

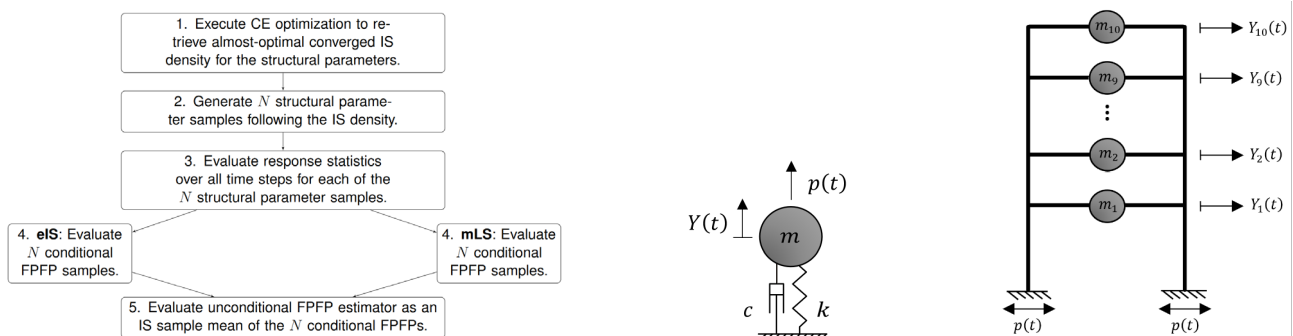
M. Sc. Thesis

# Efficient Estimation of Reliability of Uncertain Linear Structural Systems under Random Dynamic Loading

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## Background

Structural engineering involves designing structures that can withstand random external excitations caused by seismic events or wind pressures. The reliability of these structures is assessed through their first-passage failure probability (FPFP). Standard methods like Direct Monte Carlo face high computational expenses for small FPFP magnitudes. Advanced sampling methods like efficient Importance Sampling (eIS) and multidomain Line Sampling (mLS) have been proposed to determine the conditional FPFP of deterministic linear structures subjected to Gaussian excitations. This thesis aims to provide a comparative study between eIS and mLS to measure their performance when applied to structures with random parameters. While eIS had been applied before in such settings, the extension of mLS to systems with random parameters is first attempted in this work. The randomness in the system parameters is tackled by integrating eIS and mLS within the framework of Cross-Entropy (CE)-based Importance Sampling (IS).



Steps for determining the unconditional FPFP (left figure) --- Single DOF linear oscillator subjected to a stationary Gaussian excitation  $p(t)$ , resulting in a displacement  $Y(t)$  and characterized by a mass  $m$ , a damping  $c$  and a stiffness  $k$  (middle figure) --- Multi DOF linear frame structure subjected to a non-stationary filtered Gaussian excitation  $p(t)$ , resulting in consecutively numbered displacements  $Y_i(t)$  of the masses  $m_i$  (right figure).

## Methodology

The methodology uses a multi-level CE optimization to gradually approach an almost-optimal IS density for the structural parameters. This IS density is subsequently used to estimate the unconditional FPFP by approximating the expectation of the conditional FPFP with an IS estimator in the reliability stage (step 5, see flowchart). The CE approach repeatedly evaluates conditional FPFP samples. Their variability is controlled by generating the samples either by an analytical approximation rooted in Rice's formula (existing) or smoothing them by Nadaraya-Watson (NW) kernel regression (novel approach).

## Conclusion

The methodology is tested on a single degree-of-freedom (DOF) linear oscillator with a low structural parameter count and a ten-story linear frame structure with a high structural parameter count (see figures). For both systems, mLS outperforms eIS when applied in the reliability stage. For the linear oscillator, Rice's formula and NW kernel regression display comparable computational efficiency when applied during the CE optimization, whereas for the linear frame, Rice's formula demonstrates considerable superiority in this aspect.