Engineering Risk Analysis Group
Prof. Dr Daniel Straub

Diffusion-Enhanced Bayesian Model Updating for Incomplete Observations

Motivation and Objectives

Nonlinear Engineering and scientific models are widely used to support design, safety assessment, monitoring, and decision-making. In practice, however, the predictive capability of these models is often limited by (i) uncertain parameters (material properties, boundary/interface characteristics, environmental factors), (ii) imperfect modeling assumptions (missing physics or simplified boundary/loading representations), and (iii) **noisy and incomplete observations**, where only a fraction of the system responses are measured and future inputs are unknown.

Bayesian inference provides a principled framework to fuse prior knowledge with observed data and obtain a posterior distribution over uncertain parameters, enabling uncertainty-aware prediction. A key challenge arises when available data are sparse and incomplete: the inverse problem can become weakly identifiable, standard Gaussian error assumptions may be overly restrictive, and uncertainty bounds can be poorly calibrated for extrapolation to new operating regimes or for future-time prediction.

Diffusion models are a class of generative models that can represent complex, high-dimensional probability distributions and generate samples through a learned denoising process. In model updating and inverse problems, diffusion models can act as flexible probabilistic representations for latent variables such as unknown future inputs, missing observations, or model discrepancy induced by missing physics. Importantly, diffusion-based sampling does not create new information; rather, it provides a stronger, prior structure and enables robust posterior predictive sampling, which can improve calibration and predictive reliability in incomplete-data settings.

Methodology

- **Define the inverse problem:** specify the forward model, the observed quantities, and the sources of uncertainty (parameters, inputs, measurement noise, model-form error).
- **Bayesian model updating:** choose priors and a likelihood model, then infer the posterior distribution of uncertain parameters using an appropriate inference method (e.g., MCMC or variational inference).
- **Train or adopt diffusion models as generative uncertainty models:** use diffusion to represent one or more latent components such as uncertain future inputs, missing/unobserved variables, or model discrepancy due to missing physics.
- **Posterior predictive sampling:** combine posterior parameter samples with diffusion-generated latent samples to produce predictions for unobserved quantities and future cases, including calibrated uncertainty intervals.
- **Validation and comparison:** evaluate predictive accuracy and uncertainty calibration on benchmark problems or held-out datasets, and compare against baseline uncertainty models and updating approaches.

Requirements

What previous knowledge and skills do you expect the student to bring to the project e.g.

- Good mathematical background (probability, optimization).
- Programming skills (**Python**) for data processing, model evaluation, and inference workflows.

- Basic knowledge of **machine learning**, including training and validating neural network models.
- Basic understanding of **Bayesian inference** (priors, likelihood, posterior, posterior predictive; familiarity with MCMC and/or variational inference is beneficial).
- Interest in uncertainty quantification, inverse problems, and probabilistic modeling.

Starting date: Flexible, as soon as possible

Supervised by

Milad Cheragh Zade (milad.cheragh-zade@tum.de), Engineering Risk Analysis Group, TUM

References

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2. Chung, H., Kim, J., McCann, M. T., Klasky, M. L., & Ye, J. C. (2024). Diffusion Posterior Sampling for General Noisy Inverse Problems. *arXiv preprint arXiv:2209.14687* (v4).
3. Luo, C. (2023). Understanding Diffusion Models: A Unified Perspective. *Proceedings of ICLR 2023*.