Value of Information from SHM via estimating deterioration jump processes with particle filtering

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ABSTRACT: A Bayesian decision analysis framework for the quantification of the Value of Information (VoI) from long-term vibration-based SHM is proposed. The framework employs a particle filter to sequentially estimate the structural deterioration state in view of continuously identified modal data. Subsequently, the estimated structural reliability is sequentially updated, serving as the basis on which maintenance decisions are made following a heuristic-based optimization of the life-cycle cost. The framework is investigated through application on a numerical model of a bridge subjected to deterioration due to sporadic shocks over its lifetime.

1. INTRODUCTION

Despite expansive developments in the field of SHM, the adoption of SHM systems on real-world structures still falls short of the mark. A need exists for offering actionable use cases of the manner in which SHM systems can inform optimal maintenance decisions over the structural life-cycle. VoI analysis offers a formal framework for quantifying the effect of SHM systems on structural life-cycle costs (Pozzi and Der Kiureghian 2011, Straub 2014).

We work on the development of a preposterior Bayesian decision analysis framework for quantification of the VoI yielded via adoption of long-term vibration-based SHM. In previous work (Kamariotis et al. 2021) we showcase its application on a two-span bridge system subjected to progressive deterioration. Here, we adapt this framework to investigate optimal monitoring-based decision support for the same benchmark bridge system for the case of deterioration (expressed as stiffness reduction) in the middle elastic support (pier) due to sporadic shocks or sudden damage occurring over its life-cycle. A numerical model serves as a simulator to sample continuous dynamic response measurements at different time instances over the deteriorating structure's lifetime, based on samples of the deterioration process. Contingent on measurements of the system's modal frequencies, identified via an operational modal analysis (OMA) procedure, a particle filter is implemented to sequentially estimate the deterioration state. This leads to an updating of the estimated structural reliability. In the preposterior decision analysis, an optimal maintenance decision is found via imposition of a heuristic threshold on the updated time-dependent reliability. The prior optimal maintenance decision is computed on the basis of the prior estimate of the time dependent structural reliability. Finally, the Value of Information (VoI) is quantified as the difference in expected total life-cycle cost (LCC) between prior and preposterior decision analysis.

2. COMPOUND POISSON PROCESS

A prior model describing structural deterioration is a prerequisite for a VoI analysis. Herein, a compound Poisson process (CPP) model is employed to describe the temporal evolution of deterioration (stiffness reduction) due to sporadic shocks (Sanchez-Silva and Klutke 2016), which is typical of, e.g., earthquakes, floods. CPP models incorporate two types of randomness: i) random times of arrival of sporadic shock occurrences and ii) random damage increase due to an occurring shock. A CPP is a continuous-time stochastic process $\{X(t), t \geq 0\}$ of the form $X(t) = \sum_{i=1}^{N(t)} D_i$, where:

1) the number of jumps $\{N(t), t \geq 0\}$ is a Poisson process with rate λ,
2) the jumps $\{D_i, i = 1, .., \}$ are i.i.d. random variables with a specified probability distribution,
3) the process $\{N(t), t \geq 0\}$ and the damage increments $\{D_i, i = 1, .., \}$ are independent.

3. ESTIMATING DETERIORATION JUMP PROCESSES WITH PARTICLE FILTERING

Installation of a long-term vibration-based SHM system on a structure allows for continuous in-operation measurement of the dynamic response of the structure. The dynamic response data (e.g., accelerations) can be processed by an output-only OMA procedure, e.g., the Stochastic Subspace Identification (SSI) algorithm, for identifying the system's modal characteristics. The goal is to establish a model that uses the SSI-identified modal data, obtained at different points in time, to update the estimate of the structural deterioration state, described by the CPP model. A discrete-time state-space model is defined, suitable for application of Bayesian filters for monitoring the CPP deterioration. The process equation is defined by discretizing the CPP model

$$X_k = X_{k-1} + \Delta X_k$$

where ΔX_k is the distribution of the jump increments within a time interval Δt, given by the following cumulative distribution function (CDF)

$$\begin{aligned} F_{\Delta X_k}(d) &= \sum_{i=0}^{\infty} \Pr(\Delta X_k \le d | N_k - N_{k-1} = i) \cdot \Pr(N_k - N_{k-1} = i) \\ &= e^{-\lambda \Delta t} + \sum_{i=1}^{\infty} \frac{(\lambda \Delta t)^i}{i!} e^{-\lambda \Delta t} \cdot F_{\sum_{j=1}^{i} D_j}(d) \end{aligned}$$

where $F_{\sum_{j=1}^{i} D_j}(d)$ is the i-fold convolution of the distribution of D with itself. The measurement equation that links the SSI-identified modal eigenvalues with the unknown true deterioration state is given by

$$\boldsymbol{Z}_k = \mathcal{G}(X_k) + \boldsymbol{\eta}_k, \qquad \boldsymbol{\eta}_k \sim N()$$

where $\boldsymbol{Z}_k$ represents the vector of the m lower SSI-identified eigenvalues $\{\tilde{\lambda}_{i,k}, i = 1, .., m\}$ at time step k. $\mathcal{G}$ represents a forward linear finite element (FE) model predicting the modal eigenvalues at time step k, which is parametrized through the deterioration state X_k. The error term $\boldsymbol{\eta}_k$ models the prediction error (measurement error and model error) in the estimation of modal eigenvalues, assumed to follow a zero-mean joint Gaussian distribution $N()$ with variance proportional to the measured eigenvalues ($c_\lambda = 2\%$)

$$\boldsymbol{\eta}_k = \boldsymbol{Z}_k - \mathcal{G}(X_k) = \prod_{i=1}^{m} N\left(\tilde{\lambda}_{i,k} - \lambda_i\big(\mathcal{G}(X_k)\big); 0, {c_\lambda}^2 {\tilde{\lambda}_{i,k}}^2\right)$$

A bootstrap particle filter (PF) algorithm (Särkkä 2013, Chatzi and Smyth 2009) is implemented to solve the Bayesian filtering state estimation problem. The parameters describing the CPP are assumed fixed and are therefore not considered as updating parameters.

4. NUMERICAL INVESTIGATION

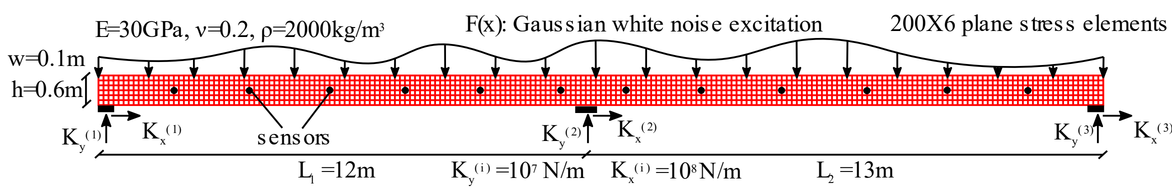

Fig. 1: Bridge system subject to damage due to deterioration (reduction of stiffness $K_y^{(2)}$) of the middle elastic support (pier).

In Fig. 1 we show the numerical benchmark model that is employed as a simulator for creating dynamic response measurement samples from the two-span bridge system, which is subjected to deterioration at the middle elastic support due to sporadic shocks (e.g., scour due to flood occurrences). Damage is introduced as a reduction of the stiffness in the y-direction of the spring $K_y^{(2)}$. The evolution of the damage over the bridge lifespan $T = 50$ years is described by the damage model below, where $K_{y,0}^{(2)}$ is the initial undamaged value, and $X(t)$ is the stiffness deterioration described by the CPP model, which contains random variables that need to be chosen a-priori. For the current investigation the rate of the Poisson process is chosen $\lambda = 4/50$, corresponding to a mean rate of 4 shock occurrences within the lifespan of 50 years, and the jumps are assumed to follow a lognormal distribution $\{D_i, i = 1, .., \} \sim LN(\mu = 1.0, \sigma = 3.0)$. The parameters are chosen to reflect sufficiently large uncertainty a-priori.

$$K_y^{(2)}(t) = K_{y,0}^{(2)}/(1 + X(t))$$

A VoI analysis is performed before the installation of an actual SHM system, with the goal of investigating the potential gains of deploying such a system in terms of total LCC. One therefore needs to first draw samples from the prior deterioration model, which describe potential realizations of the damage over the life-cycle of the structure. For the CPP model, one such realization can be seen in Fig.3a (black curve). For each CPP sample, one further needs to simulate the monitoring measurements that one expects to obtain from the SHM system. Herein, the investigated SHM system consists of 12 uniformly distributed sensors measuring vertical accelerations, as shown in Fig. 1. The process from acceleration data generation to identification of the SSI modal data is shown in Fig. 2. We execute this process for any time step k when a measurement is to be obtained. Here, the SSI always identifies between m = 4 to 6 lower eigenvalues.

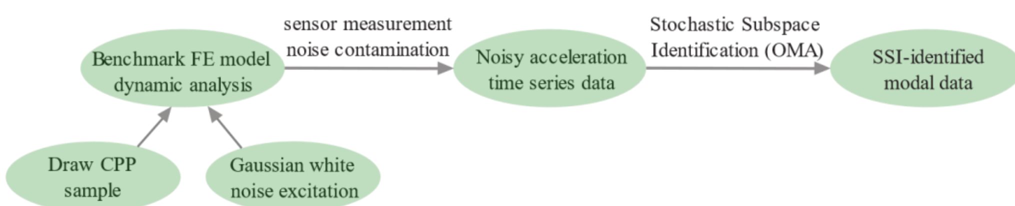

Fig. 2: Process for generating the SHM data for a given CPP sample

The FE model $\mathcal{G}$, used in a forward manner within the PF estimation, is the same FE model as the one in Fig. 1 used for generating the synthetic monitoring data (a surrogate of $\mathcal{G}$ is used within our VoI analysis).

For illustration purposes, we assume that the black line in Fig. 3a represents one underlying "true" realization of the CPP damage evolution over time. For this CPP sample, we generate one set of SSI-identified modal eigenvalues per year over the $T = 50$ years of the lifetime, and then we sequentially feed the data obtained at different points in time in the PF estimation to obtain the filtering estimate of the deterioration state. In blue, the mean state estimated via weighted particles is plotted, and the red shaded area represents the 99.9% credible interval for the estimation. The PF proves effective in estimating the evolution of deterioration. For the damage scenario assumed herein, the increase in the normal stresses at the middle of the two spans is the considered safety critical indicator for the structural reliability analysis.

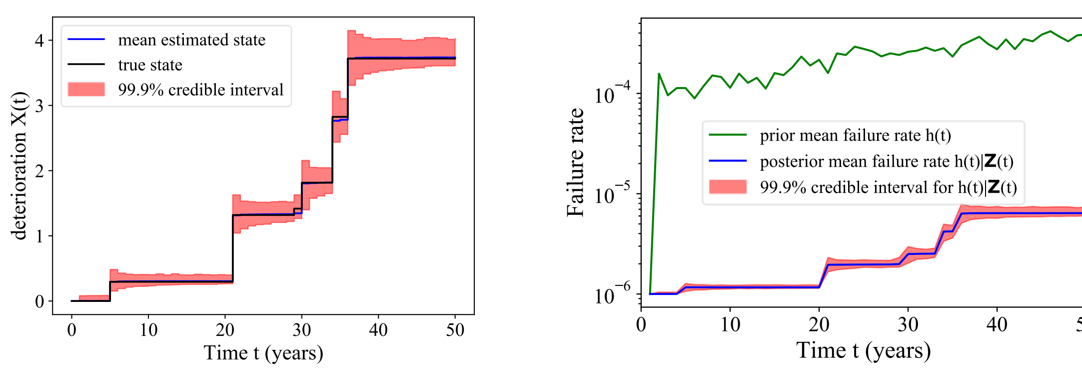

Fig. 3.a. Filtering estimation of one underlying true realization (in black) of the deterioration process using PF with one modal measurement per year. **3b**. in green: prior mean estimate of the failure rate using 10^4 Monte Carlo samples, in blue: posterior mean estimate of the failure rate, updated with measurements corresponding to the underlying true deterioration in black in Fig. 3.a.

In Fig.3b., we plot the prior mean estimate of the failure rate, and the filtering estimate of the failure rate, updated with particles from the PF deterioration estimation.

We introduce a simple heuristic for the solution of the decision problem: a repair action on the structure must be performed at the time instance when the estimated failure rate exceeds a predefined threshold. This threshold then becomes the heuristic parameter w that we seek to optimize, which results in the optimal time to perform a repair t_{repair}, for which the total expected LCC is minimum. For the VoI investigation in this paper, we assume that a failure event will lead to a cost $\hat{c}_F = 10^7$units while a large repair on the structure will cost $\hat{c}_R = 10^5$units. Costs are discounted with a discount rate of 2% per annum.

We draw 1000 samples of the CPP to compute the optimal expected LCC in both prior and preposterior analyses. We find that in the prior case the single optimal decision $t_{repair}^{pr,opt}$ is to not perform any repair throughout the structure's lifetime. In the preposterior case, for each prior CPP sample, we generate the corresponding SHM data, and we obtain the posterior filtering estimate of the failure rate, based on which we compute an optimal t_{repair} value (one for each deterioration and monitoring realization), potentially different than the $t_{repair}^{pr,opt}$. We then average over all the posterior optimal solutions to obtain the preposterior optimal expected LCC. For this investigation, the VoI indicates a potential benefit of installing the SHM system, which can trigger reaction more promptly than in the case of no SHM for some damage realizations. The VoI value must be further compared with the LCC of the SHM system itself. Any SHM system with cost less than the VoI will be cost-benefit.

Table 1: Bayesian decision analysis and VoI results

Prior decision analysis			Preposterior decision analysis		VoI analysis
w_0^{opt}	Total expected LCC	$t_{repair}^{pr,opt}$	w_{monit}^{opt}	Total expected LCC	VoI
$\geq 7\text{x}10^{-4}$	$7.37\text{x}10^4$units	no repair	$2\text{x}10^{-3}$	$4.95\text{x}10^3$units	$6.87\text{x}10^4$units

5. CONCLUSIONS

We develop a framework for quantifying the VoI from long-term vibration-based SHM, which is here applied to a bridge system subjected to damages from sporadic shocks, described by a compound Poisson process model. We presented the detailed modeling of the SHM process from data generation to processing, particle filter deterioration state estimation and reliability calculation, embedded within a VoI analysis. The framework quantifies the expected gains that the SHM system provides, i.e., it quantifies the VoI.

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