Engineering Risk Analysis Group
Prof. Dr. Daniel Straub

MSc thesis

Bayesian updating of numerical models: Comparison of the BUSS and DREAM algorithm

Florian Senft, April 2015

Background

Uncertainties in parameters driving numerical models can be reduced by observations. Bayesian inference provides a consistent mathematical framework to formalize this learning process. For complex models, the Bayesian inference can be performed only numerically.
We compare two advanced inference algorithms: BUSS (Bayesian Updating with Subset Simulation; Straub and Papaioannou, 2015) and DREAM (DiffeRential Evolution Adaptive Metropolis algorithm; Vrugt et al., 2009).

Investigated example problems

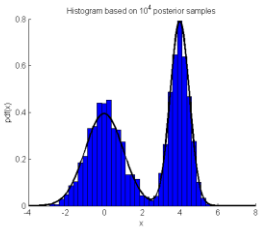
BUSS and DREAM are compared primarily in terms of the first two moments of the generated posterior samples. Additionally, a visual comparison of the posterior sample distribution is performed. We look only at the posterior samples; the number of subset-levels in BUSS and the length of the burn-in period in DREAM are not considered.
Four numerical examples are investigated: a one-dimensional bi-modal likelihood, a two-dimensional banana-shaped likelihood, a two-dimensional circular likelihood function, and a real-world hydrological problem (HYMOD). For the first three problems listed, an analytical reference solution can be derived.

Summary

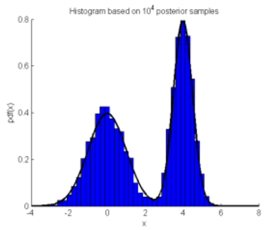
For the bi-modal problem, the posterior, approximated with BUSS and DREAM, matches well with the analytical solution. Concerning the problem with the banana-shaped likelihood, BUSS performs slightly better than DREAM.
However, for the circular likelihood, the posterior samples generated with BUSS exhibit a larger artificial clustering, whereas the samples of DREAM are distributed more homogeneously.
For the HYMOD problem, the posterior samples of BUSS and DREAM do not match well. On the one hand, the average likelihood of the samples generated with DREAM is considerably larger than the one of the samples obtained with BUSS. On the other hand DREAM produces samples of parameter C_{max} that exhibit a larger posterior standard deviation compared to BUSS. At first glance this appears surprising, as a larger average likelihood is typically associated with smaller posterior standard deviations. However, as a reference solution for this example was missing, the exact interpretation is left for further studies.

Selected Result-Plots

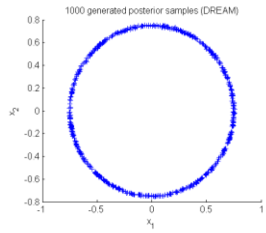
Bimodal: DREAM



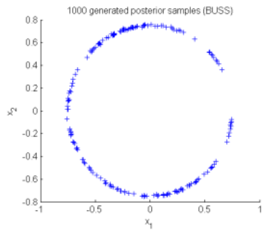
BUSS



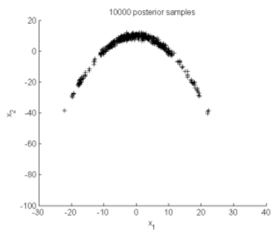
Circular: DREAM



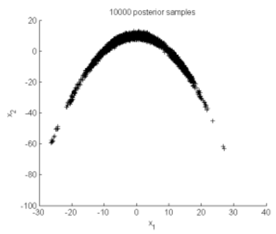
BUSS



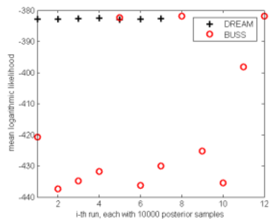
Banana: DREAM



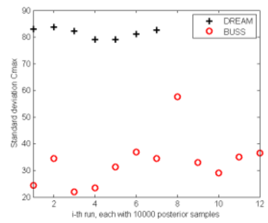
BUSS



HYMOD: mean log likelihood



std. C_{max}



Selected result plots of the 4 investigated numerical examples

Supervised by Wolfgang Betz, Prof. Daniel Straub