Engineering Risk Analysis Group
Prof. Dr Daniel Straub

MSc thesis proposal

Combining dimensionality reduction and surrogate modeling: cheap solutions of Bayesian inverse problems

Motivation and Objectives

The Bayesian approach to solving inverse problems updates a prior distribution to a posterior distribution of the parameters through data (Dashti and Stuart 2015). The likelihood of the data needed to obtain the posterior distribution depends on a computational model for the numerical solution of a partial differential equation. Standard procedures for solving the Bayesian inverse problem are based on multiple evaluations of the likelihood function and the underlying computational model. For resource demanding models, the application of surrogate models enables cheap solutions of the inverse problem.

Gaussian processes (GP) are a common surrogate modeling method used in Bayesian inference to approximate the computational model (Dinkel et al. 2023). The surrogate tries to learn the relationship between the unknown parameter and obtained measurements based on data. Like many surrogate modeling methods, GPs suffer from the curse of dimensionality – leading to a large number of samples needed to train the model in high dimensions. Dimensionality reduction tackles this problem by reducing the dimensions to an effective input dimension resulting in a reduction of the costs necessary to obtain an accurate surrogate. In the context of Bayesian inference, this can be achieved by reducing the parameter space to the likelihood-informed subspace (LIS) (Spantini et al. 2015).

This project explores efficient combination of LIS-dimensionality reduction and GP surrogate modelling for approximating the solution of Bayesian inverse problems. The project combines method development and numerical implementations to compare the method with existing approaches (Scheffels et al. 2025). The objectives, as well as the workflow, can be adjusted to the students' ideas, progress, results, etc.

Methodology

The suggested workflow is as follows:

- Literature review on linear Bayesian inverse problems and Gaussian processes
- Implementation of Gaussian process regression (Matlab/Python)
- Combining GPs with LIS-dimensionality reduction
- Benchmarking by comparison with other surrogate models
- Extension to nonlinear inverse problems (optional)

Requirements

What previous knowledge and skills do you expect the student to bring to the project e.g.

- Python or Matlab
- Advanced mathematical knowledge
- Experience with probabilistic modelling and stochastic finite element methods, e.g., via completion of "Stochastic finite element methods"

Starting date: Flexible, as soon as possible

Supervised by

This thesis is supervised by Jakob Scheffels (jakob.scheffels@tum.de, Engineering Risk Analysis Group, TUM). If you are interested in this topic, please contact me with your transcript of records and preferred approximate starting date.

References

Dashti, Masoumeh, and Andrew M. Stuart. 2015. “The Bayesian Approach To Inverse Problems.” arXiv:1302.6989. Preprint, arXiv, July 2. https://doi.org/10.48550/arXiv.1302.6989.

Dinkel, Maximilian, Carolin M. Geitner, Gil Robalo Rei, Jonas Nitzler, and Wolfgang A. Wall. 2023. “Solving Bayesian Inverse Problems With Expensive Likelihoods Using Constrained Gaussian Processes and Active Learning.” arXiv:2312.08085. Preprint, arXiv, December 13. https://doi.org/10.48550/arXiv.2312.08085.

Scheffels, Jakob, Elizabeth Qian, Iason Papaioannou, and Elisabeth Ullmann. 2025. “Likelihood-Informed Model Reduction for Bayesian Inference of Static Structural Loads.” arXiv:2510.07950. Preprint, arXiv, October 9. https://doi.org/10.48550/arXiv.2510.07950.

Spantini, Alessio, Antti Solonen, Tiangang Cui, James Martin, Luis Tenorio, and Youssef Marzouk. 2015. “Optimal Low-Rank Approximations of Bayesian Linear Inverse Problems.” *SIAM Journal on Scientific Computing* 37 (6): A2451–87. https://doi.org/10.1137/140977308.