Non-linear structural models and the partial safety factor concept

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Abstract

Most modern structural design codes are based on the partial safety factor concept. The partial safety factors are calibrated on linear limit states. Structural design codes like the Eurocode provide simplified rules on the application of partial safety factors to non-linear limit states. This paper investigates these rules and their effect on the structural reliability for various non-linear limit states. Moreover, we motivate adaptations of the current design rules. We focus on non-linear structural response functions, i.e., the non-linear relation between actions and their effects. In order to characterize the non-linearity of a structural response function, we introduce a new measure of non-linearity. We conduct a detailed parametric study and investigate two example structures. Our results show that for the case of a single dominant action current design rules lead to sufficiently safe or only marginally unsafe structures. However, they can lead to a strong over-design.

1 Introduction

The vast majority of modern structural design codes is based on the semi-probabilistic partial safety factor (PSF) concept [1–4]. The PSFs ensure sufficient structural reliability of the resulting design. They are calibrated in such a way that on average a desired target reliability is achieved for the case of linear models [5–7]. In practice they are also applied to non-linear models. This is in agreement with the PSF concept [5,8]. Except for extreme cases of non-linearities, the PSF concept would result in sufficiently safe structures if each each quantity would have its own calibrated PSF. However, in practice PSFs cover the uncertainty of multiple quantities. This raises the questions how these PSFs should be applied in the presents of non-linear models and if a sufficiently safe design can still be achieved.

The application of a PSF to a non-linear model can in principle be done in two different ways: The PSF can be applied to the argument or to the responses of the non-linear function. Both basic options can lead to reliabilities below as well as above the target reliability. Structural design codes typically try to overcome this issue by choosing the more conservative of the two design options (e.g., [9]). In some cases, this may lead to over-design. In other cases, the more conservative of the two options might still lead to insufficient reliability. This issue is the research question of this paper: How do non-linear models affect structural reliability for the two basic design options? We address this research question by a generic general parameter study and through two example applications. In order to measure the effect of non-linearity on the structural reliability we introduce a new measure of non-linearity. The proposed measure is defined such that it can be included in the PSF concept to provide assistance on what design option to choose. However, we do not explicitly propose such an inclusion, as this would require an in-depth code calibration that is beyond the scope of this work. Potential future inclusions of the measure within the PSF concept are indicated in the discussion.

Previous research on non-linear models applied within the PSF concept focuses mainly on reinforced concrete structures. The reinforced concrete research-community developed multiple methods to adapt PSF design and thereby provide alternatives to the two above mentioned design options. The most popular method is the estimated coefficient of variation method (ECOV) [10]. It is based on an estimate of the coefficient of variation of the resistance via the mean and the characteristic material strength. Other methods can be found in [11–14]. These methods are well investigated through various application studies (e.g., [15–18]). More abstract and material independent investigations on effects of non-linear models on the reliability are not known to the authors. The purpose of this paper is to provide such abstract and material independent investigations. Thereby, we focus on the two above mentioned basic design options. Alternative design options, such as those offered by the reinforced concrete research community, could be generalized and investigated as well, but this is beyond the scope of this publication.

The paper is structured as follows: We first briefly review the PSF concept and discuss challenges of the PSF concept in connection with structural non-linear models (Section 2). We then provide a probabilistic view of non-linear structural models in Section 3. In Section 4, we review existing measures to characterize the non-linearity of structural response functions and introduce an enhanced measure. Based on the introduced measure, we perform general and abstract parameter studies about the effect of non-linear structural response functions on structural reliability in Section 5. Subsequently, we give two application examples (a truss dome and a membrane structure) and classify them within the context of the parameter study (Section 6). Finally, we discuss our results and motivate various adaptations of the PSF concept.

2 Non-linearities in the partial safety factor concept

We adopt the nomenclature of the PSF concept implemented in EN1990:2002 [9], but the investigations and the results of this paper can be transferred to other semi-probabilistic design codes (e.g., [14, 19–21]).

In a Eurocode design, four different models can be identified (see Figure 1): The action model, the structural model, the material model, and the resistance model. The action model and the material model are typically probabilistic, hence, they are represented via distributions of the action and the material strength. In order to make the design process deterministic, quantile values or moments of these distributions are chosen as characteristic actions l_k and characteristic material properties m_k. For this reason, the PSF concept is called semi-probabilistic. To ensure a safe design, these characteristic values are multiplied, respectively divided, by the PSFs γ_f and γ_m. The modified values are the input to functions t_S and t_R corresponding to the structural model and the resistance model. The outcome of these functions are the action effect and the resistance. The action effect and the resistance are again multiplied, respectively divided, by PSFs γ_{Sd} and γ_{Rd}. Eventually, the design values e_d and r_d are obtained[1][2]:

$$e_d = \gamma_f \cdot t_S(l_k \cdot \gamma_{Sd}) \tag{1}$$

$$r_d = \frac{t_R\left(\frac{m_k}{\gamma_m}\right)}{\gamma_{Rd}} \tag{2}$$

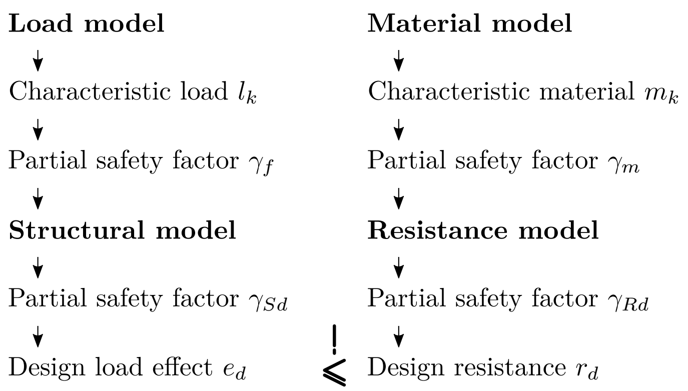

Figure 1: Overview of the Eurocode design approach.

A design is verified, if the following inequality is fulfilled:

$$e_d \leq r_d \tag{3}$$

For the sake of simplicity, the Eurocode merges the partial safety factors of the action and the resistance side:

$$\gamma_F = \gamma_f \times \gamma_{Sd} \tag{4}$$

$$\gamma_M = \gamma_m \times \gamma_{Rd} \tag{5}$$

Although this merge simplifies the design process, it raises the question if γ_F and γ_M should be applied to the characteristic values l_k and m_k directly or to $t_S(l_k)$ and $t_R(m_k)$.

[1] The calculation of design values can include combination coefficients among various actions. Here, we only consider the case of a singular action, hence combination coefficients are not included.

[2] In some cases the resistance model can depend on actions and the structural model can depend on material properties. Moreover, the resistance model and the structural model may be combined in a single function. Such cases are not covered within the scope of this work.

As long as t_S and t_R are linear functions through the origin, both options lead to the same design values; however, if t_S and t_R are non-linear functions or do not pass through the origin, the two alternatives result in different design values and, therefore, in different structural reliabilities.

In case of the structural model, the two alternatives to calculate the design action effect e_d are:

$$\text{Design option (1) (prior to } t_S\text{):} \qquad e_d = t_S(\gamma_F \cdot l_k) \tag{6}$$

$$\text{Design option (2) (posterior to } t_S\text{):} \qquad e_d = \gamma_F \cdot t_S(l_k) \tag{7}$$

We refer to these two options as design option (1) and design option (2) for the remainder of this paper.

Similar design options can be formalized for the resistance model; however, EN1990:2002 only covers non-linear structural models. We also focus on the action side only and assume t_R to be a linear linear function through the origin for the remainder of this paper. Investigations of non-linear resistance models can be found, e.g., in [11, 22–24].

EN1990:2002 [9] provides a rule when to chose design option (1)/(2) based on whether "actions effects increase more or less than the actions" (Paragraph 6.3.2.(4)). The background document *Designers' Guide to Eurocode: Basis of Structural Design* [25] specifies this mathematically as follows:

$$\text{use option (1) if:} \qquad t_S(\gamma_F \cdot l_k) > \gamma_F \cdot t_S(l_k) \tag{8}$$

$$\text{use option (2) if:} \qquad t_S(\gamma_F \cdot l_k) < \gamma_F \cdot t_S(l_k) \tag{9}$$

analog für R defenieren

The instructions of Eurocode lead to some open questions when it comes to the classification of non-linearities:

One question is how to deal with initial actions such as prestress: The relationship between actions and their effects might be linear for values of actions above 0; however, under initial actions t_S is highly non-linear at the origin (see Figure 2).[3] According to EN1990:2002 [9] this case is interpreted as linear. The background document of Eurocode [25] implies a non-linearity, leading to design option (2).

Another ambiguity arises if t_S has a change of curvature (see Figure 3). This can e.g. be the case, when a structure is dominated by softening effects at lower load levels, but is dominated by hardening effects at higher loads. Here, EN1990:2002 [9] does not provide a classification of t_S. The background document of Eurocode [25] can lead to both design options, depending on the value at which the function has the change of curvature.

A third question is how to treat the case of multiple actions. This case is not covered by the EN1990:2002 [9] nor the background document [25]. Some national annexes provide

[3] Note, that in Figure 2 the abscissa represents the action and the ordinate represent the action effect. This is reverse to the load displacement curves usually shown in non-linear structural analysis (e.g., [26]).

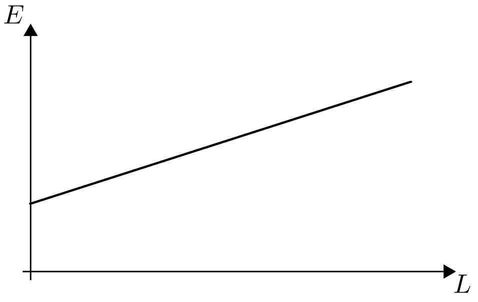

Figure 2: Relationship between actions and their effects in presence of an initial action.

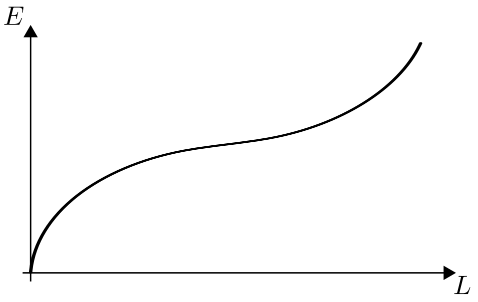

Figure 3: Relationship between actions and their effects in presence of a change in curvature.

simplified rules for non-linear design in case of multiple actions (e.g., [27]). We do not further investigate the multidimensional action case in this paper, but add some discussion on this issue.

3 Probabilistic view of non-linear models

In the following we review the probabilistic view behind the PSF design when non-linear structural models are used.

Let L and M be random variables describing the action and the material property. The distribution of the action effect E is determined by applying t_S to L:

$$E = t_S(L) \tag{10}$$

The distribution of the resistance is determined by applying t_R to M. In this study, t_R is assumed to be a linear function through the origin, hence, it can be written as

$$R = t_R(M) = p \cdot M \tag{11}$$

with $p \in \mathbb{R}$ being a constant.

The value of p is found by applying the PSF concept as follows. Let l_k and m_k be the characteristic action and the characteristic material property and γ_F and γ_M the respective

PSFs. The design resistance r_d is determined as

$$r_d = p \cdot \frac{m_k}{\gamma_M} \tag{12}$$

The optimized design is chosen, i.e., $r_d = e_d$. Depending on the chosen design option in the determination of e_d (Equation 6 or 7) p follows as

$$r_d = e_d \tag{13}$$

$$\Leftrightarrow \; p \cdot \frac{m_k}{\gamma_M} = \begin{cases} t_S(\gamma_F \cdot l_k) & \text{Option (1)} \\ \gamma_F \cdot t_S(l_k) & \text{Option (2)} \end{cases} \tag{14}$$

$$\Leftrightarrow \; p = \begin{cases} \frac{\gamma_M \cdot t_S(\gamma_F \cdot l_k)}{m_k} & \text{Option (1)} \\ \frac{\gamma_M \cdot \gamma_F \cdot t_S(l_k)}{m_k} & \text{Option (2)} \end{cases} \tag{15}$$

The probability of failure is

$$\Pr(F) = \int_{\{g<0\}} f_{LM}(m,l) \, \mathrm{d}m \, \mathrm{d}l \tag{16}$$

where f_{LM} is the joint probability density function (PDF) of M and L and g is the following limit state function (LSF)

$$g = p \cdot M - t_S(L) \tag{17}$$

In case of independent L and M, the probability of failure can be calculated via the following convolution

$$\Pr(F) = \int_{\Omega_L} F_M\left(\frac{1}{p} \cdot t_S(l)\right) \cdot f_L(l) \, \mathrm{d}l \tag{18}$$

where Ω_L is the sample space of L, F_M is the cumulative distribution function (CDF) of M and f_L is the PDF of L.

Another approximative way to calculate the probability of failure is the First Order Reliability Method (FORM). In a nutshell, FORM transforms the limit state surface into standard normal space and approximates it by its tangent hyper plane at the point closest to the origin (called FORM design point). A detailed description of FORM can be found e.g., in [5, 8, 28, 29]. FORM is the historical basis of the PSF concept. The design point resulting from a PSF design (e_d, r_d) should be close to the FORM design point [5].

Given the probability of failure, the reliability index β is determined as:

$$\beta = -\Phi^{-1}(\Pr(F)) \tag{19}$$

In order to properly interpret the parameter studies in Section 5 one should note that the probability of failure is invariant to scaling of t_S. This signifies that t_S can be redefined and replaced by $\widetilde{t}_S(x) := c \cdot t_S(x)$ (where $c \in \mathbb{R}$ is a constant) without affecting the resulting probability of failure. This property follows from the fact that the LSF (Eq. 17) can be multiplied by any constant without changing the limit state surface; hence, without changing the probability of failure. Because of this property, the subsequent numerical results do not only hold for the chosen t_S but for any scaled version of t_S.

4 Measures of non-linearities

In order to investigate and account for the effects of non-linear structural response functions on the structural reliability, a measure of non-linearity is needed. We aim at a measure that is also applicable in PSF design. A good measure should be:

1. Straightforward to evaluate within the PSF designing process.

2. Unambiguous and easy to interpret.

3. A good predictor of reliability. I.e., design situations with the same measure and the same design approach should lead to a similar structural reliability.

4.1 Existing measure of non-linearity

Multiple measures of the non-linearity of a function can be found in literature, e.g., [30–34]; however, proposals for measures applicable within the PSF concept are sparse. We are aware only of two measures: One measure introduced by Uhlemann [35] and one measure introduced by Bakeer [36].

The first measure n introduced by Uhlemann was further investigated by [37] and eventually included in the background document of the Eurocode *Prospect for European Guidance for the Structural Design of Tensile Membrane Structures* [38] as follows:

$$n = \frac{t_S(f \cdot l_k)}{f \cdot t_S(l_k)} \tag{20}$$

Here, f is an arbitrary load increase factor. Based on the value of n different design options are recommended [38]:

$$n \begin{cases} = 1 & \text{use option (1) or option (2) (linear case)} \\ > 1 & \text{use option (1)} \\ < 1 & \text{use option (2)} \end{cases} \tag{21}$$

If $f = \gamma_F$, the rules for which design option to chose are equivalent for the Designers' Guide (Equation 8, 9) and Uhlemann (Equation 20).

The second measure n_F introduced by Bakeer is called the degree of homogeneity. It is derived via a first order Taylor series expansion of t_S mapped into log-space at the design point. This results in a measure of the relative change of the effect of action to the relative change of the action at the design point:

$$n_F = \frac{\gamma_F \cdot l_k}{t_S(\gamma_F \cdot l_k)} \cdot \frac{\mathrm{d}t_S(\gamma_F \cdot l_k)}{\mathrm{d}l} \tag{22}$$

which can be approximated via

$$n_F \approx \frac{1}{\ln(\gamma_F)} \cdot \ln\left(\frac{t_S(\gamma_F \cdot l_k)}{t_S(l_k)}\right) \tag{23}$$

If $n_F = 1$ the measure indicates t_S to be linear. If $n_F > 1$ the measure indicates $\gamma_F \cdot t_S(l_k) < t_S(\gamma_F \cdot l_k)$ which is linked to design option (1). If $0 < n_F < 1$ the measure indicates $t_S(\gamma_F \cdot l_k) < \gamma_F \cdot t_S(l_k)$ which is linked to design option (2).

The measure can analogously be defined on the resistance side to measure the non-linearity of t_R. Moreover, the measure has the advantage that it is also applicable in case of multiple actions, leading to a measure of the partial degree of homogeneity per applied action.

Both, the measure n and the measure n_F are straightforward to apply within the designing process and unambiguous and easy to interpret. Hence, the first two of the above requirements to a measure of non-linearities are fulfilled. The third requirement of similar structural reliability given the same measure is not fulfilled. To visualize this issue, Figure 4 shows different t_S which share the same measure n and n_F respectively. These different t_S may result in very different structural reliabilities. However, in defense of both measures, it should be noted that it is impossible to fully satisfy the third requirement without including probabilistic quantities (which would conflict with the first requirement of applicability within the PSF design). Why this is the case can be seen from the parameter studies of Section 5.

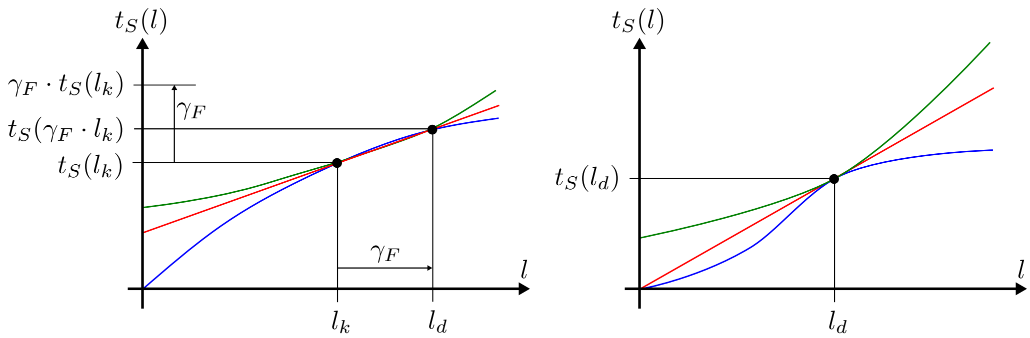

Figure 4: Different non-linear t_S that share the same measure of non-linearity n (left) and n_F (right). The t_S in the left illustration result in the same measure n since they share the same action effect at characteristic and design action. The t_S in the right illustration result in the same measure n_F since they share the same action effect and the same gradient at design action.

4.2 Proposal of a new measure of non-linearity

We propose a novel semi-probabilistic measure of non-linearity of t_S. It is based on values of the PSF-concept namely the characteristic action l_k and design action $l_d = \gamma_L \cdot l_k$ and their respective effects via t_S. The measure consists of two components, the offset measure y_0 and the curvature measure κ:

$$y_0 = \frac{e_0}{e_k} \tag{24}$$

$$\kappa = \frac{m_2}{m_1} \tag{25}$$

where $e_0 = t_S(0)$, $e_k = t_S(l_k)$, $e_d = t_S(l_d)$, $m_1 = \frac{e_k - e_0}{l_k}$ and $m_2 = \frac{e_d - e_k}{l_d - l_k}$. Figure 5 illustrates y_0 and κ.[4]

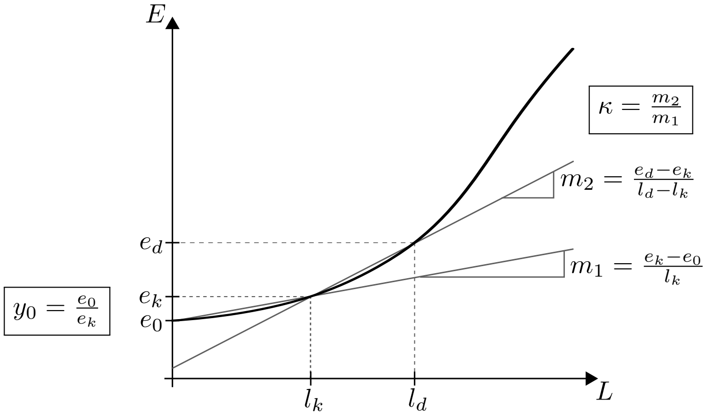

Figure 5: Measures y_0 and κ to classify the non-linearity of t_S.

The major difference of this measure to the measures given in Equation 20 and 22 is that the evaluation of $t_S(0)$ is taken into account by y_0.

y_0 and κ fulfill the first requirement: They are straightforward to calculate within the design process, since they are based on a few evaluations of the structural response function t_S. More advanced measures (e.g., derivative-based measures) may be more accurate; however, they would need more knowledge/evaluations of t_S.

The second requirement is fulfilled too: y_0 and κ are unambiguous, since they are explicitly mathematically defined. Moreover, their interpretation is straightforward: y_0 is a measure of the amount of initial actions (e.g., due to prestress). If $y_0 = 0$ no initial action is present. If $y_0 = 1$ the action effect of the initial action is equal to the action effect of the characteristic action. κ is the ratio of two secants with slope m_1 and m_2. m_1 is an approximation of the gradient of t_S between 0 and l_k and m_2 is an approximation of the gradient of t_S between l_k and l_d. Therefore, κ is an measure of the curvature at the characteristic action. If $\kappa > 1$ then t_S is approximated to be convex, if $\kappa = 1$ then t_S is approximated to be without curvature and if $\kappa < 1$ then t_S is approximated to be concave.

Overall the third requirement is not fulfilled as can be seen from the numerical investigations of Section 5. The main reason for this is that the probability of failure does not only depend on t_S but also on t_S interacting with (semi)-probabilistic properties, i.e., the choice of characteristic values, the PSF and the distributions of the actions and material strengths. The third requirement can therefore only be satisfied by a probabilistic measure, but this would contradict the first two requirements. However, we argue that the proposed measure is a better predictor of reliability than the measures of Equation 20 and 22 for the following two reasons: First, κ is based on evaluations of t_S at three points (0, l_k and l_d). This captures the non-linear behavior of t_S more globally than the measure

[4]Note that κ is not an approximation of the curvature of a function in the classical sense defined via the second derivative. E.g., the second derivative of an linear function is 0, whereas κ is 1.

n which is based on evaluations at l_k and l_d only and the measure n_F which is based on evaluations (inducing the first derivative) at l_d only. Secondly, y_0 accounts for different starting conditions at zero load level. In contrast, the measure n ignores different starting conditions and the measure n_F induces them only implicitly trough the term $\frac{\gamma_F \cdot l_k}{t_S(\gamma_F \cdot l_k)}$.

5 Parameter studies

We investigate how the reliability indices of design options (1) and (2) vary for different non-linear structural response functions t_S and different (semi)-probabilistic setups. We first investigate a base case. Afterwards, we vary properties one at a time.

All considered studies are calibrated such that a target reliability index of $\beta_{TRG} = 4.3$ is achieved in the linear case ($\kappa = 1$ and $y_0 = 0$). $\beta_{TRG} = 4.3$ is in the common range of structural reliability index targets [7, 39]. Following [7] β_{TRG} is defined with respect to a reverence period of 1 year. However, the subsequent parameter studies are not very sensitive to the value of β_{TRG}. Similar results would be obtained, e.g., for the target reliability index following EN1990:2002 [9] of 4.7 (1 year) or 3.8 (50 years).

If the structural response functions t_S is non-linear, the resulting structural reliability can deviate from the target value. This can be the case for both design options (1) and (2). If the resulting reliability indices are above/below $\beta_{TRG} = 4.3$ one can consider the design to be conservative/non-conservative. The main focus of the subsequent studies is to investigate systematically under which conditions which design option is conservative or non conservative.

5.1 Base case

In the base case, we investigate a bi-linear functional form of t_S defined with respect to different values of the curvature measure κ between 0 and 2. The offset measure y_0 is set to 0. The investigated t_S are shown in Figure 6. Such bi-linear functional forms can, for example, occur in structures which are analyzed by first order plastic hinge theory. In general, non-linear structural response functions typically have a much more complex functional form. However, as it will be shown in the subsequent Section 5.2 (where we replace the bi-linear form with a quadratic one) the structural reliability is not very sensitive to the exact functional form. This is also confirmed by [40], where the structural response function of a membrane is compared to a quadratic approximation of the structural response function. Hence, it is not critical that the utilized functional forms do not exactly cover structural response function used in practice, but only approximate their non-linear behavior.

In the base case, we assume log-normally distributed material strength M with c. o. v.$[M] = 0.1$ and a Gumbel distributed action L with c. o. v.$[L] = 0.3$.[5]

[5]The probability of failure is invariant to the choice of the mean values; hence, the mean values can be chosen arbitrary.

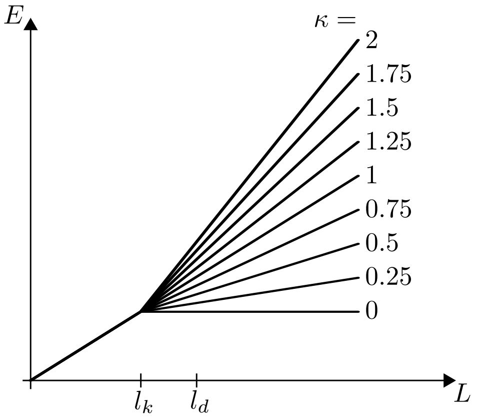

Figure 6: Bi-linear functional form of t_s of the base case for $\kappa = 0, 0.25, 0.5, \ldots, 2$.

Given a specific design situation, we find the design in the following way: The characteristic action l_k is chosen as the 98% quantile of L and the PSF of the action side γ_F is 1.5. The characteristic material strength m_k is chosen as the 5% quantile of M. These choices are common for variable actions and most materials. The PSF of the resistance side γ_M is calculated such that a target reliability index β_{TRG} is achieved in the linear case.

Figure 7 shows the reliability indices for the base case designed following design options (1) or (2). In the base case, both design options are conservative for $\kappa < 0$ and non-conservative for $\kappa > 0$. The approach of EN1990:2002 [9] chooses the more conservative of the two design options in both cases. For $\kappa < 0$ this would result in strong over-design, for $\kappa > 0$ in slight under-design.

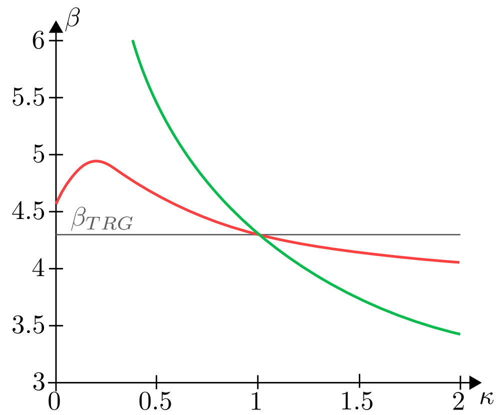

Figure 7: Reliability indices for the base case designed following design options (1) (red) or (2) (green).

Figure 8 shows the limit state surfaces in standard normal space and compares the FORM design points to the design points implied by the PSF concept for different values of κ. Values of κ above 1 do not lead to strong non-linearities of the limit state surface. For values of κ below 1 the limit state surface becomes strongly non-linear.[6] The FORM design points and the design points implied by the PSF differ significantly in most cases, including the linear case. This hints at a non ideal choice of PSFs; however, PSFs are often suboptimal in specific design situations, hence, this is not unrealistic.

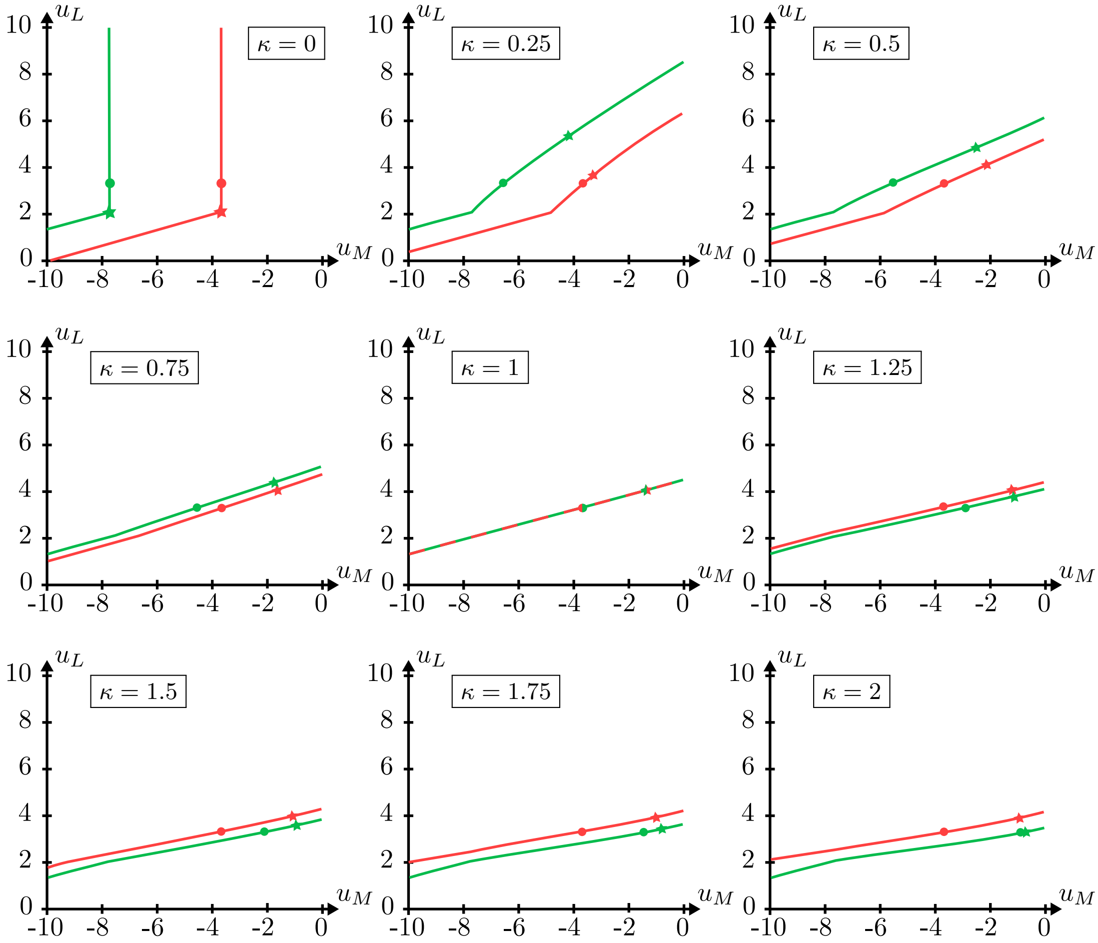

Figure 8: Limit state surfaces in standard normal space of the base case for different values of κ following design options (1) (red) or 2 (green). Stars represent the FORM design points, dots represent the design points implied by the PSF concept.

5.2 Effect of the functional form of the structural response function

To investigate the effect of the structural response function, we alter the functional form of t_S from bi-linear to quadratic. Figure 9 shows the resulting functions t_S for different values of κ. For $\kappa < 1$ the resulting t_S have a maximum and, therefore, drop to zero and become negative at higher load levels. If κ is only slightly below 1, the decreasing/negative part of t_S is at rather high load levels, which are too unlikely to be of interest. However, when κ becomes lower, the decreasing/negative part of t_S occurs at load levels likely enough

[6] Although the limit state surface looks linear in case of $\kappa = 1$, it is slightly non-linear in standard normal space. In the original space this limit state function is exactly linear.

to be of interest. These t_S might be unrealistic. However, for the sake of coherence and comparability with the bi-linear case, we also cover these cases.

Figure 10 shows the resulting reliability indices. For $\kappa < 1$ both design options are more conservative than in the base case with bi-linear t_S and for $\kappa > 1$ more non-conservative; however the differences are only marginal. Figure 11 shows the limit state surfaces for different values of κ. The limit state surfaces are again almost linear for $\kappa > 1$ and highly non-linear for $\kappa < 1$. Overall, the functional form of t_S has little effect on the structural reliability for given κ and $e_0 = 0$. We observed similar trends for other functional forms.

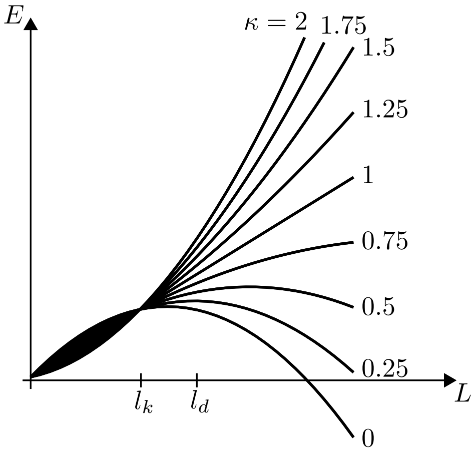

Figure 9: Quadratic functional form of t_s for $\kappa = 0, 0.25, 0.5, \ldots, 2$.

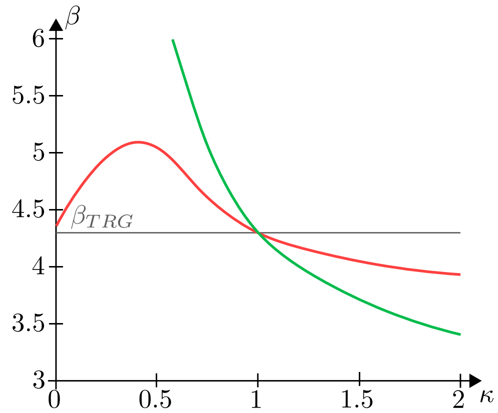

Figure 10: Reliability indices in case of quadratic t_S following design options 1 (red) or 2 (green).

5.3 Effect of initial actions

To evaluate the effect of initial actions we consider different values of the offset measure $y_0 = 0.2, 0.4, 0.6$ or 0.8. Initial actions can, for example, be caused by prestressing. Moreover, deterministic permanent actions (e.g., dead weight) can be interpreted

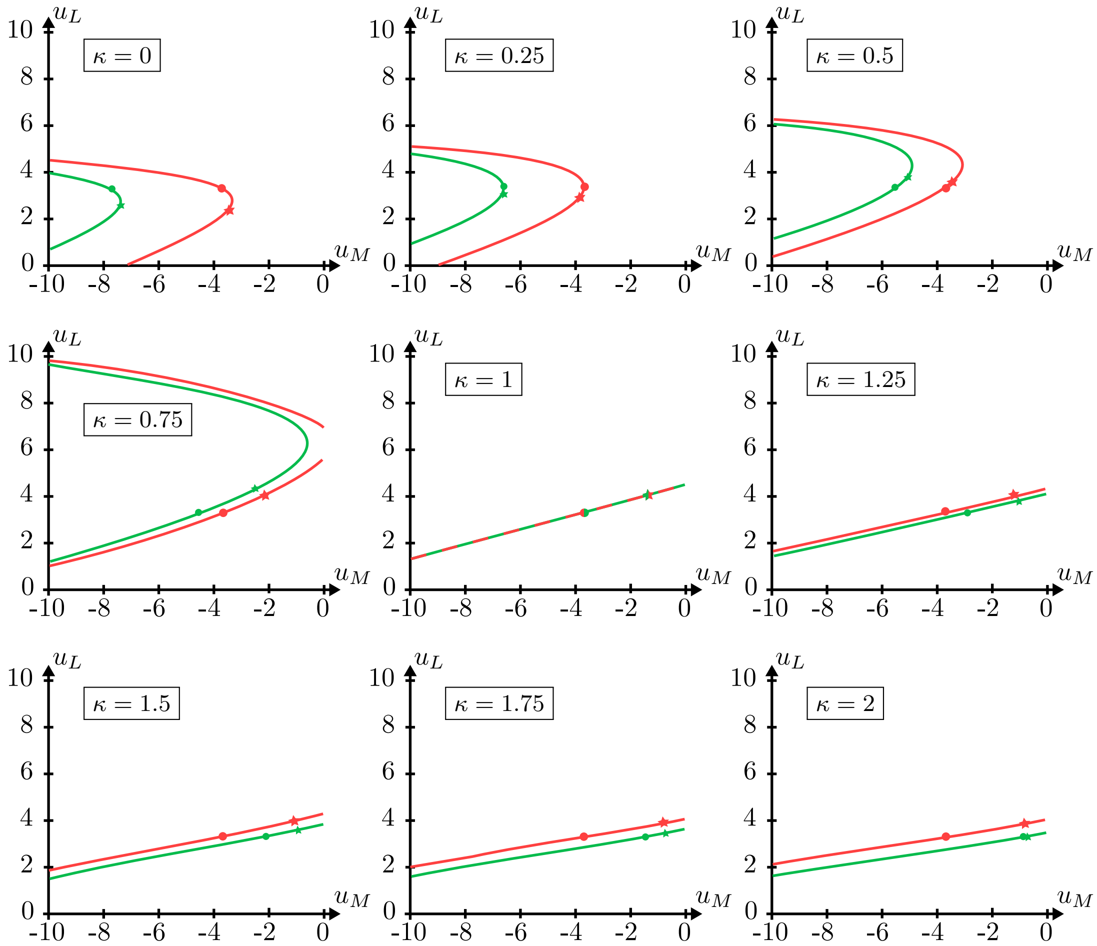

Figure 11: Limit state surfaces in standard normal space in case of quadratic t_S for $\kappa = 0, 0.25, 0.5, \ldots, 2$ following design options 1 (red) or 2 (green). Stars represent the FORM design points, dots represent the design points implied by the PSF concept.

as initial actions. This includes permanent loads deterministically, which is in contrast to EN1990:2002 that includes permanent loads semi-probabilistically; however, since the uncertainties of permanent loads are typically small this can be considered as a good approximation.

Figure 12 shows the resulting reliability indices. High values of y_0 indicate large initial actions. With increasing y_0, both design options become significantly more conservative, leading to strong over-design. The only exception occurs with design option (1) when the value of κ is rather low ($\kappa \lesssim 0.5$), however, this is only relative to the base case. In absolute terms, the resulting reliability indices are still conservative. Overall the amount of initial actions has significant effect on the reliability. This issue is typically not covered by PSF codes (e.g., [9, 19]).

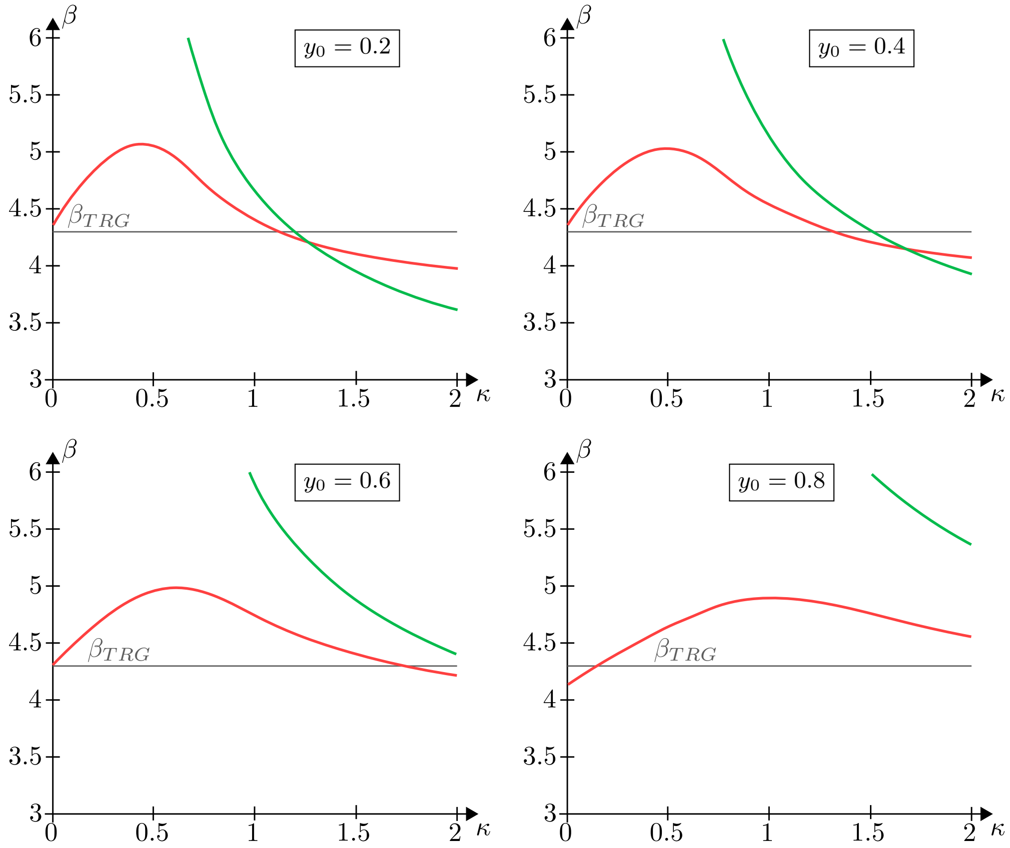

Figure 12: Reliability indices in case different y_0 following design options 1 (red) or 2 (green).

5.4 Effect of the distribution types

We alter the distribution type of the action L and the material strength M of the base case from Gumbel and log-normal to both being normal. The resulting reliability indices shown in Figure 13 strongly differ from the base case. Now design option (1) is non-conservative for $\kappa < 1$ and conservative for $\kappa > 1$. Design option (2) is still conservative for $\kappa < 1$ and non-conservative for $\kappa > 1$; however the conservatism is significantly less than the base case if $\kappa < 1$. In this case the approach of EN1990:2002 [9] to respectively choose the more conservative of the two design options is satisfactory.

5.5 Effect of the uncertainty of action and material

To investigate the uncertainty of action and material we alter the coefficients of variation of the action L between 0.1, 0.3 and 0.5 and the coefficients of variation of the material strength M between 0.05, 0.10 and 0.15. This covers the typical range of coefficients of variation of the action and resistance side [39]. The resulting reliability indices are very sensitive to these changes (see Figure 14). In general, the more the design situation is dominated by the uncertainty of the action side ($\mathrm{c.o.v.}[L] >> \mathrm{c.o.v.}[M]$), the greater the

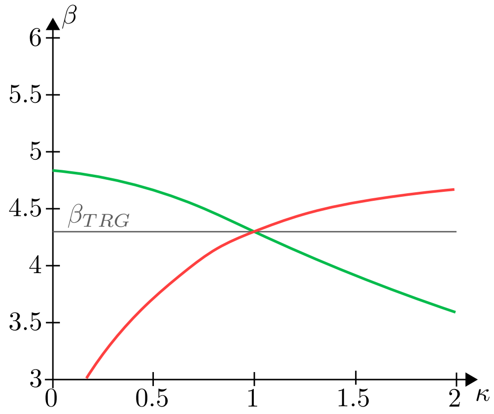

Figure 13: Reliability indices in case of L and M being normally distributed (design options 1 (red) and 2 (green)).

deviation from target reliability β_{TRG} is.

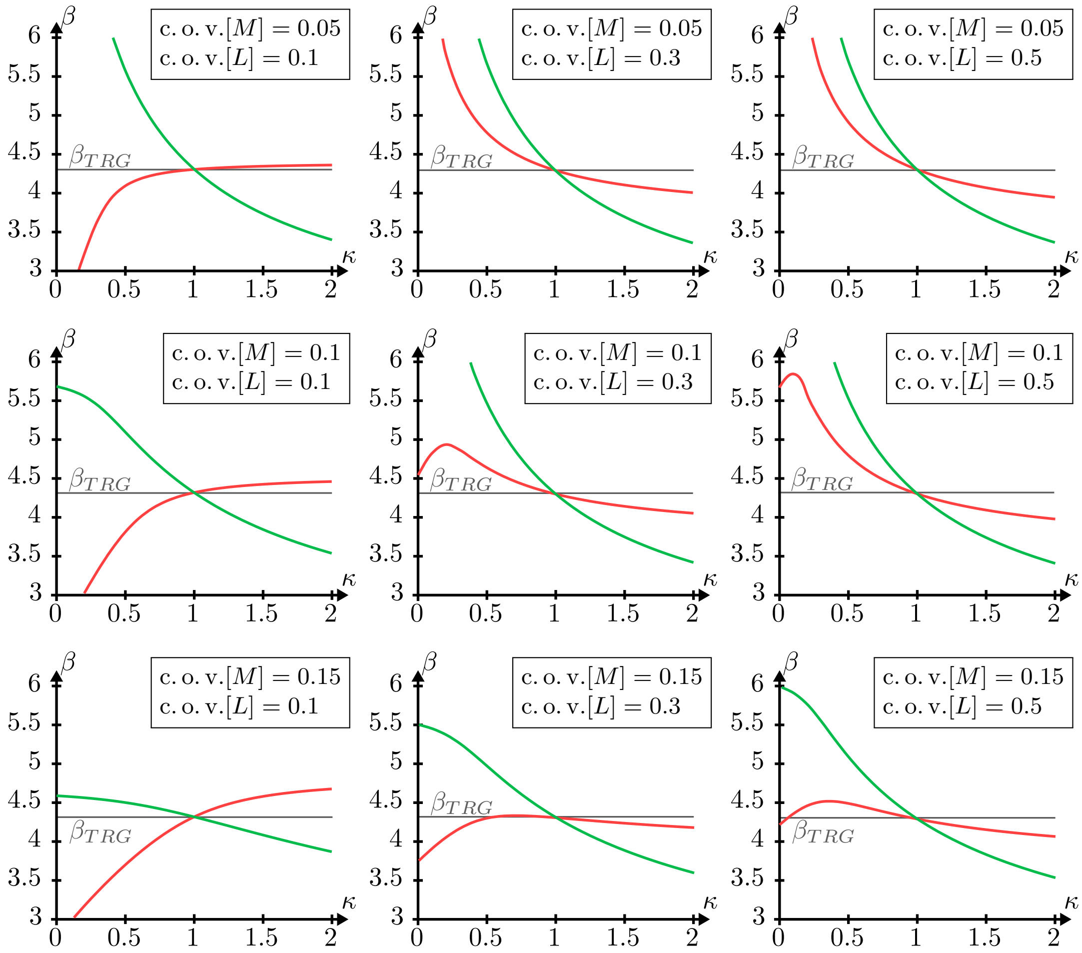

Figure 14: Reliability indices in case of $\mathrm{c.o.v.}[L] = 0.1, 0.2, ..., 0.5$ and $\mathrm{c.o.v.}[M] = 0.05, 0.1, ..., 0.25$ (design options 1 (red) and 2 (green)).

Especially the reliability resulting from design option (1) shows high sensitivity to the values of c. o. v.$[L]$ and c. o. v.$[M]$: The ranges of κ for which option (1) is conservative or non-conservative can switch: If the design situation is dominated by the uncertainty of the action design, option (1) is conservative for $\kappa < 1$ and non conservative for $\kappa > 1$. With less domination of the action (e.g., graph in the left column of Figure 14) this characteristic switches and design option (1) is non-conservative for $\kappa < 1$ and conservative for $\kappa > 1$.

Structural design codes typically only provide different PSF for different coefficients of variation of action and resistance, but do not provide different non-linear design procedures [9, 19]. The next section shows the case if the PSF are adjusted.

5.6 Effect of the values of the partial safety factors

We alter the PSFs – without changing the target reliability – such that the design point and the FORM design point coincide in the linear case ($\kappa = 1$ and $y_0 = 0$); hence, the dot and the star in the middle row and middle column of Figure 8 lie on top of each other. This can be interpreted as a more ideal choice of PSFs. We do for different combinations of the coefficients of variation of the action and the material strength as in Section 5.5. Figure 15 shows the resulting PSFs and the resulting reliability indices.

The resulting reliability indices of design option (1) differ significantly from those observed with fixed PSFs (Figure 14). For $\kappa \lesssim 0.5$ design option (1) is non-conservative in all considered cases. If $\kappa \gtrsim 0.5$ the resulting reliability indices are very close to the target reliability index. This "convergence" is more rapid if c. o. v.$[L]$ >> c. o. v.$[M]$.

The resulting reliability indices of design option (2) are unaffected by the values of the PSFs compared to the case of less ideal PSFs (Figure 14). This is because only the individual values of γ_M and γ_L differ but not their product $\gamma_M \cdot \gamma_L$; hence, the design resulting from option (2) is unaffected.

5.7 Summery of the parameter study

The parameter study answers our research question – how non-linear models affect structural reliability given one of the two design options – as follows: The degree of non-linearity of the structural response function t_S – measured via κ and y_0 – has great effect on the structural reliability. In case of design option (1), the reliability index is either increasing with increasing κ (e.g., bottom left of Figure 14) or the reliability index is first increasing with increasing κ reaching a maximum at $0 < \kappa < 1$ and then decreasing with increasing κ (e.g., bottom right of Figure 14). Moreover, the reliability index is increasing with increasing y_0. In case of design option (2) the reliability index decreases with increasing κ and increases with increasing y_0. The exact functional form of t_S (e.g., bi-linear or quadratic) only plays a minor role.

However, not only the non-linearity of the structural response function but also the interaction of the non-linear structural response functions with (semi)-probabilistic properties

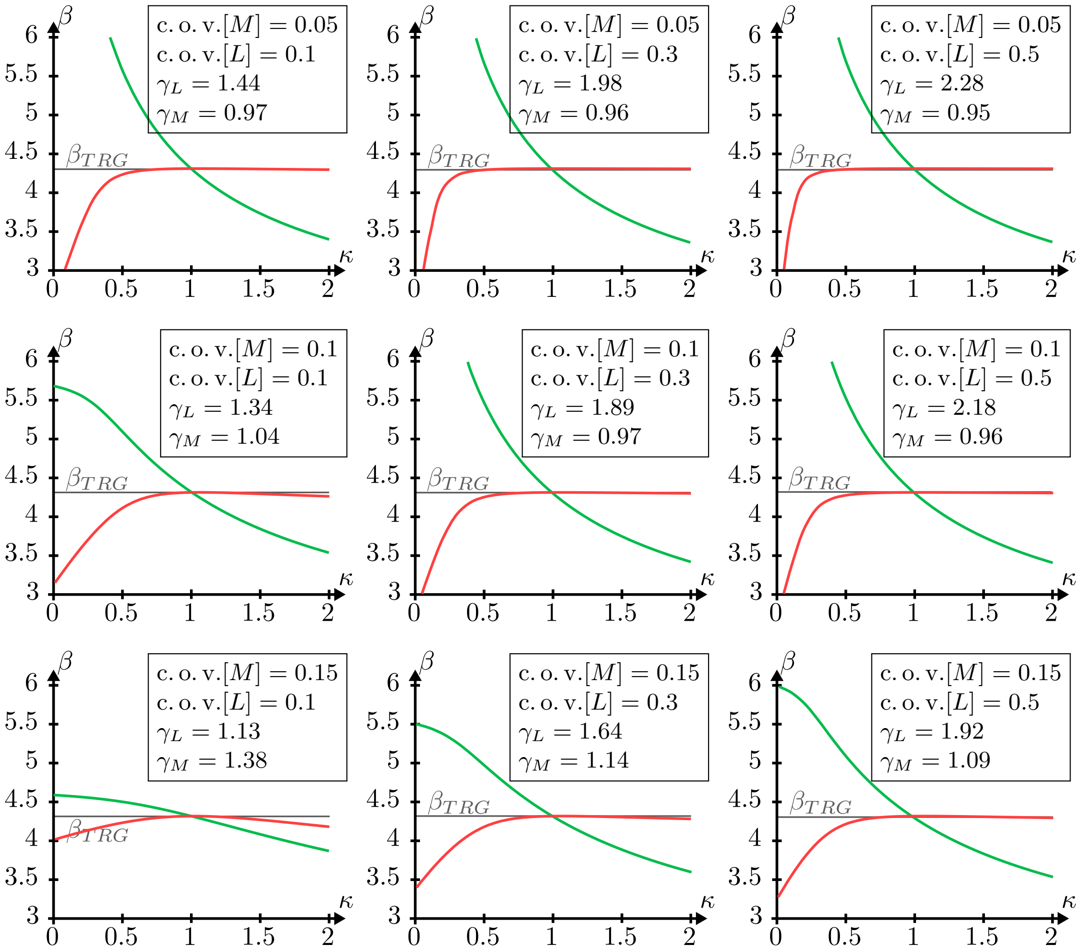

Figure 15: Reliability indices in case of PSF chosen such that the FORM design point and design point implied by the PSF concept coincide in the linear case (design options 1 (red) and 2 (green)).

strongly affects the structural reliability. These (semi-)probabilistic quantities include the choice of the PSF as well as the distribution types and distribution parameters of the action and material strength.

6 Example structures

In this section we transfer the insights from the theoretical investigations to two example structures: A dome space truss structure and a membrane structure. We show how one can derive the measure of non-linearity (κ and y_0) for these examples and how one can use the measure of non-linearity to classify the example structures within the general parameter study.

6.1 24-bar dome space truss structure

Dome-like space truss structures are regularly utilized examples for investigations in the presence of geometrical non-linearity. The observed 24-bar dome truss (cf. Figure 16) is a slightly modified version of the structure proposed in [26]. Most of the trusses (indicated as "truss 1" (solid lines) and "truss 3" (dashed-dotted lines) in Figure 16) are tensioned and instability of the overall structure can be avoided. We assume that local buckling of the compressed members (indicated as "truss 2" (dashed lines) in Figure 16) is prevented constructively.

For the ultimate limit state design, the situation is considered in which the maximal stress in the trusses exceeds the yield strength. Steel S355 with a characteristic yield strength of f_y=355 MPa is chosen. We assume a Gumbel distributed action L with mean $\mathrm{E}[L] = 0.0375\ MN$ and $\mathrm{c.o.v.}[L] = 0.3$ and a log-normally distributed yield strength M with mean $\mathrm{E}[M] = 412.8\ MPa$ and $\mathrm{c.o.v.}[M] = 0.07$ [39].

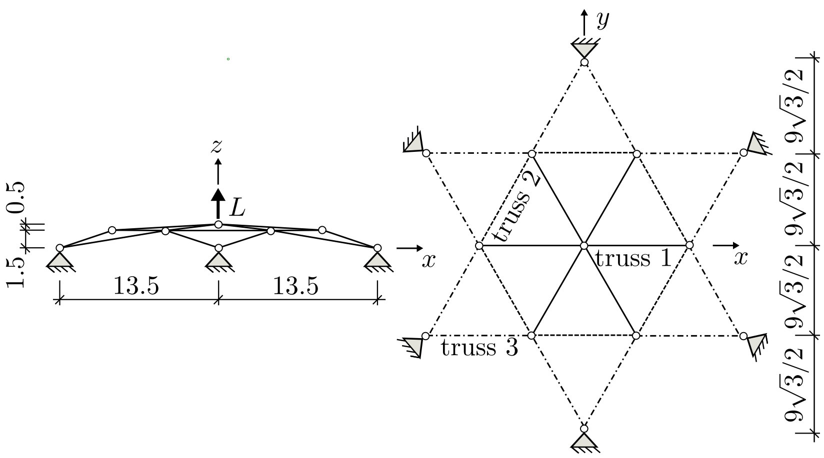

Figure 16: Observed 24-bar dome space truss structure shown in side view (left) and top view (right) with action L acting on the center node. The solid, dashed and dashed-dotted lines indicate the three different truss types. The dimensions are given in meters.

6.1.1 Partial safety factor design

The cross-sectional design of the steel trusses follows the rules of EN1990:2002 [9] and EN1993:2005 [41]. The utilized partial safety factors are

$$\gamma_F = 1.5 \tag{26}$$

$$\gamma_{M0} = 1.0 \tag{27}$$

with γ_{M0} as the partial safety factor for the stressability of cross sections. The characteristic values are chosen based on [7] as

$$l_k = F_L^{-1}(0.98) = 0.0667\ MN \tag{28}$$

$$m_k = f_y = E[M] - 2 \cdot \sigma_M = 355.0 \; MPa \tag{29}$$

with σ_M being the standard deviation of M.

Due to the symmetry of the structure and loading, only the cross section of three trusses must be designed, which are indicated as "truss 1-3" in Figure 16. Hence, the goal of the structural design is to determine the cross sections $\mathbf{A} = [A_1, \, A_2, \, A_3]$. Based on the two design options of Equation 8 and 9, the PSF designs

$$(1) \quad N_{truss\; i}\,(\gamma_F \cdot l_k, \, \mathbf{A}) = A_i \cdot \frac{f_y}{\gamma_{M0}} \quad \text{or} \quad (2) \quad \gamma_F \cdot N_{truss\; i}\,(l_k, \, \mathbf{A}) = A_i \cdot \frac{f_y}{\gamma_{M0}} \tag{30}$$

are obtained for the ith truss member. The normal force $N_{truss\; i}$ represents the structural response function t_S and the cross section $A_i \in \mathbf{A}$ can be interpreted as the design parameter p (cf. Section 3). The transformation of Equation 30 leads to

$$(1) \quad A_i = \frac{\gamma_{M0} \cdot N_{truss\; i}\,(\gamma_F \cdot l_k, \, \mathbf{A})}{f_y} \quad \text{or} \quad (2) \quad A_i = \frac{\gamma_{M0} \cdot \gamma_F \cdot N_{truss\; i}\,(l_k, \, \mathbf{A})}{f_y} \tag{31}$$

as calculation rule for the cross section A_i.

Equation 30 indicates that the normal forces depend on the cross sections $\mathbf{A}$ of the members as well. As the cross sections are changing based on Equation 31, the PSF design needs to be executed iteratively. Consequently, also the function $N_{truss\; i}\,(l, \, \mathbf{A})$ will be different for the design option (1) and (2). This can can be seen in Figure 17, which shows the progress of the normal forces for an increasing action as an action - effect of action diagram. The graphs of $N_{truss\; i}\,(l, \, \mathbf{A})$ are based on the final designs, i.e., after the iteration process to determine the cross sections has successfully converged. For the structure at hand, the PSF design based on option (2) leads to larger absolute values of the normal forces for each truss.

6.1.2 Measure of non-linearity and classification in parameter study

The curvature measure κ (cf. Section 4.2) to classify the non-linearity of $N_{truss\; i}\,(l, \, \mathbf{A})$ is (i) changing during the iteratively design process and is (ii) slightly different when design options (1) or (2) are utilized. Table 1 summarizes the values of κ of the three investigated trusses based on the two design options. These result indicate that κ can be very different for different members of the same structure.

Table 1: κ-values for the three observed truss members designed according to PSF option (1) and (2).

Design option	truss 1	truss 2	truss 3
(1)	0.727	0.637	0.965
(2)	0.734	0.651	0.964

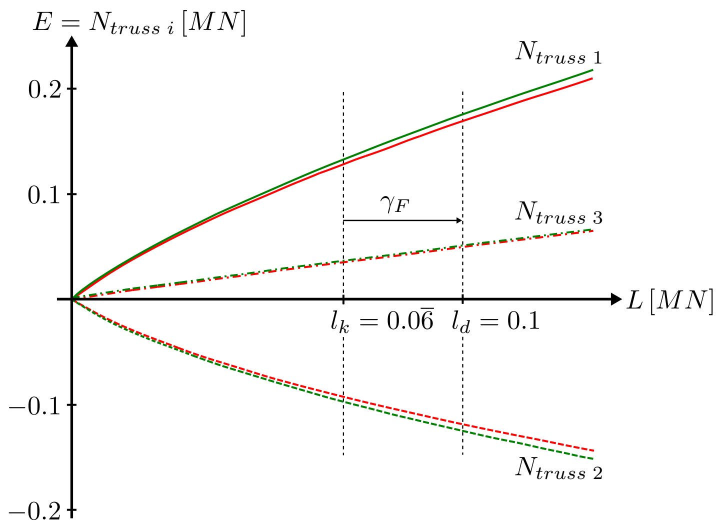

Figure 17: Action-effect of action diagrams of the truss dome normal forces $N_{truss\ i}$ (cf. Figure 16). The red lines correspond to the design based on PSF option (1) and the green lines are according to design option (2).

The classification within the parameter study is not straightforward since the parameter study of Section 5 only covers a finite set of possible structural designs. For the structure at hand, the following conditions differ from the parameter study: The partial safety factor γ_{M0} was taken from the standard and was not specifically computed to achieve a target value for the linear case. The truss dome is therefore not at the same reliability level as the parameter study. Moreover, the characteristic material strength, the functions $N_{truss\ i}$ and the coefficient of variation of the material strength are not exactly covered by the parameter study.

The case of the parameter study which comes closest to the truss dome example is the case of bi-linear t_S with log-normally distributed M with $\text{c.o.v.}[M] = 0.05$ and Gumbel distributed L with $\text{c.o.v.}[L] = 0.3$ shown in Figure 14 in the first row and the second column. The Figure shows that both design options are conservative for $\kappa < 1$ compared to the linear case; hence, the parameter study suggests that all three trusses are conservative compared to the linear case. This is verified in the reliability analysis of the next Section 6.1.3.

6.1.3 Reliability Analysis

Based on the cross sections A_i (Equation 31), the limit state function for the ith structural member can formulated following Equation 17:

$$g_i = A_i \cdot M - N_{truss\ i}(L) = \begin{cases} \frac{\gamma_{M0} \cdot N_{truss\ i}(\gamma_F \cdot l_k)}{f_y} \cdot M - N_{truss\ i}(L) & \text{option (1)} \\ \frac{\gamma_{M0} \cdot \gamma_F \cdot N_{truss\ i}(l_k)}{f_y} \cdot M - N_{truss\ i}(L) & \text{option (2)} \end{cases} \quad (32)$$

Here, the dependency of $N_{truss\ i}$ on $\mathbf{A}$ is dropped for better readability.

Table 2 shows the resulting reliability indices calculated following Equation 18. The difference between design option (1) and (2) is larger when the curvature measure κ differs from 1. This can be seen in particular by comparing the results of trusses 2 and 3. Furthermore, the reliability indices based on designs determined with PSF option (1) show less sensitivity to a varying degree of non-linearity as the designs according to option (2).

Table 2: Reliability indices β for the three observed truss members designed according to PSF option (1) and (2).

Design option	truss 1	truss 2	truss 3
(1)	3.79	3.83	3.72
(2)	4.18	4.40	3.76

The limit state surface in standard normal space of both design options, together with the design points implied by the PSF concept and the FORM design points, are shown in Figure 18. It can be seen that the difference between the limit state surfaces based on the two design options is increasing the more κ varies from 1. Figure 18 indicates also that all limit state surfaces are only marginally non-linear in standard normal space despite the low κ values for trusses 1 and 2.

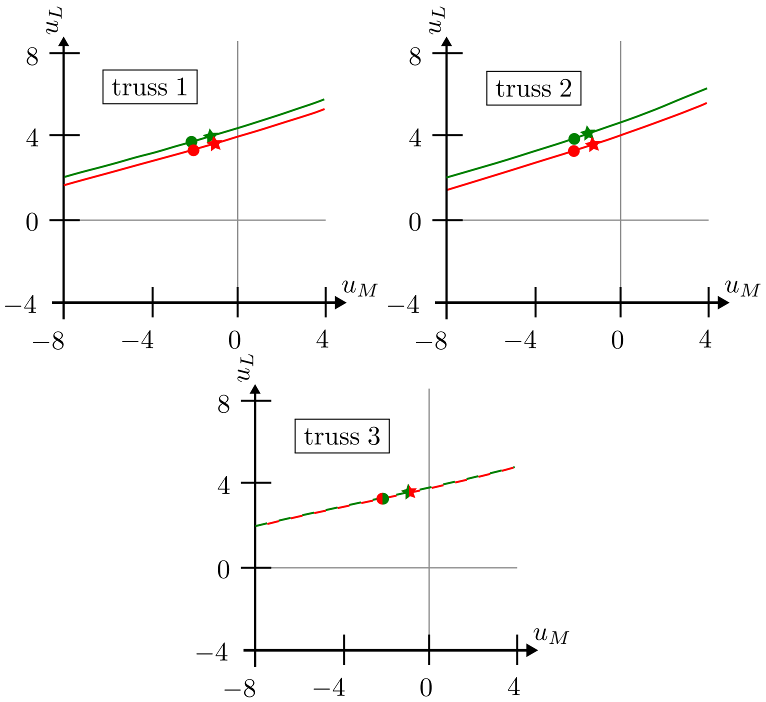

Figure 18: Limit state surface of the truss members 1-3 (cf. Figure 16) designed following option (1) (red) and option (2) (green) in standard normal space. The bullet points indicate the design points implied by the PSF concept. The stars indicate the FORM design points.

To verify the classification of Section 6.1.2, we repeated the reliability analysis with the hypothetical case of linear functions for the normal forces. The resultant reliability index is 3.70 for all trusses. Since 3.70 is below all values of Table 2, the classification as conservative design by the parameter study is confirmed.

6.2 Membrane structure

The investigated membrane structure is a hyperbolic paraboloid (hypar), which is shown in Figure 19. It is a slightly modified version of the hypar presented in the Round Robin Exercise 4 of [42] (RR4). The structure was already investigated by the authors in [43] where the effect of the non-linearity of membranes on the reliability was discussed. The structure has a base area of 6×6 m and a height of 2 m (cf. coordinates of edge points in Figure 19) and is subjected to a snow load, which is acting in negative z-direction. The membrane and its edge cables are fixed at the low and high points. The Young's moduli in warp and fill direction are $E_{warp/fill}$ = 600 kN/m (pre-integrated over the thickness), the shear modulus is G=30 kN/m (pre-integrated over the thickness) and the Poisson's ratio is ν=0.4. The edge cables have a Young's modulus of 205 kN/mm^2 and a diameter of 12 mm. The membrane is subjected to an isotropic pre-stress of 3.0 kN/m and the edge cables are pre-stressed by 30 kN.

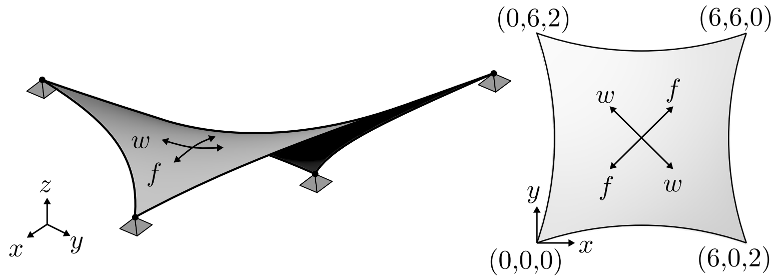

Figure 19: Observed membrane structure with indication of warp (w) and fill (f) direction.

We assume a Gumbel distributed snow load L with mean $\mathrm{E}[L] = 0.34\ kN/m^2$ and $\mathrm{c.o.v.}[L] = 0.3$ and a log-normally distributed tensile strength M with mean $\mathrm{E}[M] = 1.0\ kN/m^2$ and $\mathrm{c.o.v.}[M] = 0.1$.

6.2.1 Partial safety factor design

The utilized partial safety factors are

$$\gamma_F = 1.5 \tag{33}$$

$$\gamma_M = 1.4 \tag{34}$$

with γ_F taken from EN1990:2002 [9] and γ_M taken from the Technical Specification [44] of CEN TC250 WG5. The characteristic values defined following [7] as the 98% und 5% fractile are

$$l_k = F_L^{-1}(0.98) = 0.60\ kN/m^2 \tag{35}$$

$$m_k = F_M^{-1}(0.05) = 0.84\ kN/m^2 \tag{36}$$

For the ultimate limit state design, the situation is considered in which the maximal stress exceeds the tensile strength of the membrane. Because the snow load is acting in negative z-direction on the membrane, the decisive stress is appearing in warp direction. The progress of the maximal stress in warp direction for an increasing snow load is shown on the left hand side of Figure 20 (blue line). On the right hand side of Figure 20 the stress distribution in warp direction due to design action $l_d = \gamma_F \cdot l_k$ and the position of the maximal stress is shown. It can be seen that the membrane is fully under tension at this stage, i.e., no wrinkling occurs.

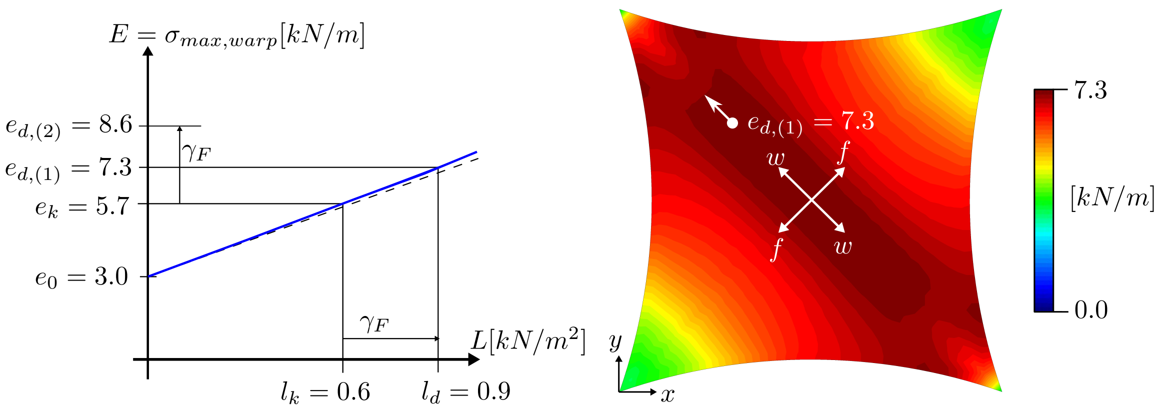

Figure 20: Left: Progress of maximal stress in warp direction of the membrane due to increasing action (blue), its tangent at zero action (dashed). Right: Distribution of stress in warp direction due to design action l_d.

The PSF design is calculated following Equation 15. In contrast to the truss dome structure, which is discussed in Section 6.1, we assume here that the choice of the design parameter p does not influence the structural behavior of the membrane, i.e., the maximal membrane stress is supposed to be independent of p. Hence, the PSF design can be computed within one step by applying Equation 31 without any further iterations.

6.2.2 Measure of non-linearity and classification in parameter study

The offset measure y_0 and the curvature measure κ are

$$y_0 = \frac{e_0}{e_k} = 0.53 \qquad\qquad \kappa = \frac{m_2}{m_1} = 1.19 \tag{37}$$

Similar to the truss dome example of Section 6.1.2, the structure cannot be exactly classified in the parameter study shown in Section 5. The case of the parameter study which comes closest to the membrane example is the case of bi-linear t_S, log-normally distributed M with c. o. v.$[M] = 0.1$, Gumbel distributed L with c. o. v.$[L] = 0.3$ and an initial action of $y_0 = 0.6$, shown in Figure 12 (bottom left). For the value of κ of the membrane of 1.19, this figure suggests that both design options are conservative compared to the linear case ($\kappa = 1$ and $y_0 = 0$).

We further investigate two more hypothetical cases: In the first case, we set κ to 1 and y_0 remains as in the original structure. By comparing this case with the original case, we can isolate the non-linear effect of the convex form of t_S. The comparison can be made within Figure 12 (bottom left). The reliability indices of both design options decrease from $\kappa = 1$ to $\kappa = 1.19$, therefore, the parameter study suggests that both design options of the original structure are non-conservative compared to this hypothetical case. From this we conclude that the convex form of t_S has a negative effect on the reliability.

Second, we investigate the hypothetical case if κ remains the same as in the original structure but y_0 is set to 0. By comparing this case with the original case, we can isolate the non-linear effect of the prestress. The comparison can be done by comparing Figure 12 (bottom left) to the base case (Figure 7). This shows that both design options of the original structure are conservative compared to this hypothetical case. From this we conclude that the prestres has a positively effect on the reliability.

Moreover, the positive effect of the prestress is greater than the negative effect of the convex form of t_S, since the comparison to the linear case showed an overall positive effect of the non-linearity.

6.2.3 Reliability analysis

The reliability analysis is conducted following Equation 18. The resulting reliability indices are shown in Table 3.

Table 3: Reliability indices of the membrane according to designs options (1) and (2).

Design option	Reliability index β
(1)	4.96
(2)	5.56

The limit state surface in the standard normal space of both design options and the design points according to PSF concept and FORM are shown in Figure 21.

The authors investigated the hypothetical linear case already in [43] and received the reliability index of 4.72 for both design options. Since 4.72 is below the resulting reliability indices (4.96 and 5.56) of Table 3, this result agrees with the conclusions from the parameter study.

Moreover, in [43] the authors performed a reliability analysis for the two above mentioned hypothetical cases (first case: $\kappa = 1$ and y_0 remains, second case: $y_0 = 0$ and κ remains). The reliability indices of the first hypothetical case are 5.30 (design option 1) and 6.20 (design option 2). The reliability indices of the second hypothetical case are 4.46 (design option 1) and 4.26 (design option 2) [43]. This confirms the positive effect of the prestress and the negative effect of the convex form of t_S suggested by the parameter study.

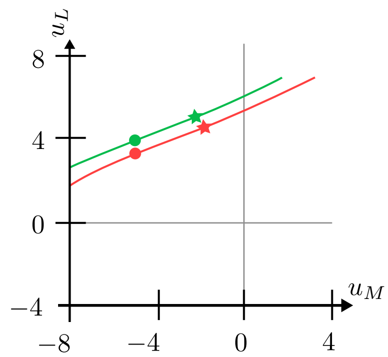

Figure 21: Limit state surface of the membrane designed with option (1) (red) and option (2) (green) in standard normal space. The bullet points indicate the design points following the PSF concept. The stars indicate the FORM design points.

7 Discussion

Semi-probabilistic structural design codes like the Eurocode typically choose the more conservative alternative from design option (1) and (2) which, obviously, leads to the larger structural reliability. This signifies that in Figures 7, 10, 12, 13 and 15 the upper of the two curves is chosen. If this curve is below/above the reliability index one would obtain with a linear t_S through the origin (in case of the parameter study this is β_{TRG}) it is unsafe/safe but has low/high resource consumption.[7] What deviation from the reliability index at the linear level is critical for safety or resource consumption is debatable and to some extent subjective. In our opinion, the policy of choosing the more conservative option leads to sufficient structural design in case of convex t_S ($\kappa > 1$) without initial force ($y_0 = 0$). If $\kappa < 1$ (concave t_S) and/or $y_0 > 0$ (initial action is present), this policy can lead to an unsustainable large over-design.

The curvature measure κ and the offset measure y_0 can be calculated as a side product of a PSF design and can therefore provide guidance to the engineer in classifying a design without having to perform a reliability analysis. Similar to the investigated examples an engineer can roughly classify a structure within the parameter study and determine if the design is on the safe side or not (see Sec. 6.1.2 and 6.2.2). However, a guidance on how to adapt the design is not given within this paper. Three possible adaptation approaches within the PSF concept are:

- The rule when to choose design option (1) or (2) could be based on ranges of values of κ and y_0. In contrast to the current policy, the design option which is leading to the smaller design value should also be valid in some cases. In particular, design option (1) can be preferable also for cases of $\kappa < 1$. This could avoid drastic over-design. Additionally, in extreme ranges of κ and y_0, further analysis or even a full probabilistic analysis could be recommended or required.

[7] In some design situations this reliability index at linear level inherently may be to high or to low; however, the reasons for this is not related to the non-linearity of the structural response function but something different (e.g., suboptimal PSFs) what should be considered separately.

- An additional PSF could be implemented. The value of the PSF would depend on the degree of the non-linearity (i.e., on the value of κ and y_0) of the design situation. The more case specific this additional PSF is derived, the better the homogenization of the reliability level would be.

- A split of the PSF with respect to the uncertainty of the action and the uncertainty of the structural response function could be conducted. This would reverse the merge of Equation 4 and values of γ_f and γ_{Sd} could be determined individually.

In general, the non-linearity of a structure can not be quantified on system level, but only the non-linearity of an action - effect of action relation corresponding to a certain limit state can be quantified. Hence, separate values of y_0 and κ need to be determined for each limit state. Therefore, each of the above adaptation approaches needs to be applied on each limit state of a structure separately.

All three adaptation approaches would homogenize the reliability level such that it would be closer to β_{TRG}. All three adaptations would only depend on κ and y_0, hence, only cover the non-linearity of the structural response function but not take the interaction of non-linear structural response functions with (semi)-probabilistic properties into account. However, the conducted parameter study shows that the interaction of (semi)-probabilistic properties with non-linear structural response functions has a great impact on the reliability. Therefore, all three adaptations would only partly homogenize the level of safety with respect to non-linearities.

The investigations of this paper can be extended in the following directions: The initial actions could be treated probabilistically. Moreover, the case of multiple actions could be considered, and the semi-probabilistic measure of non-linearity could be extended to the multidimensional case. This extension should not only cover the non-linearity of each individual action - effect of action relation, but also the interaction of the effects of different actions.

8 Conclusion

We systematically investigated the effects of non-linear structural response functions within a PSF design on the structural reliability. The conducted parameter study and the two application examples reveal some of the effects. We showed that not only the degree of non-linearity of the structural response function but also the interaction of non-linear structural response functions with (semi)-probabilistic properties has a strong effect on the structural reliability. For this reason, it is impossible to homogenize the safety level perfectly with respect to non-linear models without leaving the scope of the PSF concept. However, there is some potential to homogenize the safety level. This seems to be especially necessary in case of strongly concave structural response functions or in cases of large initial force (e.g., prestress), which both can lead to heavy over-design. Cases of under-design are possible if the structural response function is convex; however, the under-design appears to be acceptable.

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