Probability Distribution Table

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Name	PDF/PMF and CDF	Support	Parameters	Mean	Standard deviation
Beta	$f_X(x) = \frac{(x-a)^{r-1}(b-x)^{s-1}}{B(r,s)(b-a)^{r+s-1}}$ $F_X(x) = I_{\frac{x-a}{b-a}}(r,s)$	$x \in (a,b)$	$r > 0$ $s > 0$ $(a < b) \in \mathbb{R}$	$\frac{as+br}{r+s}$	$\frac{b-a}{r+s}\sqrt{\frac{rs}{r+s+1}}$
Binomial	$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$ $F_X(x) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$	$x \in \{0, ..., n\}$	$n \in \mathbb{N}_0$ $p \in [0,1]$	np	$\sqrt{np(1-p)}$
Chi-squared	$f_X(x) = \frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)} x^{\left(\frac{k}{2}-1\right)} \exp\left(-\frac{x}{2}\right)$ $F_X(x) = \frac{\gamma\left(\frac{k}{2}, \frac{x}{2}\right)}{\Gamma\left(\frac{k}{2}\right)}$	$x \in [0, \infty)$	$k \in \mathbb{N}_{>0}$	k	$\sqrt{2k}$
Exponential	$f_X(x) = \lambda \exp(-\lambda x)$ $F_X(x) = 1 - \exp(-\lambda x)$	$x \in [0, \infty)$	$\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$

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Fréchet	$f_X(x) = \frac{k}{a_n}\left(\frac{a_n}{x}\right)^{k+1}\exp\left(-\left(\frac{a_n}{x}\right)^k\right)$ $F_X(x) = \exp\left(-\left(\frac{a_n}{x}\right)^k\right)$	$x \in [0, \infty)$	$a_n \in (0, \infty)$ $k \in (0, \infty)$	$a_n\Gamma\left(1-\frac{1}{k}\right)$ for $k > 1$ ∞ for $k \leq 1$	$a_n\left[\Gamma\left(1-\frac{2}{k}\right) - \Gamma^2\left(1-\frac{1}{k}\right)\right]^{1/2}$ for $k > 2$ ∞ for $k \leq 2$
Gamma	$f_X(x) = \frac{\lambda^k x^{k-1}\exp(-\lambda x)}{\Gamma(k)}$ $F_X(x) = \frac{\gamma(k,\, \lambda x)}{\Gamma(k)}$	$x \in [0, \infty)$	$k > 0$ $\lambda > 0$	$\frac{k}{\lambda}$	$\sqrt{\frac{k}{\lambda^2}}$
Geometric	$p_X(x) = (1-p)^{x-1}p$ $F_X(x) = 1-(1-p)^x$	$x \in \{1, 2, 3, ...\}$	$p \in (0, 1]$	$\frac{1}{p}$	$\sqrt{\frac{1-p}{p^2}}$
GEV	$f_X(x) = \frac{1}{\alpha}\,(t(x))^{\beta+1}\exp(-t(x))$ $F_X(x) = \exp(-t(x))$ with $t(x) = \left(1+\beta(\frac{x-\epsilon}{\alpha})\right)^{-1/\beta}$	$x \in [\epsilon - \frac{\alpha}{\beta}, \infty)$ for $\beta > 0$ $x \in (-\infty, \epsilon - \frac{\alpha}{\beta}]$ for $\beta < 0$	$\alpha > 0$ $\beta \in \mathbb{R}$ $\epsilon \in \mathbb{R}$	$\epsilon + \alpha\frac{\Gamma(1-\beta)-1}{\beta}$ for $\beta \neq 0, \beta < 1$ ∞ for $\beta \geq 1$	$\frac{\alpha}{\beta}\sqrt{\Gamma(1-2\beta)-\Gamma(1-\beta)^2}$ for $\beta \neq 0, \beta < 1/2$ ∞ for $\beta \geq \frac{1}{2}$

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GEV Min (mirror image of GEV around ϵ)	$f_X(x) = \frac{1}{\alpha}(t(x))^{\beta+1}\exp(-t(x))$ $F_X(x) = 1 - \exp(-t(x))$ with $t(x) = \left(1 - \beta(\frac{x-\epsilon}{\alpha})\right)^{-1/\beta}$	$x \in [\epsilon + \frac{\alpha}{\beta}, \infty)$ for $\beta < 0$ $x \in (-\infty, \epsilon + \frac{\alpha}{\beta}]$ for $\beta > 0$	$\alpha > 0$ $\beta \in \mathbb{R}$ $\epsilon \in \mathbb{R}$	$\epsilon - \alpha\frac{\Gamma(1-\beta)-1}{\beta}$ for $\beta \neq 0, \beta < 1$ ∞ for $\beta \geq 1$	$\frac{\alpha}{\beta}\sqrt{\Gamma(1-2\beta) - \Gamma(1-\beta)^2}$ for $\beta \neq 0, \beta < 1/2$ ∞ for $\beta \geq \frac{1}{2}$
Gumbel	$f_X(x) = \frac{1}{a_n}\exp(-z - \exp(-z))$ $F_X(x) = \exp(-\exp(-z))$ with $z = \frac{x - b_n}{a_n}$	$x \in (-\infty, \infty)$	$a_n > 0$ $b_n \in \mathbb{R}$	$b_n + a_n\gamma$ where $\gamma \approx 0.577216$	$\frac{\pi a_n}{\sqrt{6}}$
Gumbel Min (mirror image of Gumbel around b_n)	$f_X(x) = \frac{1}{a_n}\exp(z - \exp(z))$ $F_X(x) = 1 - \exp(-\exp(z))$ with $z = \frac{x - b_n}{a_n}$	$x \in (-\infty, \infty)$	$a_n > 0$ $b_n \in \mathbb{R}$	$b_n - a_n\gamma$ where $\gamma \approx 0.577216$	$\frac{\pi a_n}{\sqrt{6}}$
Log-normal	$f_X(x) = \frac{1}{x\sigma_{lnX}\sqrt{2\pi}}\exp\left(-\frac{(\ln(x) - \mu_{lnX})^2}{2\sigma_{lnX}^2}\right)$ $F_X(x) = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left[\frac{\ln x - \mu_{lnX}}{\sqrt{2}\sigma_{lnX}}\right]$	$x \in (0, \infty)$	$\mu_{lnX} \in \mathbb{R}$ $\sigma_{lnX} > 0$	$\exp\left(\mu_{lnX} + \frac{\sigma_{lnX}^2}{2}\right)$	$\exp\left(\mu_{lnX} + \frac{\sigma_{lnX}^2}{2}\right)\sqrt{\exp(\sigma_{lnX}^2) - 1}$

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Negative binomial	$p_X(x) = \binom{x-1}{k-1}(1-p)^{x-k}p^k$ $F_X(x) = \sum_{i=k}^{x}\binom{i-1}{k-1}(1-p)^{i-k}p^k$	$x \in \{k, k+1, ...\}$	$k \in \mathbb{N}$ $p \in (0,1)$	$\frac{k}{p}$	$\sqrt{\frac{k(1-p)}{p^2}}$
Normal	$f_X(x) = \frac{1}{\sqrt{2\sigma^2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ $F_X(x) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$	$x \in \mathbb{R}$	$\mu \in \mathbb{R}$ $\sigma > 0$	μ	σ
Pareto	$f_X(x) = \frac{\alpha\, x_{\mathrm{m}}^{\alpha}}{x^{\alpha+1}}$ $F_X(x) = 1 - \left(\frac{x_{\mathrm{m}}}{x}\right)^{\alpha}$	$x \in [x_m, \infty)$	$x_m > 0$ $\alpha > 0$	$\frac{\alpha\, x_{\mathrm{m}}}{\alpha - 1}$ for $\alpha > 1$	$\sqrt{\frac{x_{\mathrm{m}}^2\alpha}{(\alpha-1)^2(\alpha-2)}}$ for $\alpha > 2$
Poisson	$p_X(x) = \frac{\lambda^x \exp(-\lambda)}{x!} = \frac{(vt)^x \exp(-vt)}{x!}$ $F_X(x) = \exp(-\lambda)\sum_{i=0}^{\lfloor x \rfloor}\frac{\lambda^i}{i!}$	$x \in \{0, 1, 2, ...\}$	$\lambda > 0$ or $v > 0, \quad t > 0$	$\lambda = vt$	$\sqrt{\lambda} = \sqrt{vt}$
Rayleigh	$f_X(x) = \frac{x}{\alpha^2}\exp(-x^2/2\alpha^2)$ $F_X(x) = 1 - \exp(-x^2/2\alpha^2)$	$x \in [0, \infty)$	$\alpha > 0$	$\alpha\sqrt{\frac{\pi}{2}}$	$\sqrt{\frac{4-\pi}{2}\alpha^2}$

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Standard normal	$\varphi(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right)$ $\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{u} \exp\left(-t^2/2\right) \mathrm{d}t$	$u \in \mathbb{R}$	–	0	1
Truncated normal	$f_X(x) = \frac{\varphi\left(\frac{x-\mu_n}{\sigma_n}\right)}{\sigma_n\left(\Phi\left(\frac{b-\mu_n}{\sigma_n}\right) - \Phi\left(\frac{a-\mu_n}{\sigma_n}\right)\right)}$ $F_X(x) = \frac{\Phi\left(\frac{x-\mu_n}{\sigma_n}\right) - \Phi\left(\frac{a-\mu_n}{\sigma_n}\right)}{\Phi\left(\frac{b-\mu_n}{\sigma_n}\right) - \Phi\left(\frac{a-\mu_n}{\sigma_n}\right)}$	$x \in [a, b]$	$\mu_n \in \mathbb{R}$ $\sigma_n > 0$ $a < b$	$\int_a^b x \cdot f_X(x)\mathrm{d}x$	$\sqrt{\int_a^b x^2 \cdot f_X(x)\mathrm{d}x - \left(\int_a^b x \cdot f_X(x)\mathrm{d}x\right)^2}$
Uniform	$f_X(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$ $F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & x \in [a, b) \\ 1 & x \geq b \end{cases}$	$x \in [a, b]$	$-\infty < a < \infty$ $-\infty < b < \infty$	$\frac{1}{2}(a + b)$	$\sqrt{\frac{1}{12}(b-a)^2}$
Weibull	$f_X(x) = \frac{k}{a_n}\left(\frac{x}{a_n}\right)^{k-1} \exp\left(-\left(\frac{x}{a_n}\right)^k\right)$ $F_X(x) = 1 - \exp\left(-\left(\frac{x}{a_n}\right)^k\right)$	$x \in [0, \infty)$	$a_n \in (0, \infty)$ $k \in (0, \infty)$	$a_n\Gamma(1 + 1/k)$	$a_n \left[\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)\right]^{1/2}$

In order to compute some of the previous expressions the following special functions are required:

- the error function,

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\mathrm{d}t$$

- The beta function,

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\mathrm{d}t$$

- The regularized beta function,

$$I_x(a,b) = \frac{B(x;a,b)}{B(a,b)} = \frac{\int_0^x t^{a-1}\,(1-t)^{b-1}\mathrm{d}t}{B(a,b)}$$

- The gamma function,

$$\Gamma(t) = \int_0^\infty x^{t-1}\exp(-x)\ \mathrm{d}x$$

- The lower incomplete gamma function,

$$\gamma(s,x) = \int_0^x t^{s-1}\exp(-t)\ \mathrm{d}t$$