Engineering Risk Analysis Group
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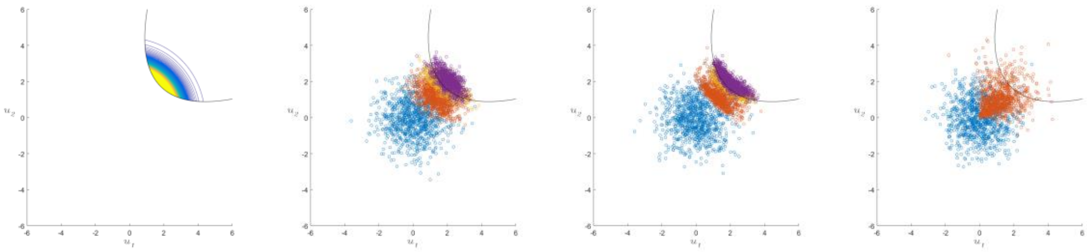
MSc thesis

Cross Entropy Method for Structural Reliability

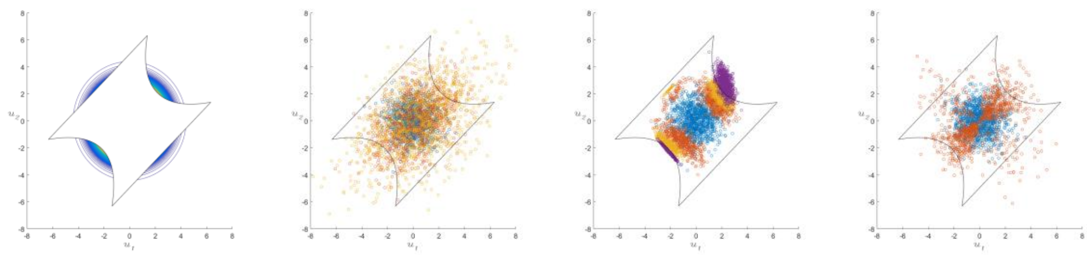
Sebastian Geyer, October 2016

Background

To estimate the failure probability of a structure with acceptable computational effort, one can make use of Importance Sampling (IS). The question of interest when using IS is how to choose the alternative sampling density. A method to find a near-optimal sampling density is the Cross Entropy (CE) method, which minimizes the Kullback-Leibler divergence between the optimal IS density and a proposed distribution type within a few steps of iteration. Different distribution types are suitable for different problem settings, e.g. component problems, system problems or problems in high dimensions.



Step-wise convergence of the multivariate normal distribution, the GM and the vMFNM to a single failure domain



Step-wise convergence of the multivariate normal distribution, the GM and the vMFNM to several failure domains

Methodology

The CE method was introduced and a general algorithm for its implementation was provided. The multivariate normal distribution, the Gaussian mixture (GM) and a new mixture distribution, the von Mises-Fisher-Nakagami mixture (vMFNM), were described and investigated regarding their suitability for IS using the CE method. For the mixture distributions an iterative updating algorithm was developed, the Expectation-Maximization (EM) algorithm, as for these distributions no closed-form updating rules exist for the parameter update in each step of the CE method. The performance was assessed by means of several example problems settings.

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Results

With the chosen simulation parameters, the vMFNM performed best regarding the bias, coefficient of variation and number of model evaluations, and was the only distribution suitable for problems in high dimensions. The multivariate normal distribution gave good results in low dimensions for different problem settings. The GM gave larger bias and coefficient of variation, and needed more steps to converge than the other two distribution types. By increasing the number of samples the performance gap to the multivariate normal distribution could be closed.