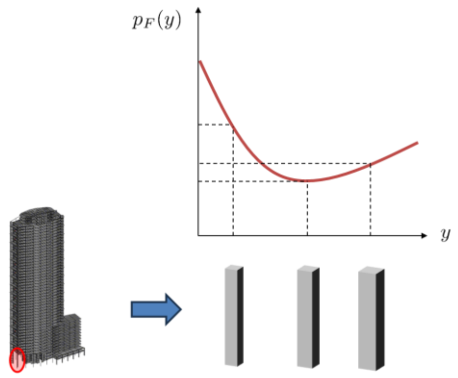


Motivation

Challenge 1 & 2: probability and sensitivity



Challenge 3: optimal design



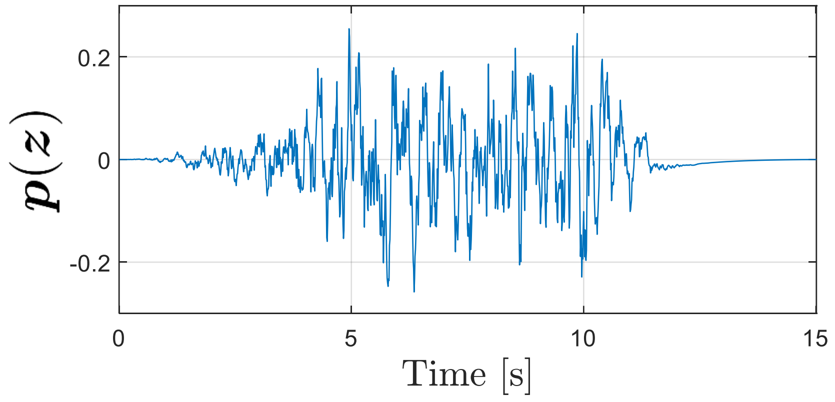
Focus:

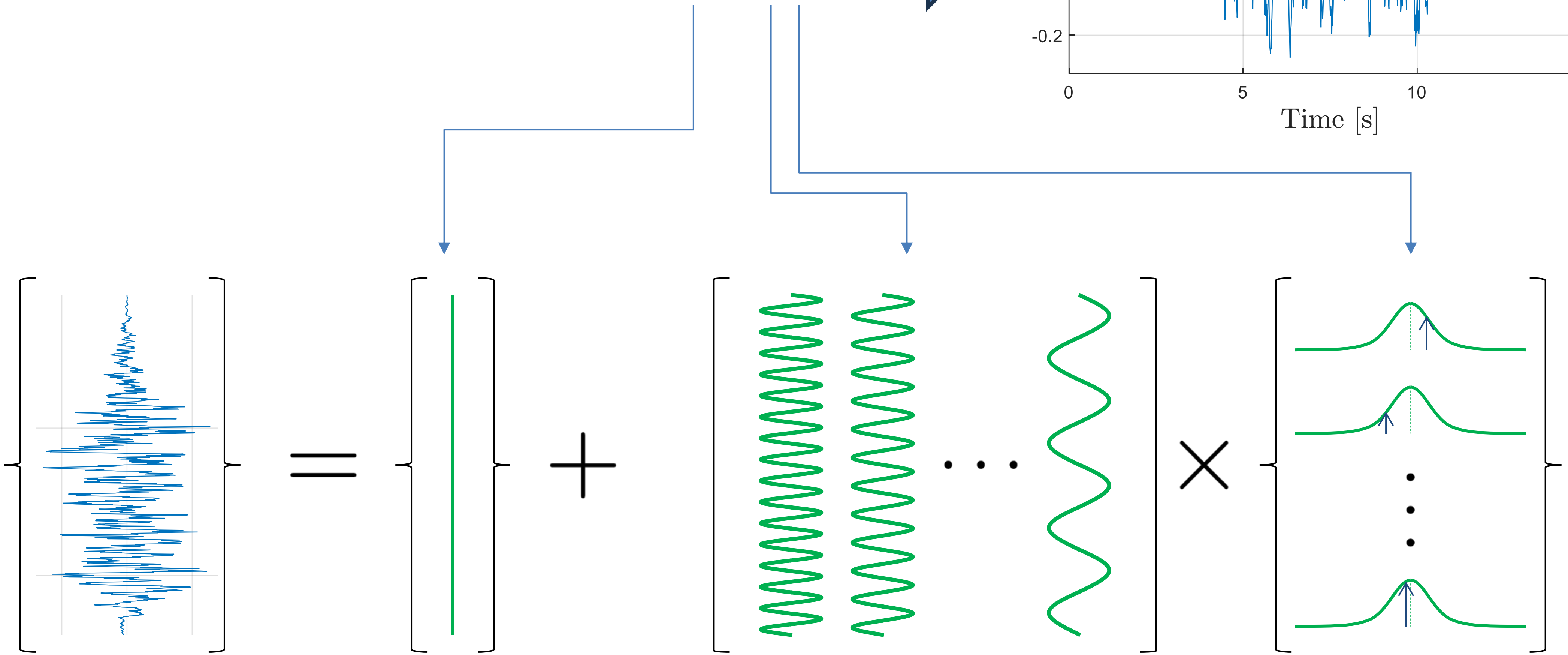
- Linear structural systems
- Gaussian stochastic loading

Challenge 1: Failure Probability

- Stochastic loading modelled as Gaussian process (assumption: $\boldsymbol{\mu} = \mathbf{0}$)

$$\boldsymbol{p}(\boldsymbol{z}) = \boldsymbol{\mu} + \boldsymbol{B}\boldsymbol{z}$$



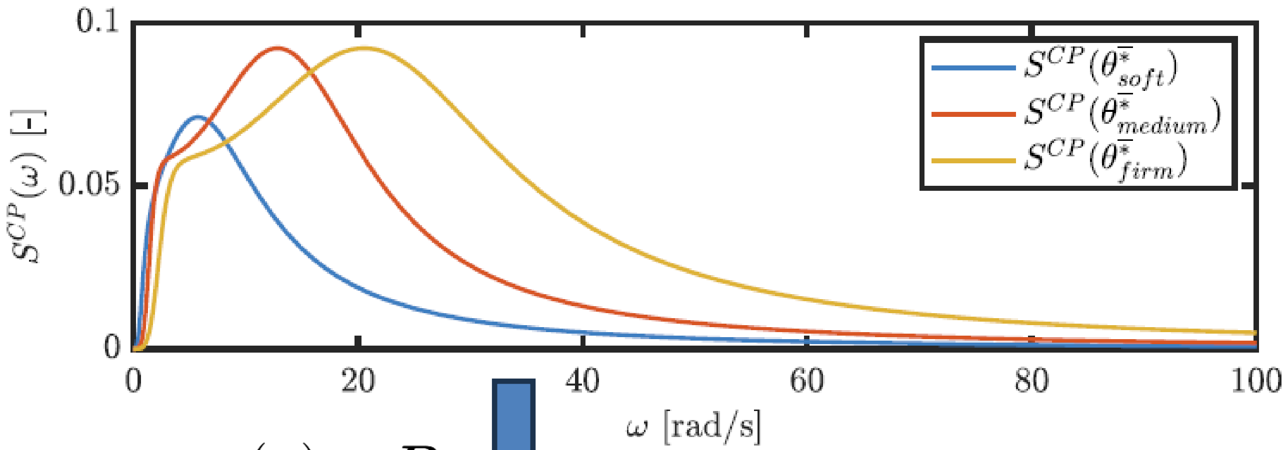


Challenge 1: Failure Probability

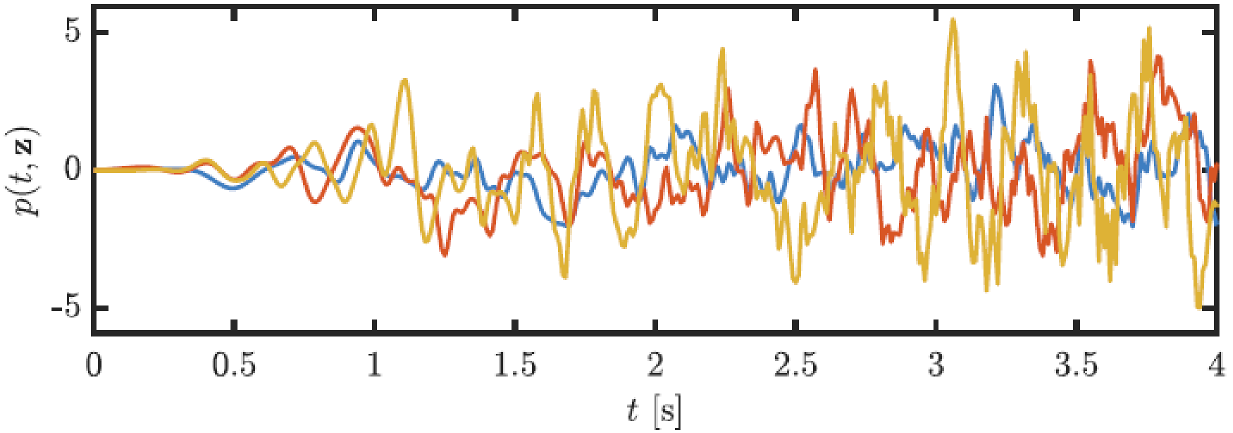
- <u>Stochastic loading</u>: Clough-Penzien power spectrum

$$S^{\mathrm{CP}}(\omega) = \frac{\omega_g^4 + (2\zeta_g\omega_g\omega)^2}{\left(\omega_g^2 - \omega^2\right)^2 + (2\zeta_g\omega_g\omega)^2} \cdot \frac{\omega^4}{\left(\omega_f^2 - \omega^2\right)^2 + (2\zeta_f\omega_f\omega)^2} \cdot S_0$$

Soil type	ω_g [rad/s]	ζ_g	ω_f [rad/s]	ζ_f
Firm	8π	0.60	0.8π	0.60
Medium	5π	0.60	0.5π	0.60
Soft	2.4π	0.85	0.24π	0.85

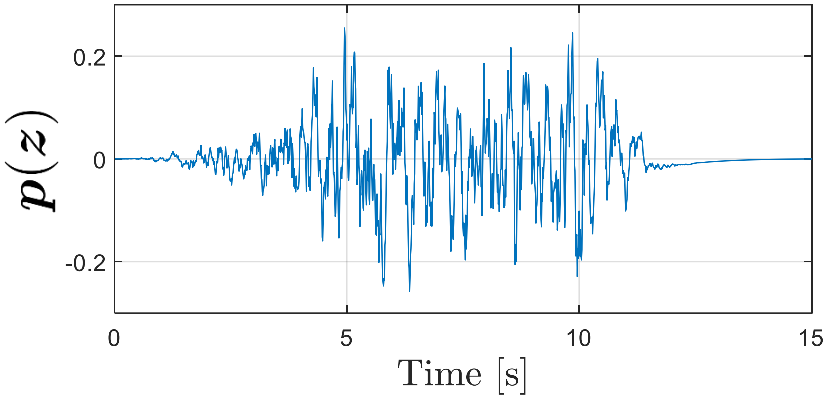


$$\boldsymbol{p}(\boldsymbol{z}) = \boldsymbol{B}\boldsymbol{z}$$

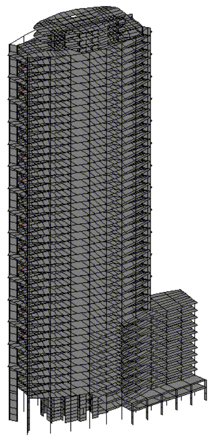


Challenge 1: Failure Probability

- <u>Structural response</u>: convolution integral

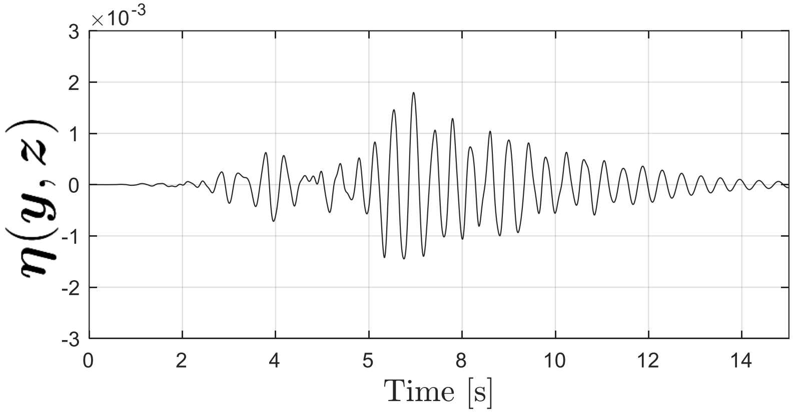


$$\boldsymbol{p}(\boldsymbol{z}) = \boldsymbol{B}\boldsymbol{z}$$



Finite element model

$\boldsymbol{y}$ (design variables)



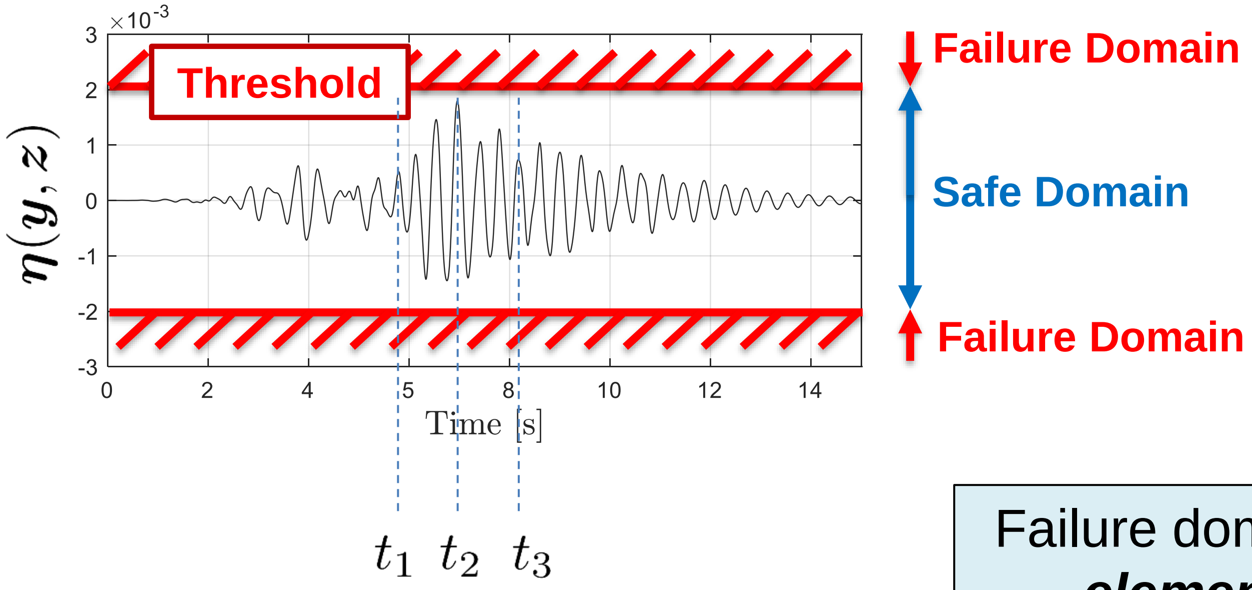
$$\eta(t, \boldsymbol{y}, \boldsymbol{z}) = \int_0^t h(\boldsymbol{y}, t - \tau) p(\tau, \boldsymbol{z}) d\tau$$

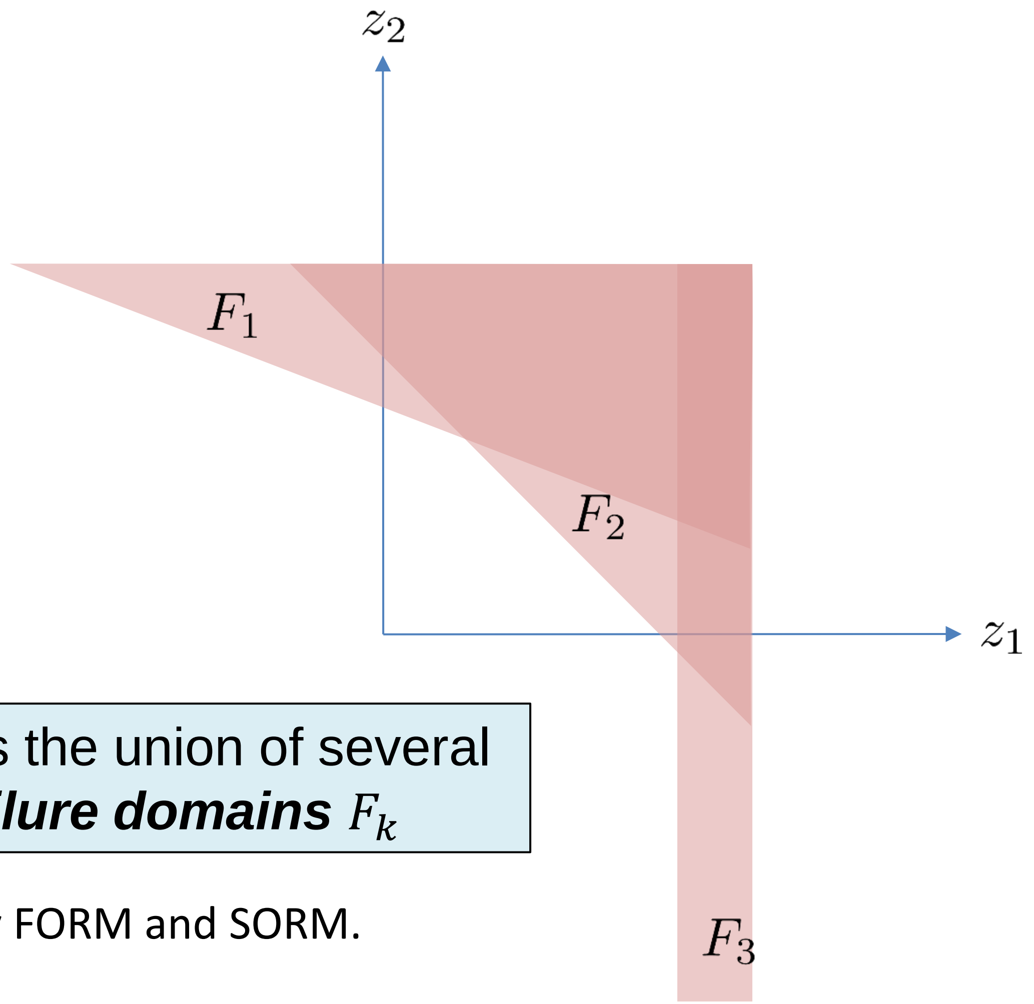
$$\boldsymbol{\eta}(\boldsymbol{y}, \boldsymbol{z}) = \underbrace{\boldsymbol{H}(\boldsymbol{y})}_{\text{spectral properties}} \underbrace{\boldsymbol{B}\boldsymbol{z}}_{\text{loading}} = \boldsymbol{A}(\boldsymbol{y})\boldsymbol{z}$$

Challenge 1: Failure Probability

- <u>Geometry of the failure domain</u>* in stochastic linear dynamics under Gaussian force

$$\eta(\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{A}(\boldsymbol{y})\boldsymbol{z}$$





Failure domain F is the union of several ***elementary failure domains*** F_k

*Der Kiureghian, A. The geometry of random vibrations and solutions by FORM and SORM. Probabilistic Engineering Mechanics, 2000 , 15 , 81-90

Challenge 1: Failure Probability

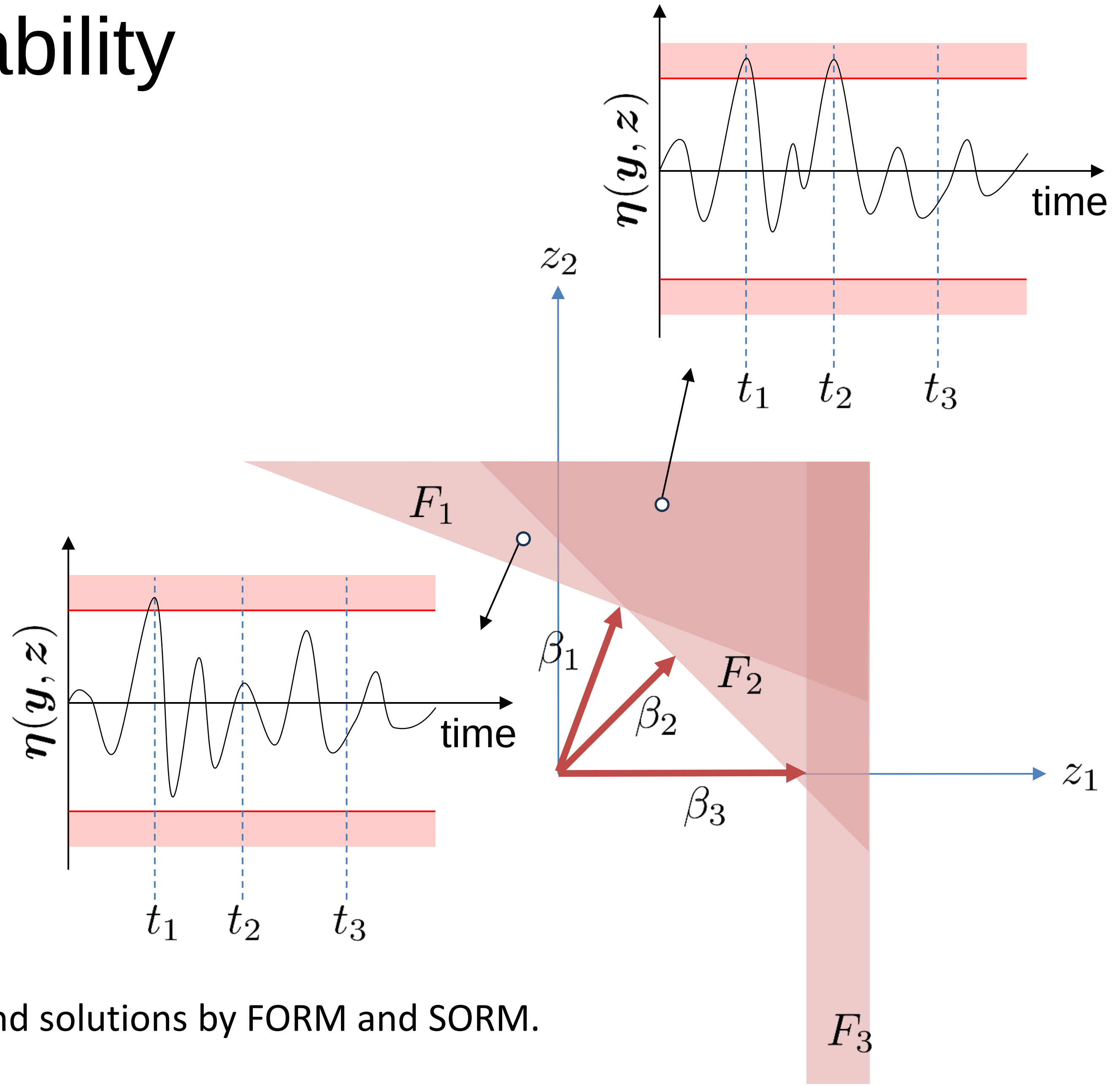
- Geometry of the failure domain in stochastic linear dynamics under Gaussian force
  - Reliability for elementary failure domain known in closed form

$$P[F_k] = \Phi(-\beta_k)$$

  - Bounds for failure probability*

$$p_F \leq \hat{P}_F = \Phi(-\beta_1) + \Phi(-\beta_2) + \Phi(-\beta_3)$$

Upper bound, ignores **interactions**



*Der Kiureghian, A. The geometry of random vibrations and solutions by FORM and SORM. Probabilistic Engineering Mechanics, 2000 , 15 , 81-90

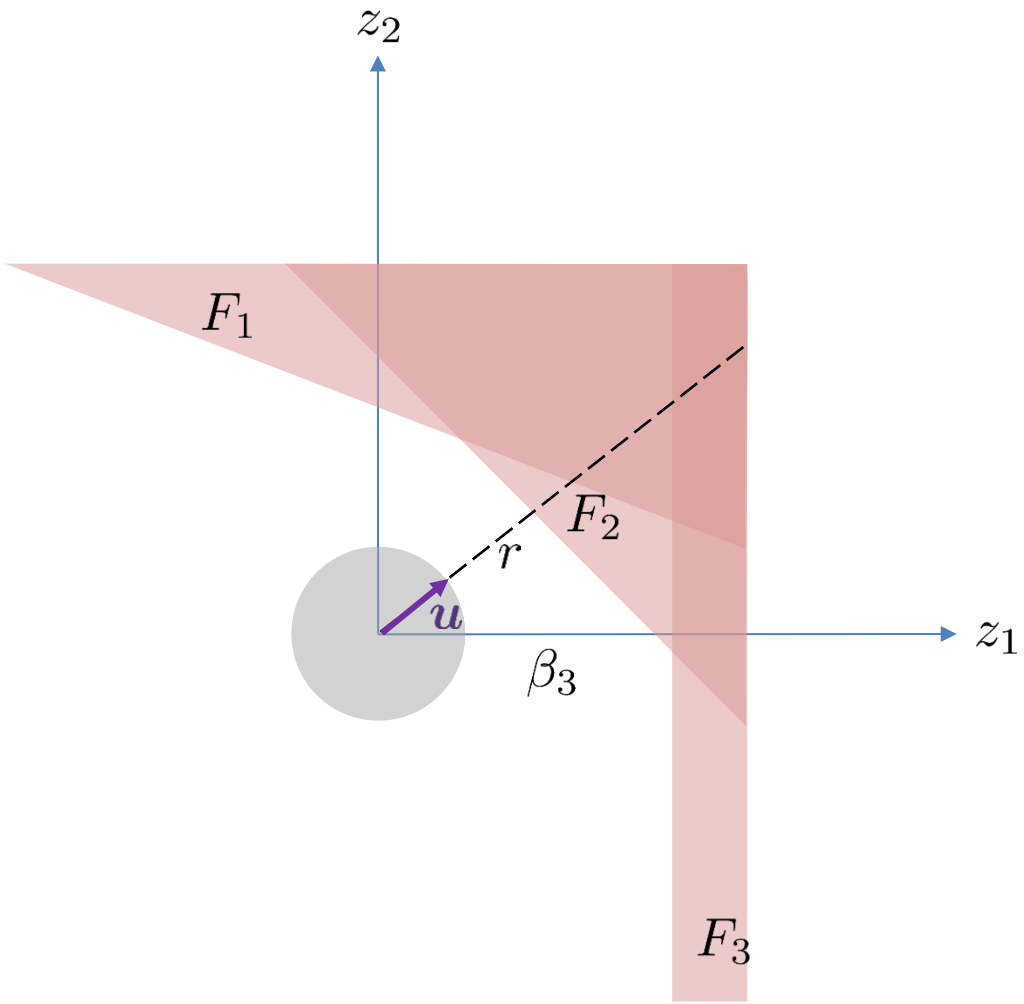
Challenge 1: Failure Probability

- **Coordinate change** of the failure probability integral and **importance sampling density function**

$$p_F(\boldsymbol{y}) = \int_{\boldsymbol{z}\in\Omega_z} I_F(\boldsymbol{y},\boldsymbol{z}) f_{\boldsymbol{Z}}(\boldsymbol{z}) d\boldsymbol{z}$$

$$p_F = \int_{\boldsymbol{u}\in\Omega_U} \int_0^\infty 2r I_F\left(\boldsymbol{y}, r\boldsymbol{u}\right) f_{R^2}\left(r^2\right) f_{\boldsymbol{U}}\left(\boldsymbol{u}\right) dr d\boldsymbol{u}$$

$$p_F = \int_{\boldsymbol{u}\in\Omega_U} \int_0^\infty 2r I_F\left(\boldsymbol{y}, r\boldsymbol{u}\right) f_{R^2}\left(r^2\right) \frac{f_{\boldsymbol{U}}\left(\boldsymbol{u}\right)}{f_{\boldsymbol{U}}^{\mathrm{IS}}\left(\boldsymbol{u}\right)} f_{\boldsymbol{U}}^{\mathrm{IS}}\left(\boldsymbol{u}\right) dr d\boldsymbol{u}$$

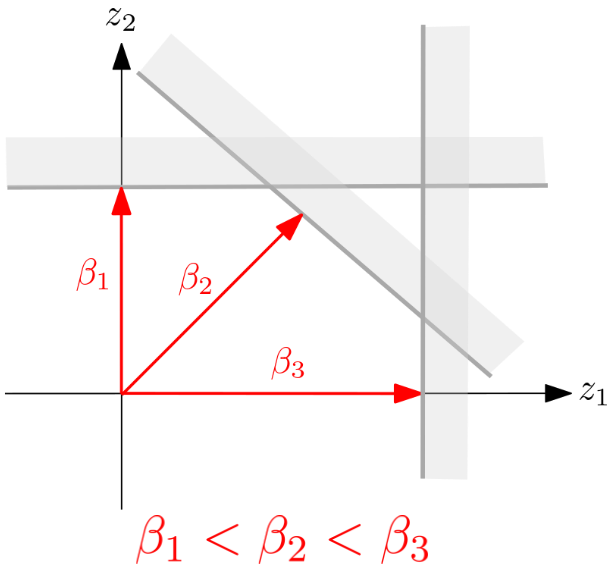


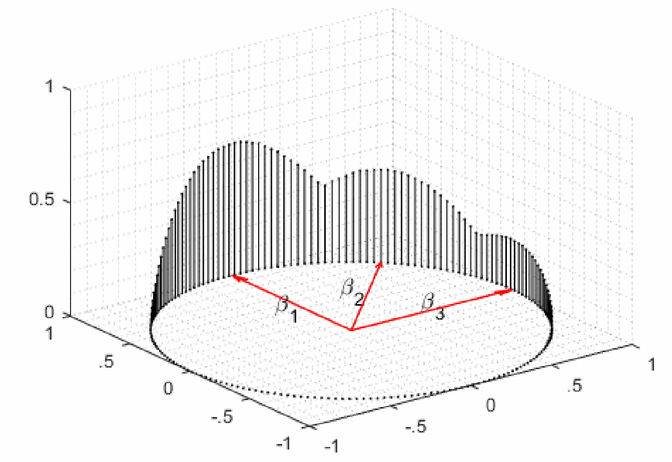
Challenge 1: Failure Probability

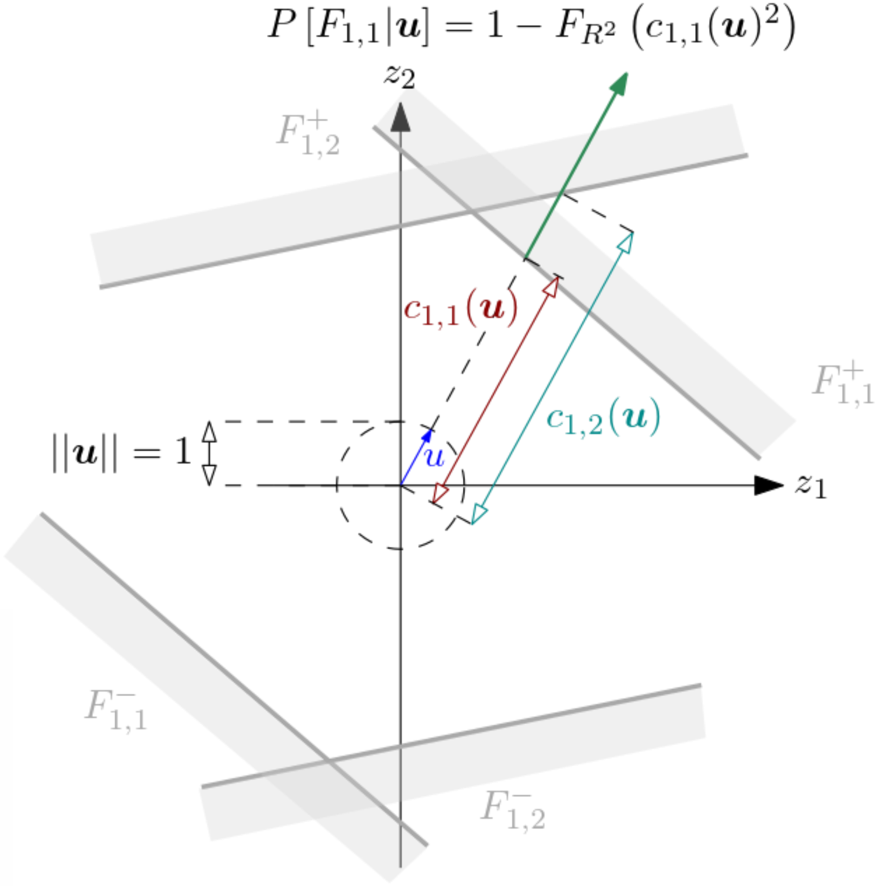
- **Importance sampling density function**

$$f_U^{\mathrm{IS}}(\boldsymbol{u}) = \sum_{i=1}^{n_\eta} \sum_{k=1}^{n_T} w_{i,k} f_U(\boldsymbol{u}/F_{i,k})$$

$$= \frac{f_U(\boldsymbol{u})}{\hat{P}_F} \sum_{i=1}^{n_\eta} \sum_{k=1}^{n_T} \left(1 - F_{R^2}\left(c_{i,k}(\boldsymbol{u})^2\right)\right)$$



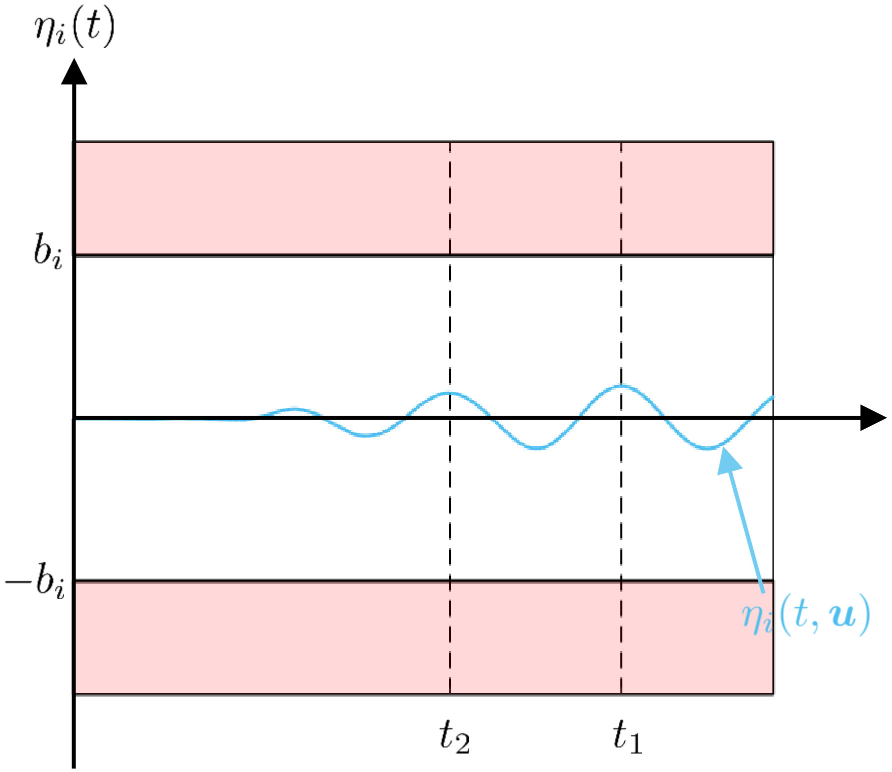




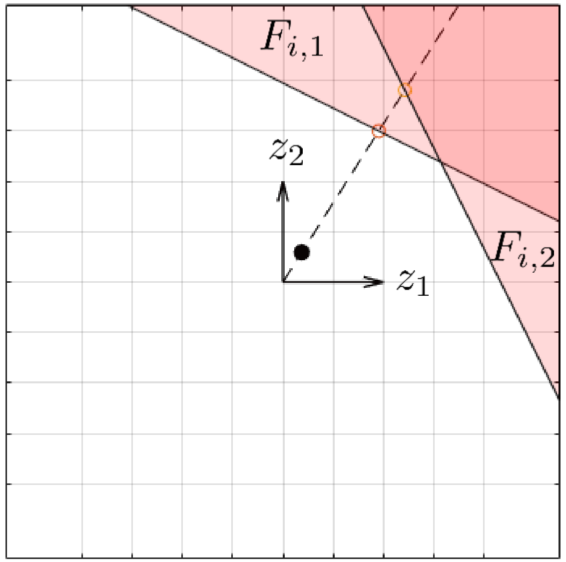
*O. Ditlevsen, P. Bjerager, R. Olesen, and A. M. Hasofer, “Directional simulation in Gaussian processes,” Probabilistic Engineering Mechanics, vol. 3, Art. no. 4, 1988.

Challenge 1: Failure Probability

- Interpretation of distances $c_{i,k}$



Response Amplification



Directional Exploration

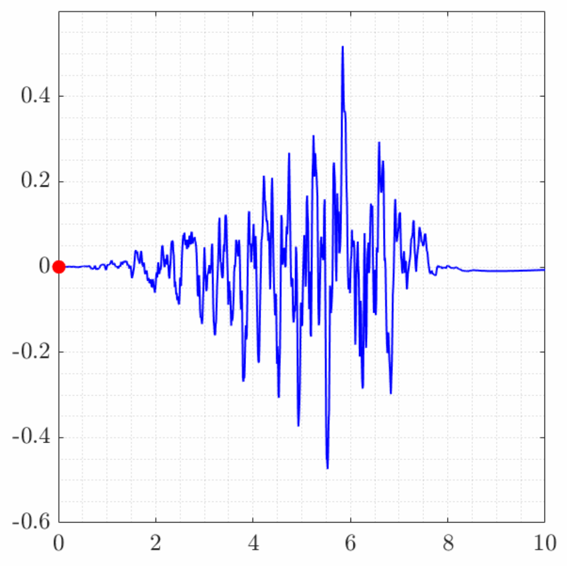
$$\tilde{p}_F^{\mathrm{DIS}} \approx \hat{P}_F \left(\frac{1}{N} \sum_{j=1}^{N} \frac{1 - f_{R^2}\left(c_{\min}\left(\boldsymbol{u}^{(j)}\right)^2\right)}{\sum_{i=1}^{n_\eta} \sum_{k=1}^{n_T} 1 - f_{R^2}\left(c_{i,k}\left(\boldsymbol{u}^{(j)}\right)^2\right)} \right)$$

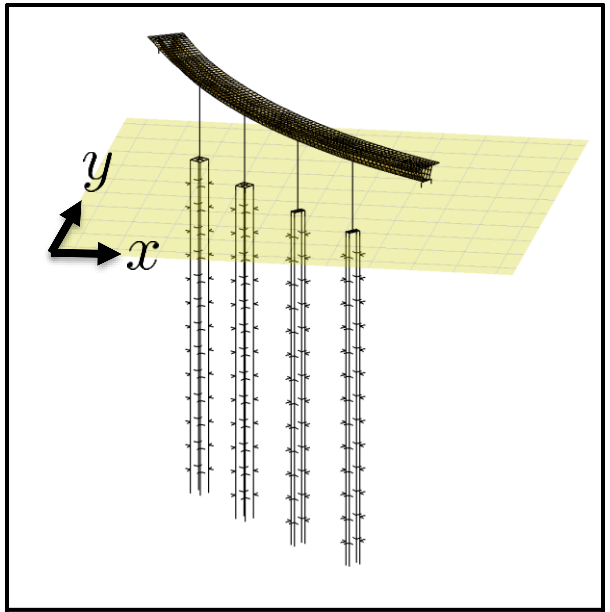
Upper bound

Value between 0 and 1, overlap

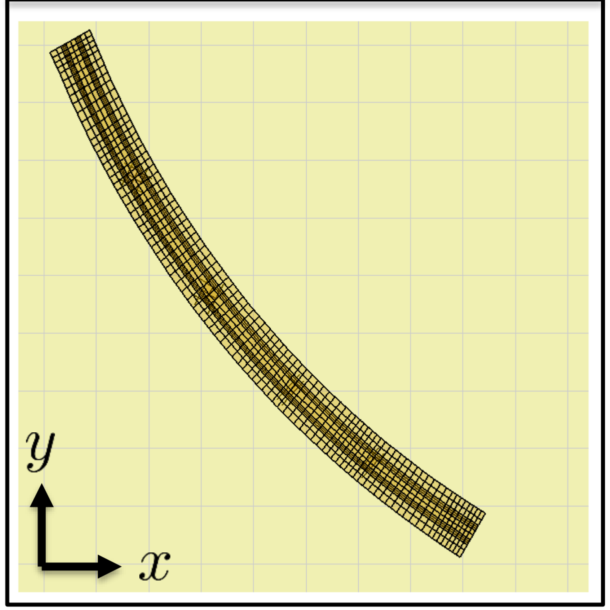
Challenge 1: Failure Probability

- Example:
  - Bridge subjected to stochastic ground acceleration, low vibration level, linear elastic range.
  - FE model involves **10068** DOF's





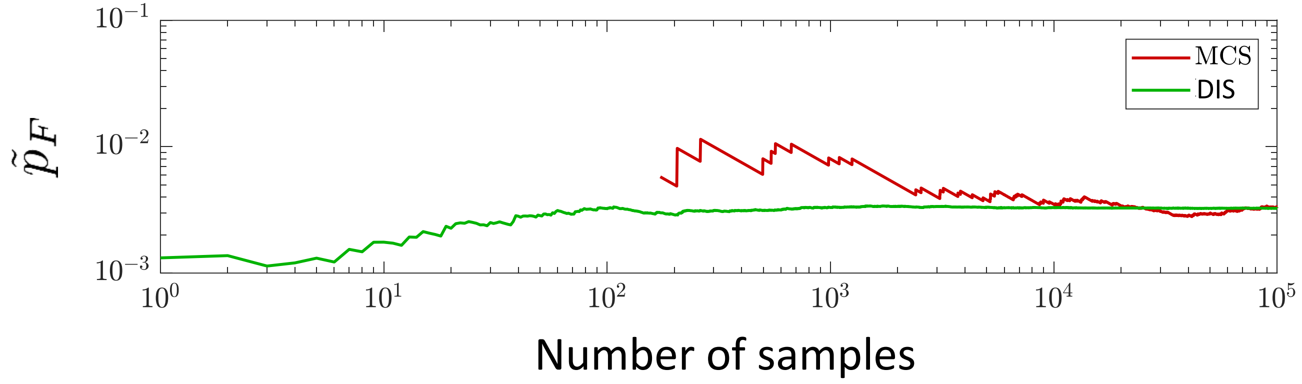
- Response of interest: first excursion, column drift (8 responses of interest)

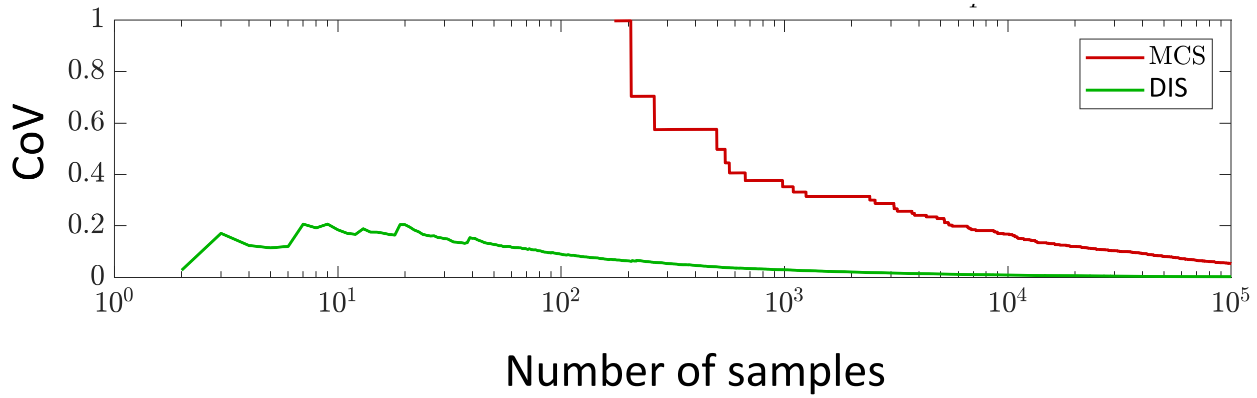


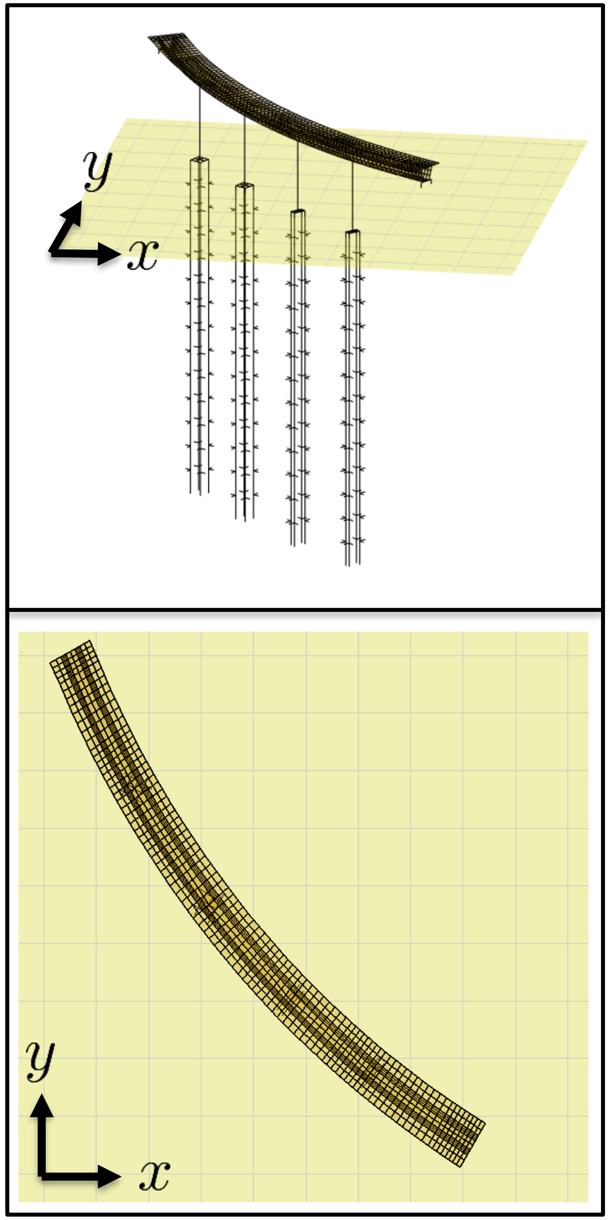
- Uncertainty:
  - Stochastic acceleration comprises **1001 random variables**
  - Discrete time representation involves **8008 elementary failure domains**

- Objective: Calculate first excursion probability

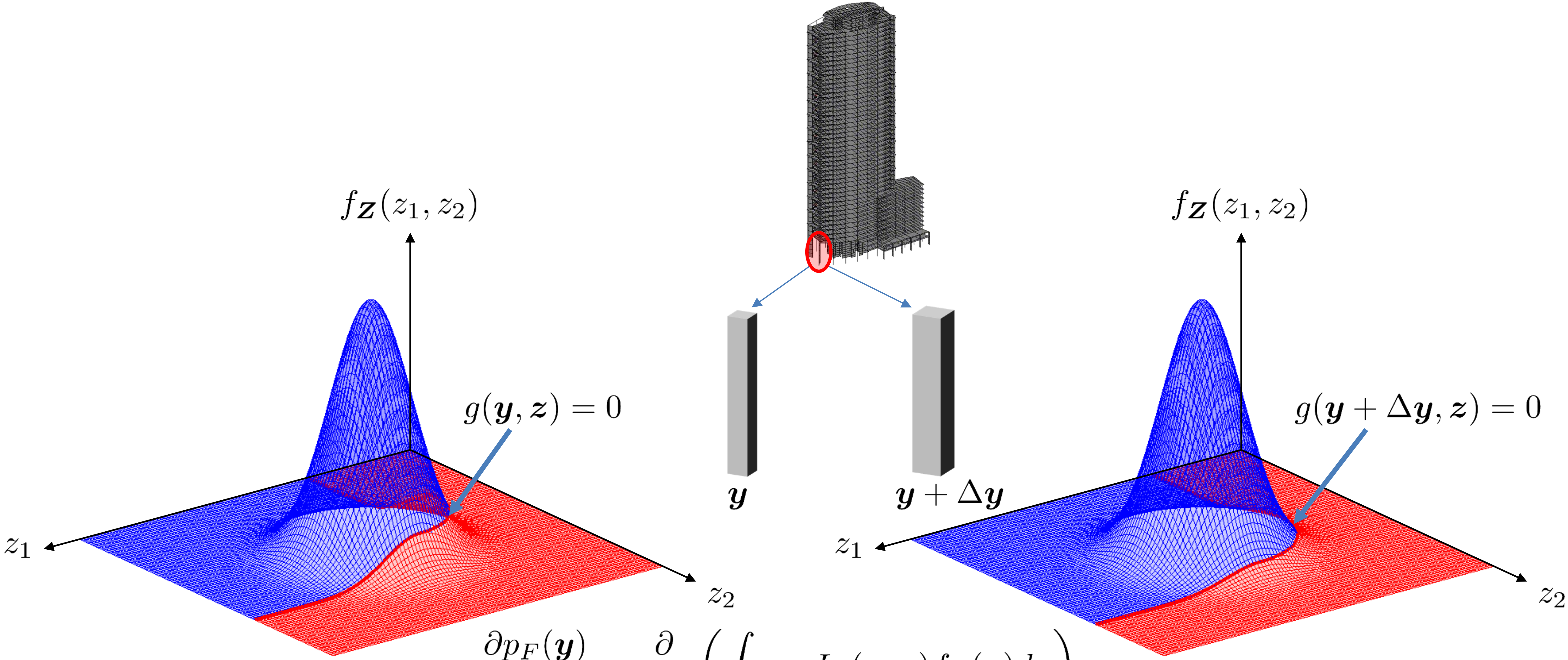
Challenge 1: Failure Probability





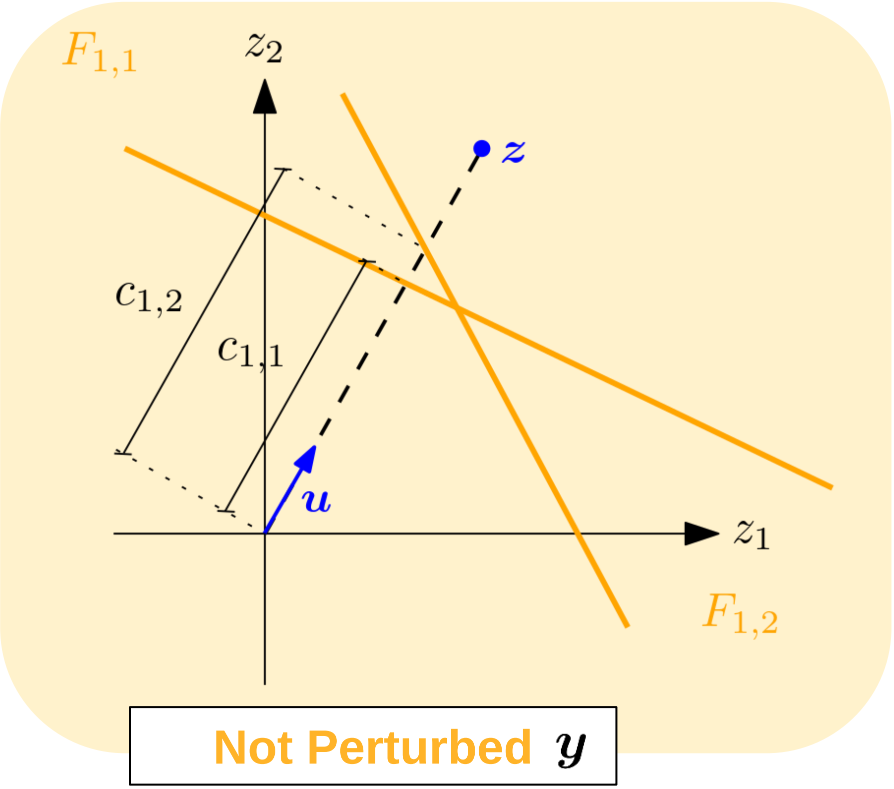


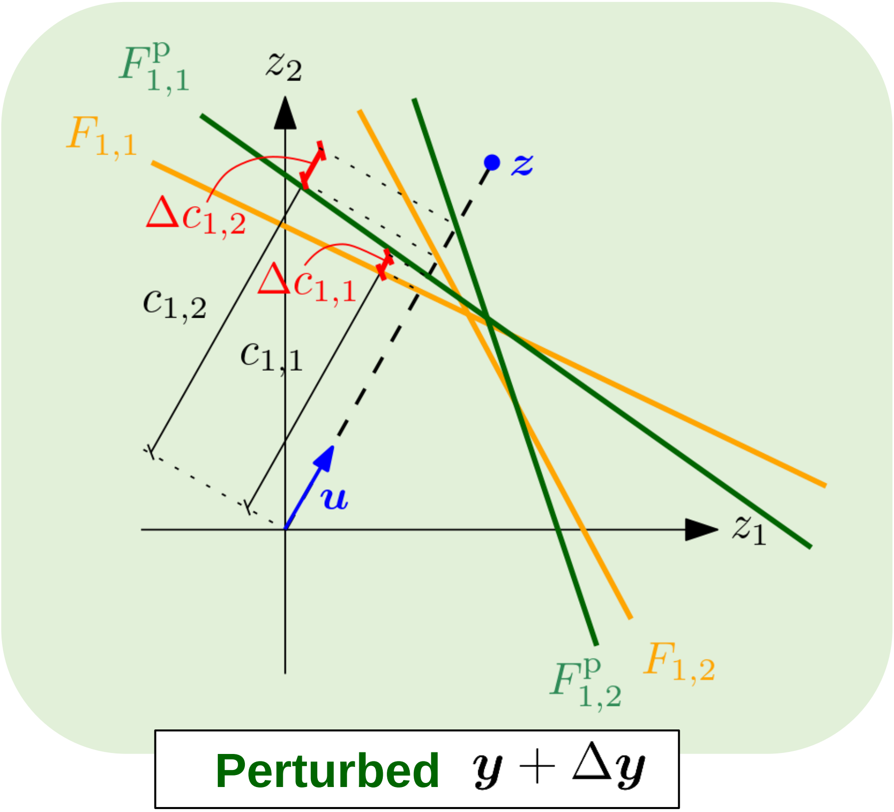
Challenge 2: Probability Sensitivity



$$\frac{\partial p_F(\boldsymbol{y})}{\partial y_j} = \frac{\partial}{\partial y_j} \left(\int_{\boldsymbol{z} \in \Omega_z} I_F(\boldsymbol{y}, \boldsymbol{z}) f_{\boldsymbol{Z}}(\boldsymbol{z}) d\boldsymbol{z} \right)$$

Challenge 2: Probability Sensitivity



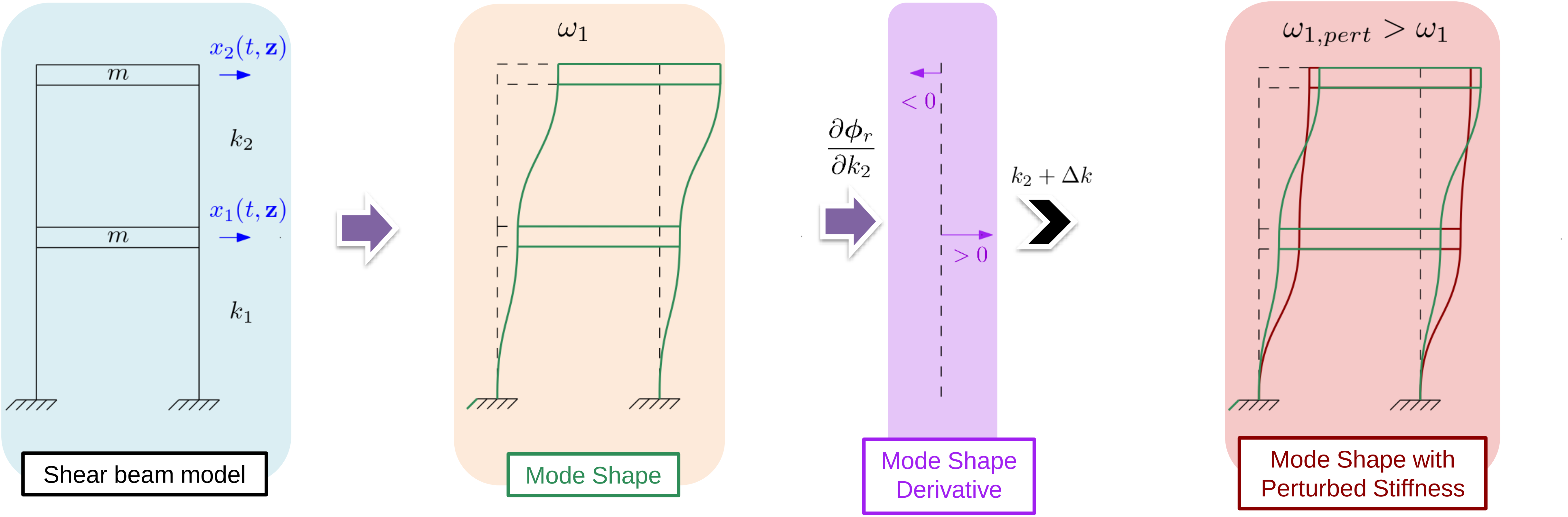


$$\Rightarrow \quad \frac{\partial \tilde{p}_F^{\mathrm{DIS}}}{\partial y_q} \approx -\frac{2\hat{P}_F}{N}\left(\sum_{j=1}^{N} \frac{f_{R^2}\left(c_{\min}\left(\boldsymbol{y},\boldsymbol{u}^{(j)}\right)^2\right) c_{\min}\left(\boldsymbol{y},\boldsymbol{u}^{(j)}\right) \frac{\partial c_{\min}\left(\boldsymbol{y},\boldsymbol{u}^{(j)}\right)}{\partial y_q}}{\sum_{i=1}^{n_\eta}\sum_{k=1}^{n_T} 1 - f_{R^2}\left(c_{i,k}\left(\boldsymbol{y},\boldsymbol{u}^{(j)}\right)^2\right)}\right)$$

Challenge 2: Probability Sensitivity

- Sensitivity of spectral properties*

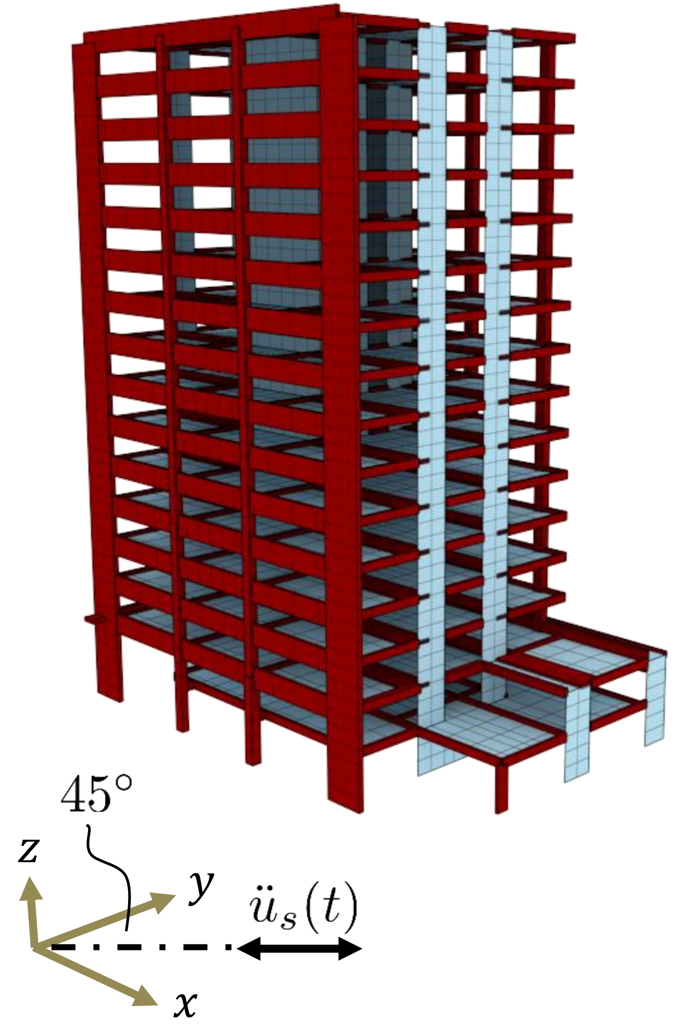
$$\begin{pmatrix} \boldsymbol{K}-\omega_r^2 \boldsymbol{M} & -\boldsymbol{M}\boldsymbol{\phi}_r \\ -\boldsymbol{\phi}_r^T \boldsymbol{M} & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial \phi_r}{\partial y_q} \\ \frac{\partial \omega_r}{\partial y_q} \end{pmatrix} = \begin{pmatrix} -\left(\frac{\partial \boldsymbol{K}}{\partial y_q} - \omega_r^2 \frac{\partial \boldsymbol{M}}{\partial y_q}\right) \\ 0.5\boldsymbol{\phi}_r^T \frac{\partial \boldsymbol{M}}{\partial y_q} \boldsymbol{\phi}_r \end{pmatrix}$$



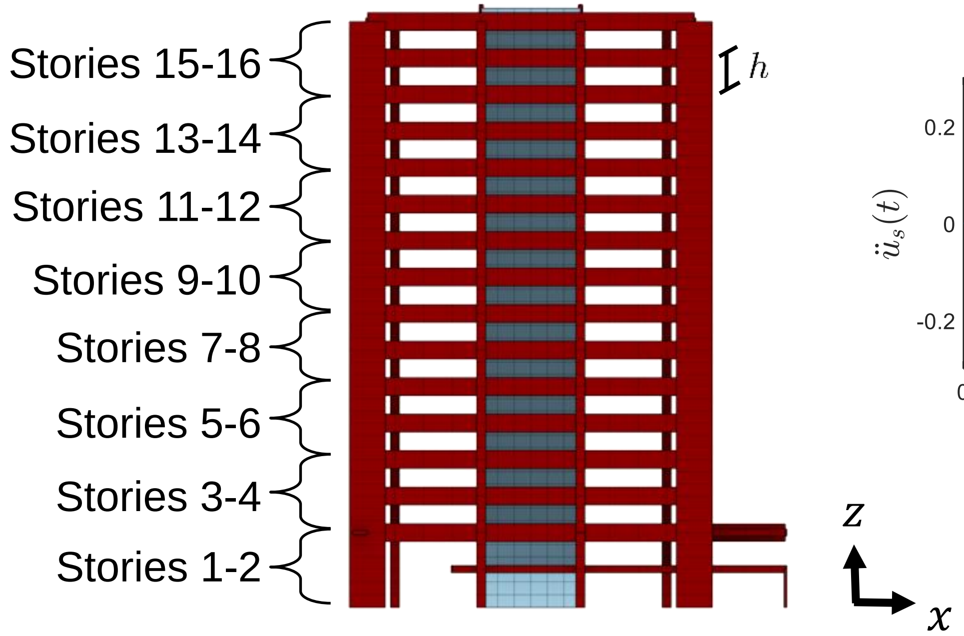
*I.-W. Lee and G.-H. Jung. An efficient algebraic method for the computation of natural frequency and mode shape sensitivity – part I, Distinct natural frequencies. *Computers & Structures,* 63(3):429-435.1997

Challenge 2: Probability Sensitivity

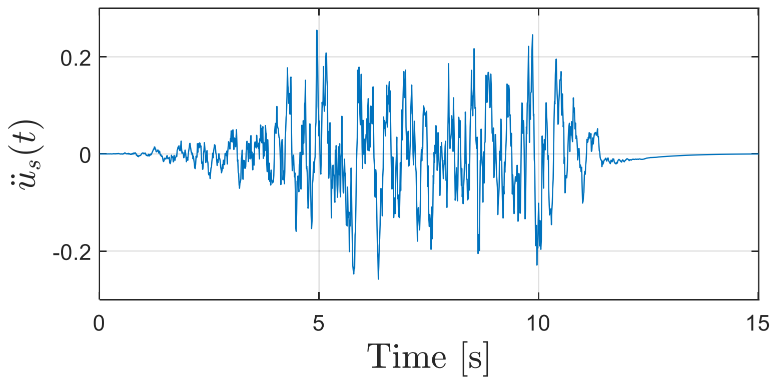
Perspective view



Elevation view

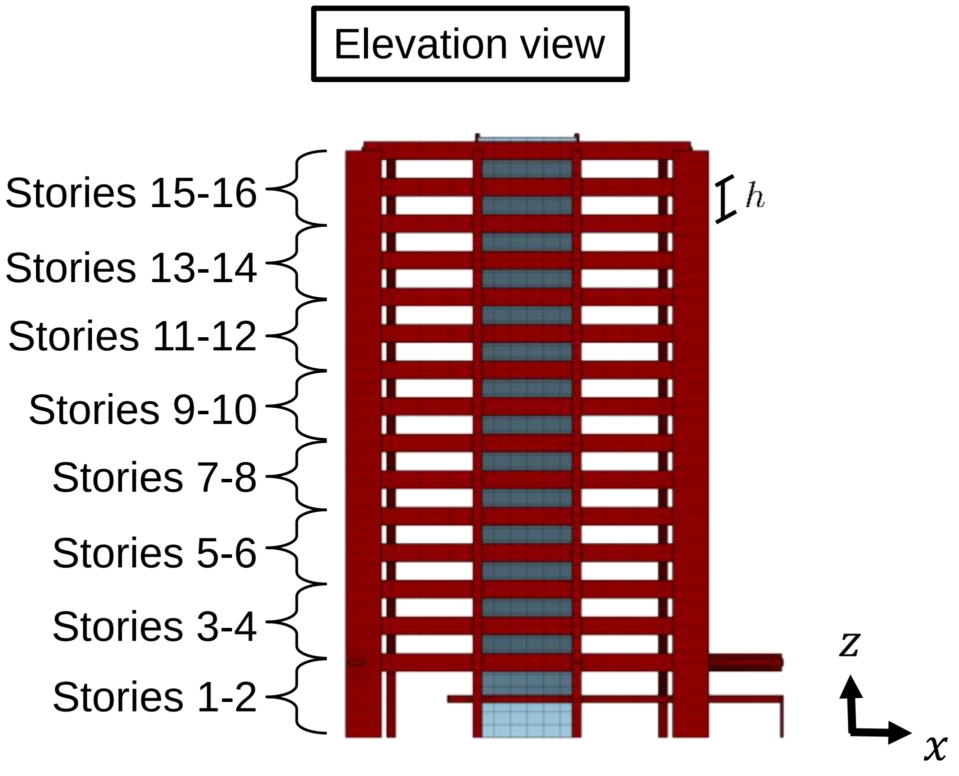


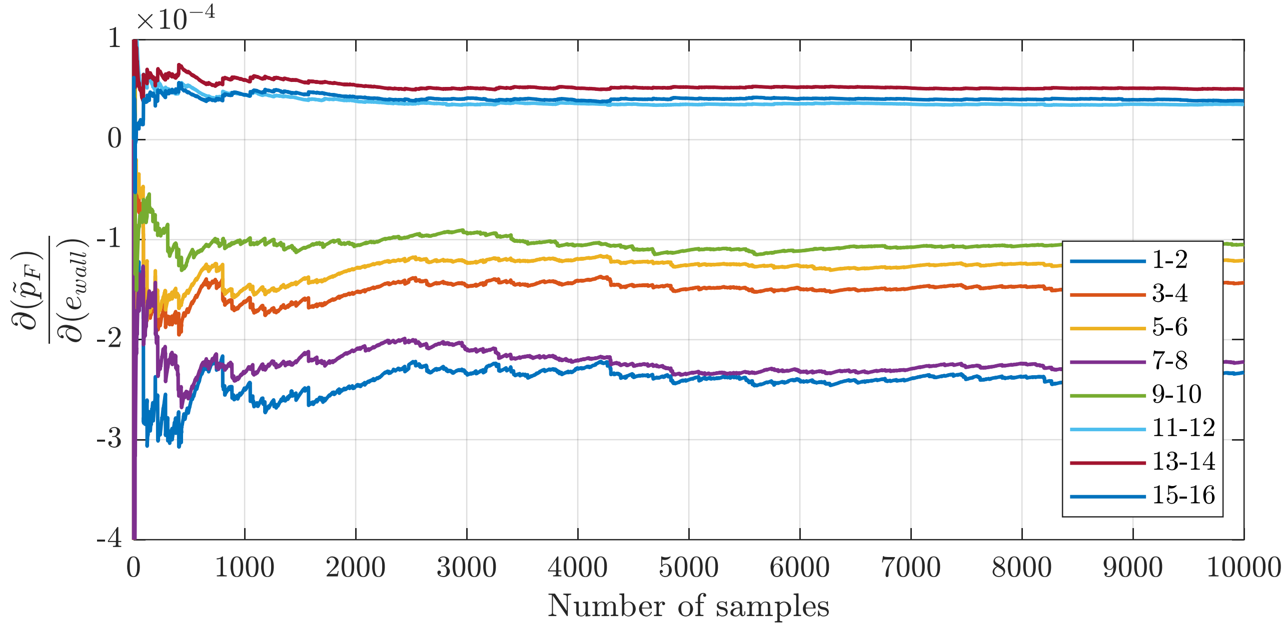
Realization of stochastic load

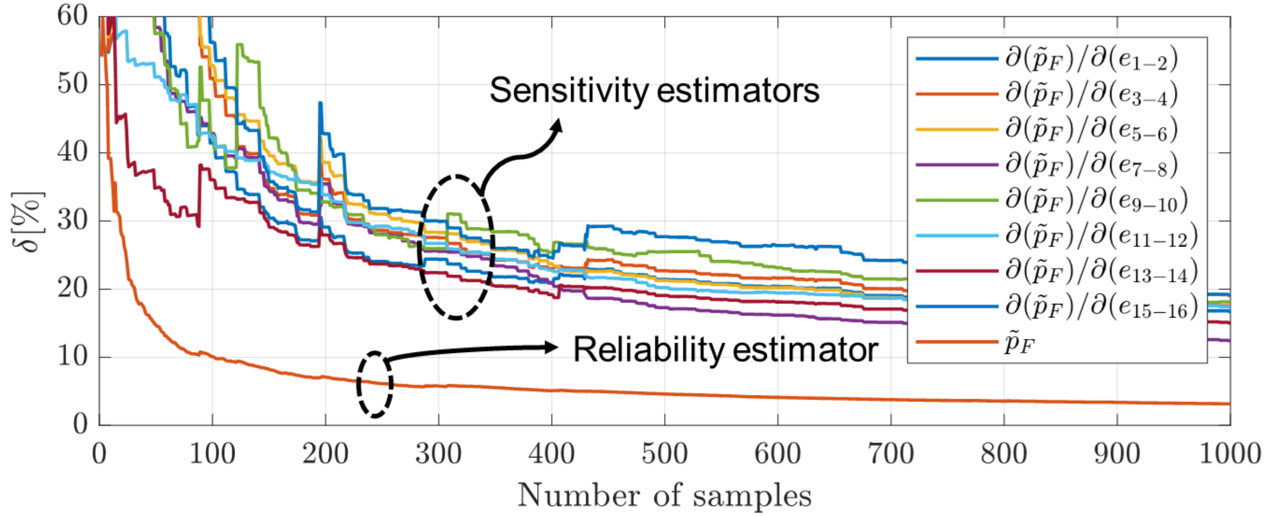


- ~ 30,000 DOFS, uncertain ground acceleration: discrete **filtered white noise**, 1001-time instants
- Failure criterion: Story drifts exceeds threshold in dir. x or y
- Objective: Calculate first excursion probability sensitivities with respect the thickness of the core shear walls

Challenge 2: Probability Sensitivity



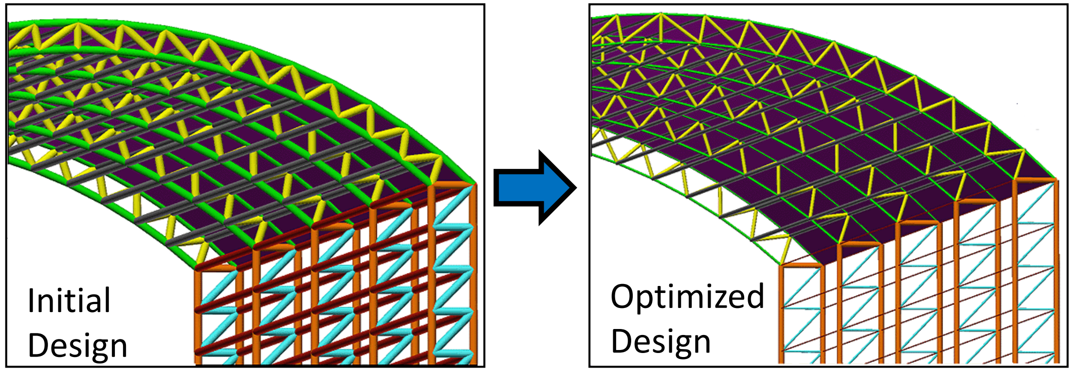




Challenge 3: Optimal Design

- <u>Failure probability</u>
  - Number between 0 and 1, quantifies level of safety
  - **What is the use of this?**

- **<u>Reliability-based optimization</u> (RBO)**
  - Minimizing costs of construction, maintenance and eventual failure considering uncertainties

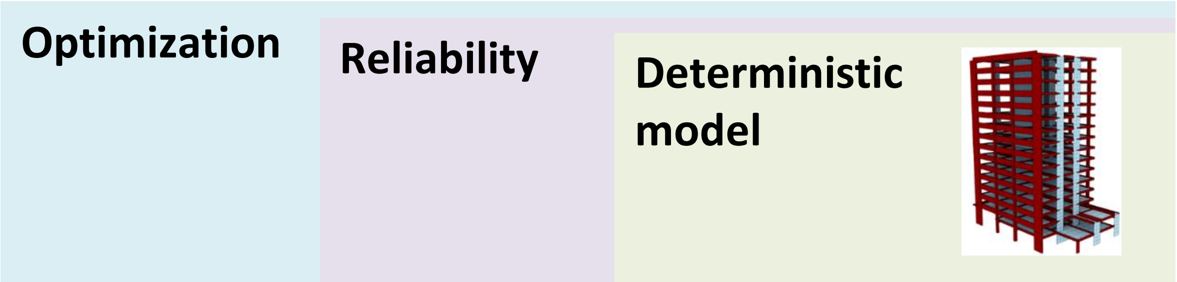


Challenge 3: Optimal Design

- Type of problem considered
  - Minimization of failure probability subject to budget constraint

$$
\begin{aligned}
&\min_{\boldsymbol{y}} C(\boldsymbol{y}) \\
&\text{subject to} \\
&p_F(\boldsymbol{y}) \leq p_F^t \\
&c_l(\boldsymbol{y}) \leq 0,\ \ l = 1, \ldots, n_C
\end{aligned}
$$

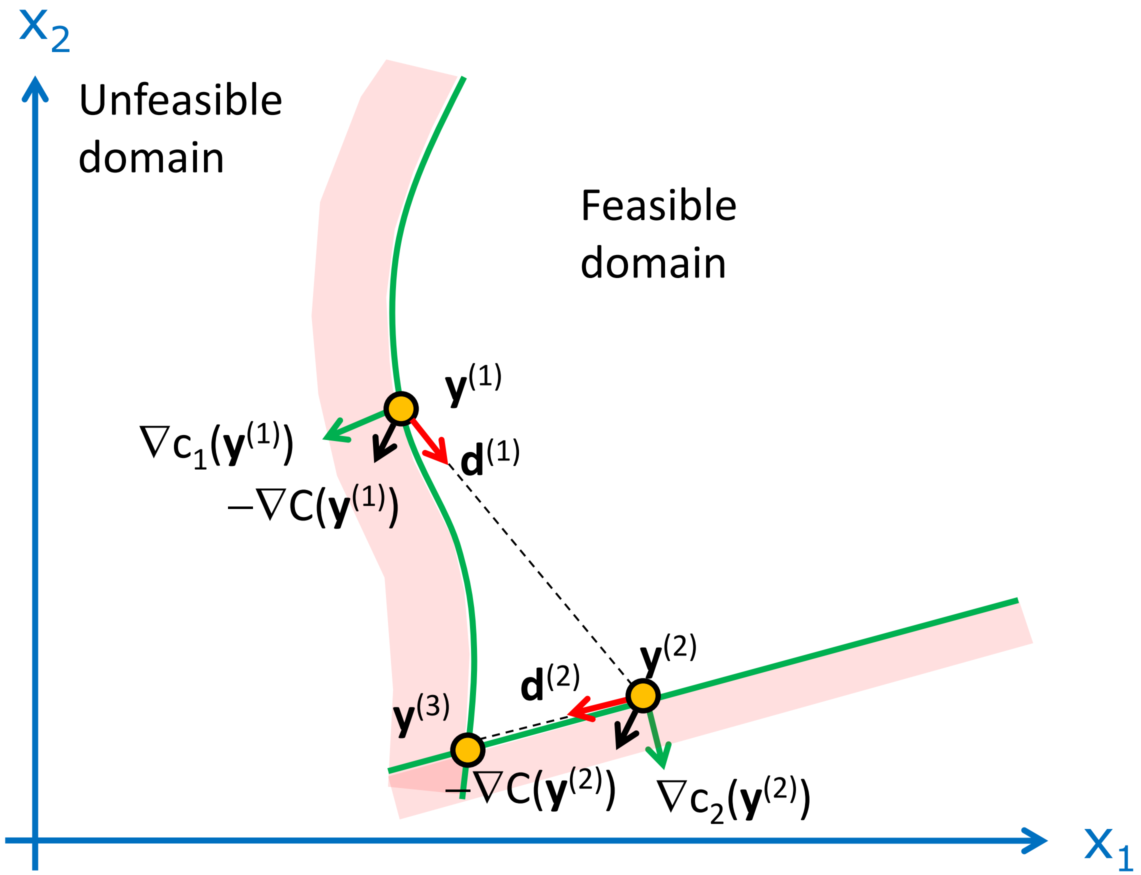
- $C(\boldsymbol{y})$: cost function
- p_F^t: maximum allowable probability
- $c_l(\boldsymbol{y})$: deterministic constraints (e.g. geometry, bounds)



Challenges:
- Double loop
- High dimensional probability integral
- Probability available through simulation

Challenge 3: Optimal Design

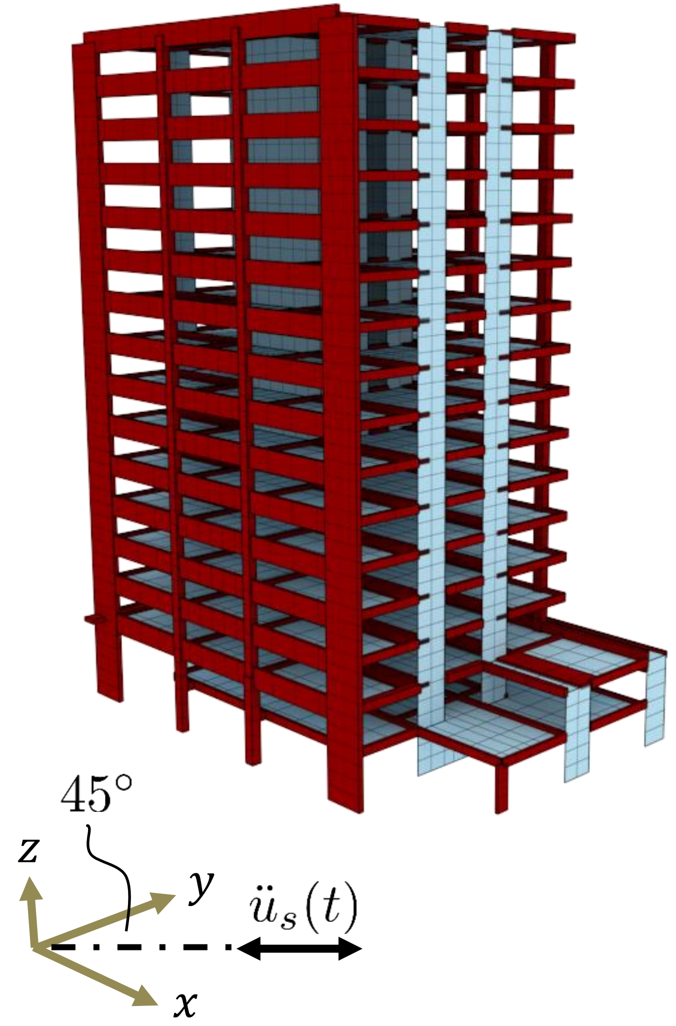
- <u>RBO strategy</u>
  - Gradient-based optimization
  - Feasible-direction interior point algorithm*
  - Line search performed with surrogate
  - Sequence of feasible, improved designs



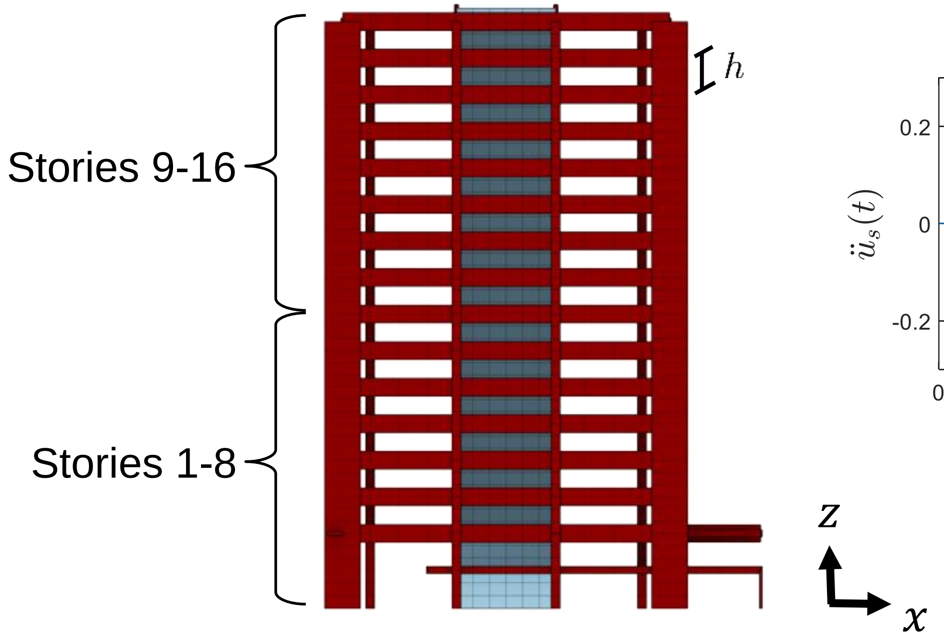
*J. Herskovits and G. Santos, “On the computer implementation of feasible direction interior point algorithms for nonlinear optimization,” Structural Optimization, vol. 14, Art. no. 2–3, 1997.

Challenge 3: Optimal Design

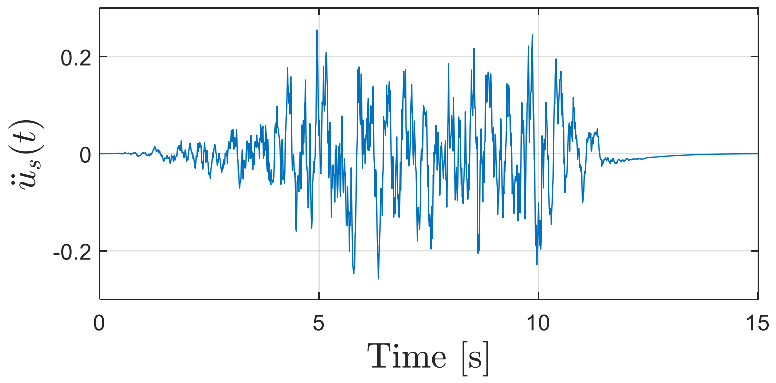
Perspective view



Elevation view

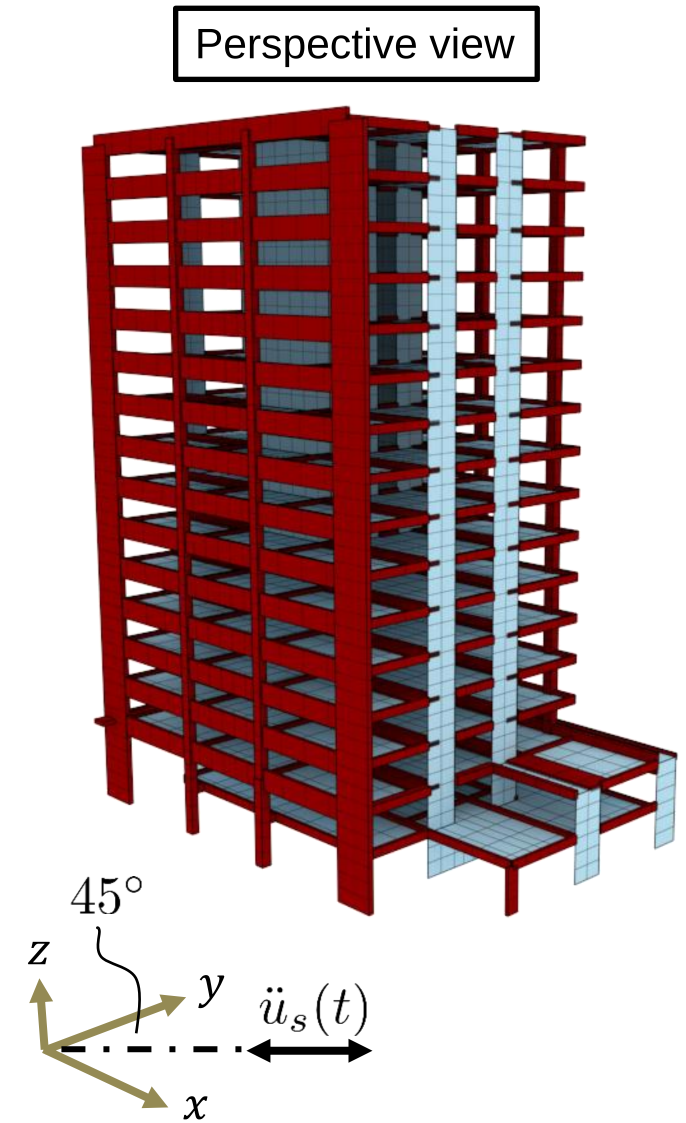


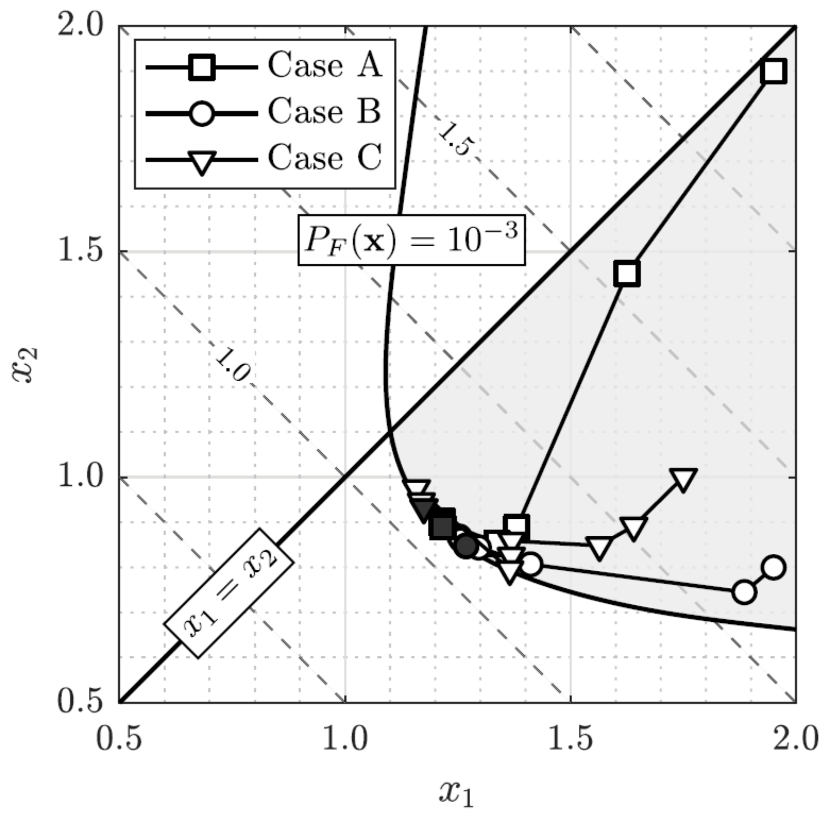
Realization of stochastic load

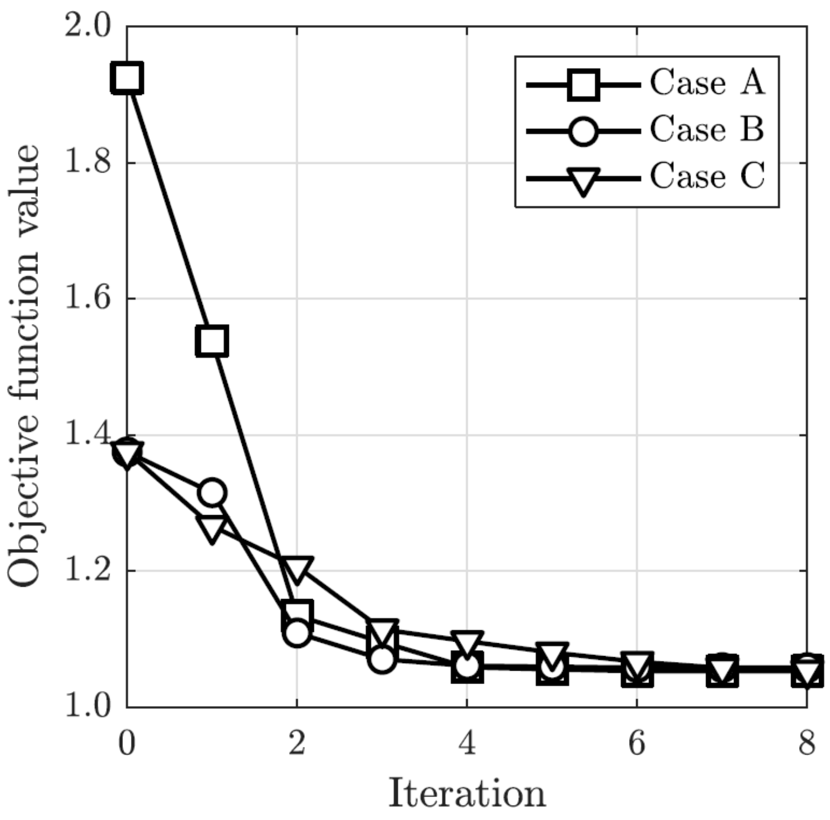


- <u>Example</u>: minimize thickness of walls (stories 1-8, 9-16) subject to probability of exceeding interstory drift threshold below 10^{-3}

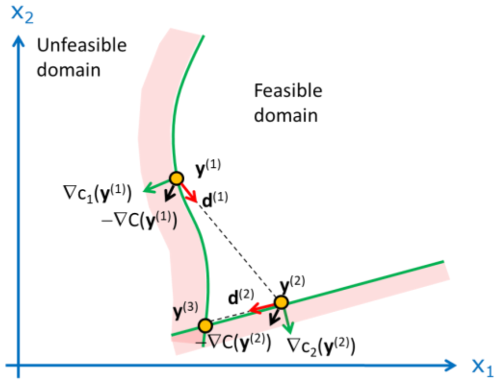
Challenge 3: Optimal Design







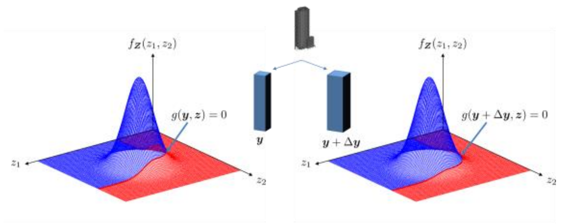
Conclusions

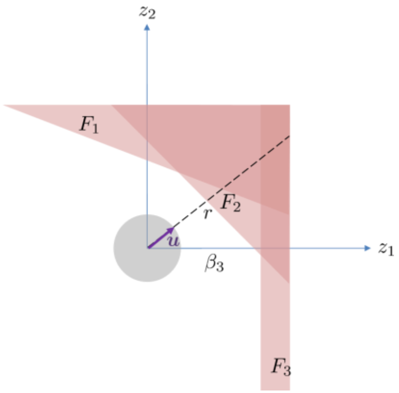


Decoupling strategies ⇨ **Stochastic linear dynamics** ⇦ Sensitivity analysis

⇧

Advanced simulation





<u>Open challenges</u>: non-Gaussianity, non-linearity

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Simulation methods for analysis and design in stochastic linear dynamics

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