Engineering Risk Analysis Group
Prof. Dr Daniel Straub

MSc thesis
First Passage Probability for Linear Stochastic Dynamical Systems

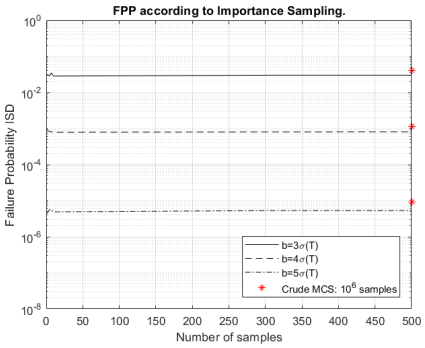
Eftychia Massali, December 2017

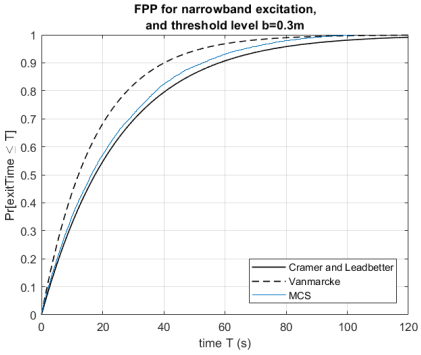
Background

Uncertain forces of nature, such as earthquakes and wind, can cause catastrophic damage to engineering structures and systems; a principal challenge for engineers is to find appropriate ways to handle them. In the design phase, engineers are interested in the load bearing capacity of structures, and especially, in the time of their first failure. Specifically, they need to predict the time when the response of the structure first passes above a prescribed threshold level. In probabilistic terms, this is called first passage probability (FPP).

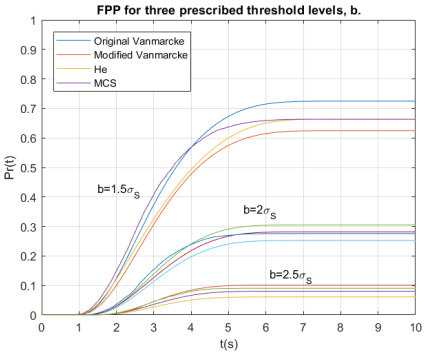
Visual representation of a subset of the implementation results:

Stationary Excitation





Non-Stationary Excitation



Methodology

To investigate the most appropriate numerical method, in terms of structural reliability, the current project examines linear dynamic systems with single degrees of freedom. Their subjected excitation is applied as both a stationary and a non-stationary Gaussian random process. In both cases, various numerical methods were selected from literature for the calculation of the FFP. Furthermore, these numerical methods are compared to the Monte Carlo simulation in order to find the most suitable approach for each practical scenario. Additionally, the Importance Sampling (IS) algorithm, proposed by Au and Beck [1], was implemented in Matlab and compared to the standard Monte Carlo simulation, in terms of the balance between the accuracy of the results and the computational effort.

Results

Based on the obtained results, a categorization of the numerical methods can be made regarding their practical application. For stationary narrowband excitation, the Cramer and Leadbetter method is most appropriate for a high prescribed threshold level due to its assumption of Poisson crossings. In contrast, the original Vanmarcke approach is the best numerical method for a long duration study and a low threshold level. For non-stationary white noise excitation, the He approach is considered the most proper method compared to the original and the modified Vanmarcke. Finally, the Importance Sampling algorithm requires considerably fewer generated samples to determine the first passage probability than the standard Monte Carlo simulation.

Supervised by Prof. Dr. Giulio Cottone, Prof. Dr. Daniel Straub (TUM)

Reference

[1] Au, S. K. and J. L. Beck (2001). “First excursion probabilities for linear systems by very efficient importance sampling”. In: Elsevier, Probabilistic Engineering Mechanics.