Engineering Risk Analysis Group
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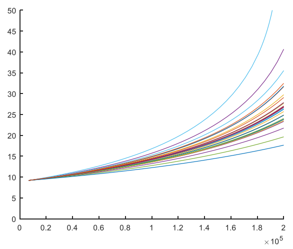
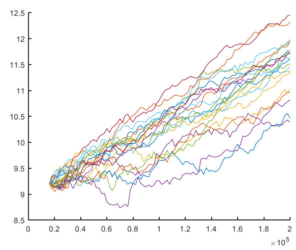
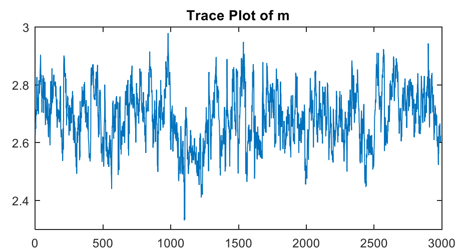
MSc thesis

Bayesian Updating of Stochastic Differential Equation

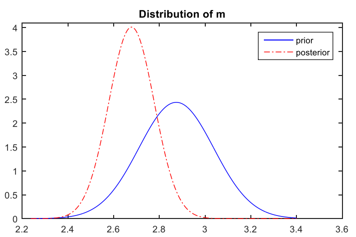
Lidan Jin, August 2015

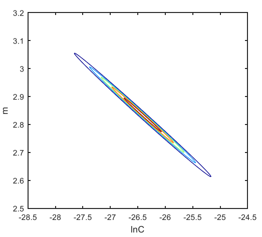
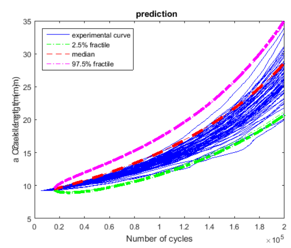
Background

Uncertainty is pervasive in engineering, which needs to be simulated and studied. The frequently used model for that purpose is the deterministic models with “randomized” parameters, i.e. parameters viewed as random variables. This behaves well in modelling and predicting engineering phenomenon with constant loads. An alternative is to use stochastic differential equation (SDE) for modelling also load uncertainty. Either choice needs proper parameters, whose distributions can be found with the Bayesian updating method.

Crack growth model with ODE; Crack growth model with SDE; MCMC simulation of one parameter



Prior and updated distributions of one parameter; Updated distribution; Updated prediction of crack growth with SDE

Methodology

Ordinary differential equation (ODE) and SDE are used to model the fatigue crack length growth of material. Bayesian updating is applied to the models using Virkler data. The likelihood function, which is the kernel of Bayesian updating, for the SDE model can be developed based on the marginal distribution of stochastic response, whose analytical form is not always available. An approximation is developed based on path integral solution.

Conclusion

Stochastic differential equation (SDE) behaves well in modelling uncertainties and randomness in engineering. Main advantage is that SDE are able to model systems under varying loads. Modelling with SDE is suited for high-frequency data scheme in the present form, and it can be extended to non-Markovian processes. Yet, in the present formulation, this method has some limitations in dealing with strong non-linear systems.

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