Engineering Risk Analysis Group
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MSc thesis

Estimating Filtering Distributions via a Cross Entropy based Importance Sampling Scheme

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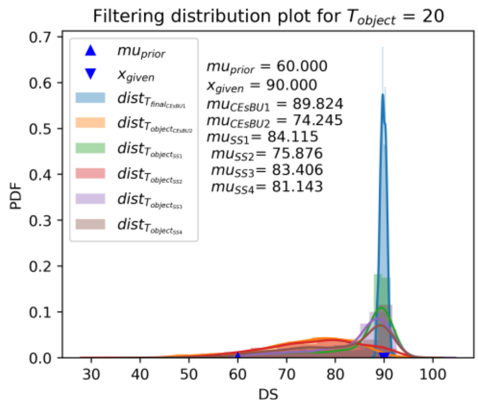
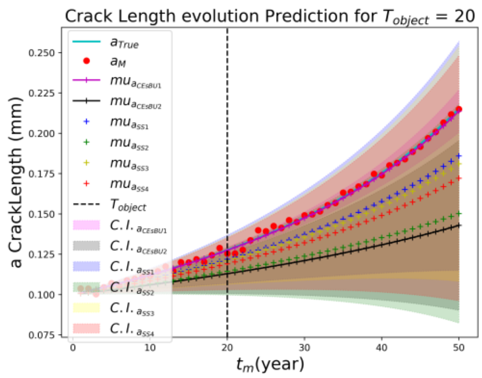
Background

Cross Entropy (CE) based Importance Sampling (IS) has recently been introduced as a scheme tailored for solving Bayesian Updating problem. This method is referred to as CEBU. Herein the CEBU method is extended to the estimation of filtering distribution for sequential Bayesian Updating problems (CEsBU). We investigate CEsBU with the example of a simplified fatigue deterioration crack growth model, with time-invariant uncertain parameters. We first perform the CEsBU for estimating the posterior distribution at the final estimation time step $\mathrm{T}_{\mathrm{final}}$, using all the available data up to $\mathrm{T}_{\mathrm{final}}$. This step is referred to as CEsBU1. The goal is to estimate the filtering distribution at any intermediate time step $\mathrm{T}_{\mathrm{object}}$. We can either perform CEsBU again or we assign new weights $\mathbf{W}_{\mathrm{finalt}}$ to the intermediate samples from the CEsBU1 within a mixture IS scheme. This novel step (we call it Sequential Step) reuses the evaluated likelihood functions of all the intermediate samples from CEsBU1 so that it saves the computation cost compared to performing the CEsBU from scratch for the intermediate time step $\mathrm{T}_{\mathrm{object}}$.



Posterior filtering distribution of model parameter for T_{object} = 20



Crack Length evolution prediction using posterior samples obtained for T_{object} = 20

Methodology

The new weights of samples $\mathbf{W}_{\mathrm{finalt}}$ can be further divided into level weights $\boldsymbol{\pi}$ and the inner-level weights $\mathbf{W}_{\mathrm{newt}}$. Four different methods of performing Sequential Step are investigated. The best method(named SS2) is defined as below: $\mathbf{W}_{\mathrm{newt}} = \mathbf{L}_{\mathrm{object}}\boldsymbol{\phi}(\mathbf{U}_{\mathrm{t}})/\mathbf{h}(\mathbf{U}_{\mathrm{t}}, \hat{\boldsymbol{v}}_{\mathrm{t}})$ (with $\mathbf{L}_{\mathrm{object}}$ being the evaluated likelihood function at $\mathrm{T}_{\mathrm{object}}$ and $\mathbf{h}(\mathbf{U}_{\mathrm{t}}, \hat{\boldsymbol{v}}_{\mathrm{t}})$ being the IS density in process CEsBU1) and $\boldsymbol{\pi} = \mathbf{nESS}/\sum_{\mathrm{t}}^{\mathrm{lv}}\mathbf{nESS}$ (with $\mathbf{nESS}$ being the normalized effective sample size of $\mathbf{W}_{\mathrm{newt}}$.)

Conclusion

Finally the second idea (SS2) is shown to perform best for learning the intermediate filtering distributions. This study helps for the case when we need to know the filtering distribution of the intermediate time steps when we already solve the filtering distribution of final time step in the sequential Bayesian Updating.

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