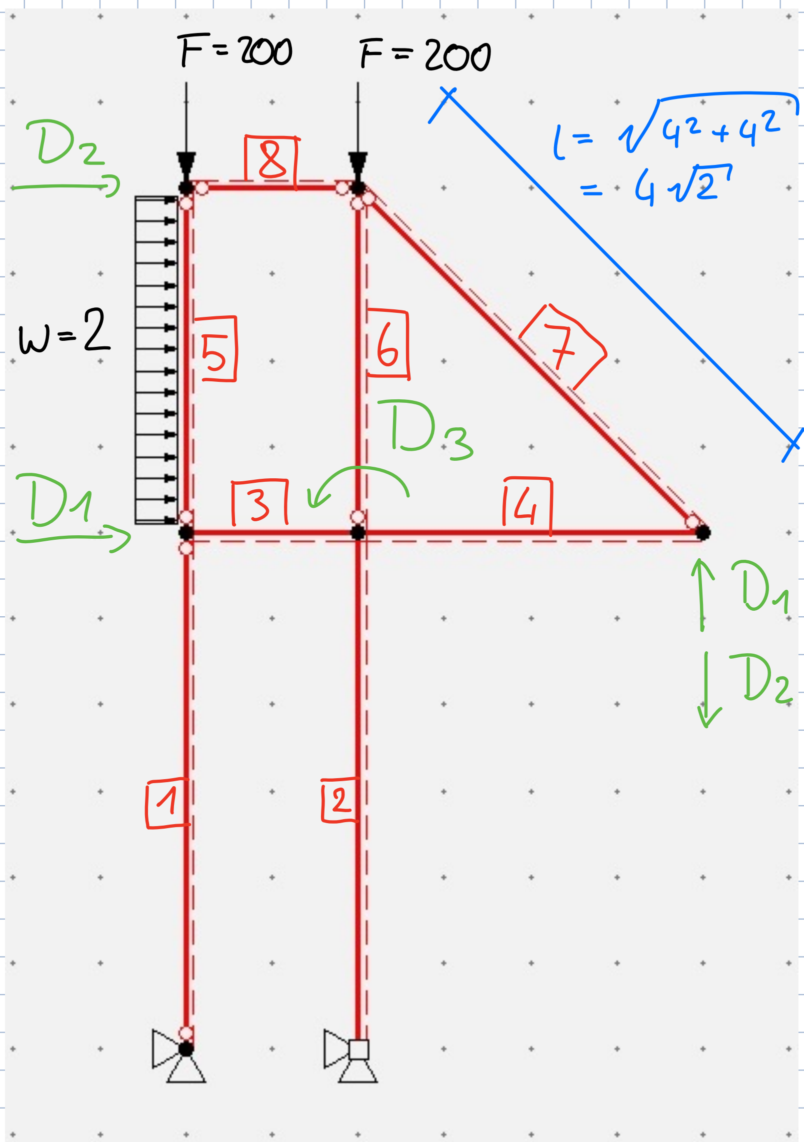
Statik EKS Musterlösung

Probeklausor 1 - Aufgabe 1



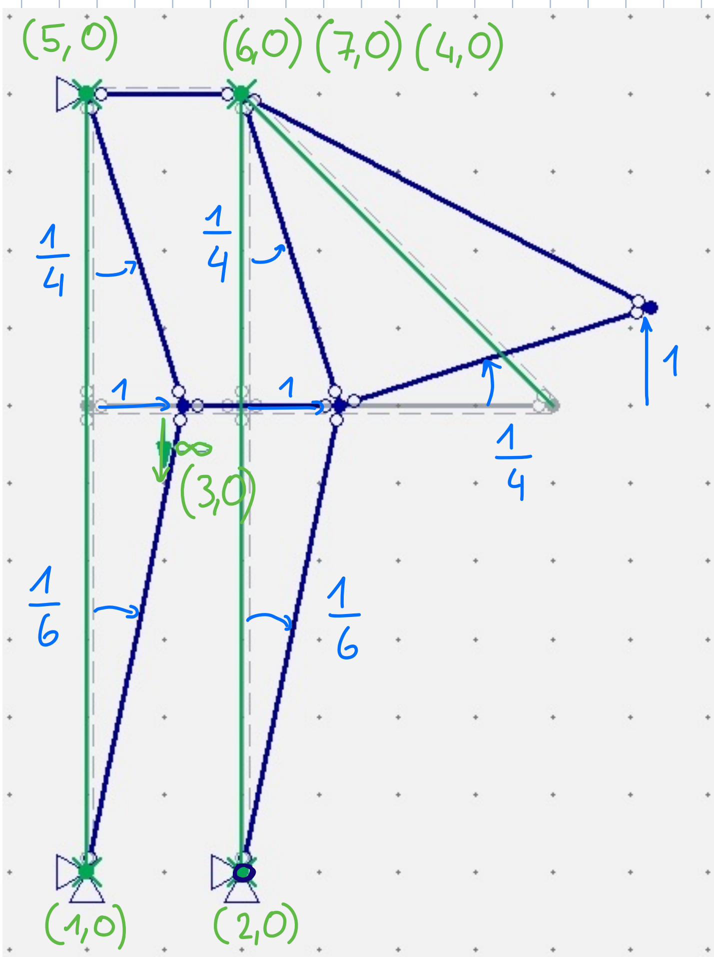
Stab	N [kN]
1	-182,00
2	-218,00
3	-4,00
4	4,00
5	-200,00
6	-196,00
7	-5,66
8	-4,00

EI = 4 000 kNm²
EA → ∞

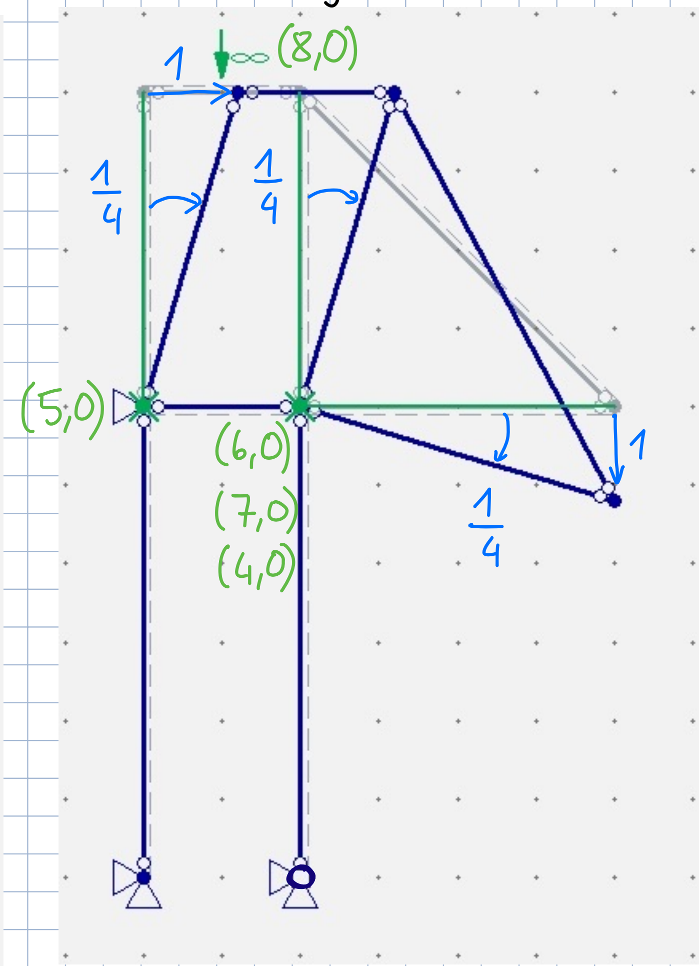
w = 2 kN/m
F = 200 kN

Gelenkfigur / Polplan:

Gelenkfigur 1:



Gelenkfigur 2:



relevante α-Werte:

$\alpha_2 = 6\sqrt{\frac{218}{4000}} = 1{,}40 \quad \begin{matrix} > 1 \\ < 2{,}5 \end{matrix}$

$\alpha_3 = 2\sqrt{\frac{4}{4000}} = 0{,}06 < 1$

$A_2' = 4 - \frac{2}{15}(1{,}40)^2 = 3{,}74$

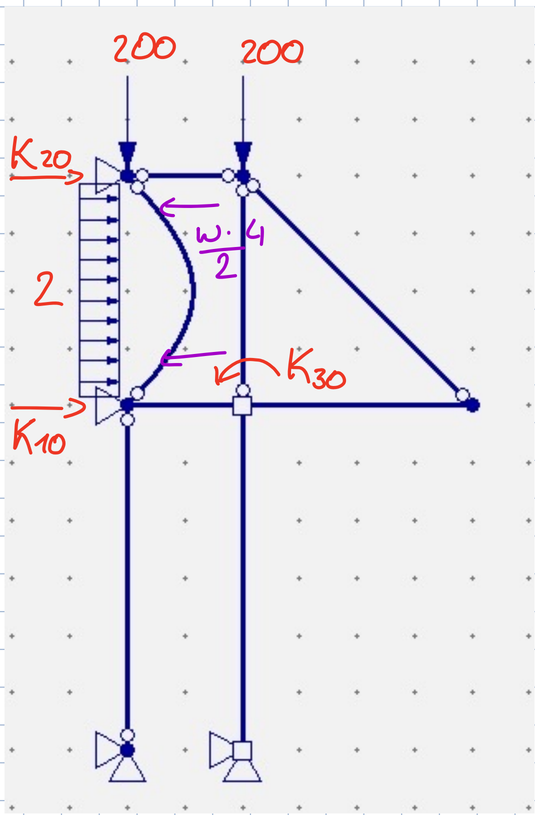
$C_3' = 3$

$B_2' = 2 + \frac{1}{30}(1{,}40)^2 = 2{,}07$

$D_2' = 6 - \frac{1}{10}(1{,}40)^2 = 5{,}80$

S_4 ist ein Zugstab $\rightarrow$ $C_4' = 3$

LZ:

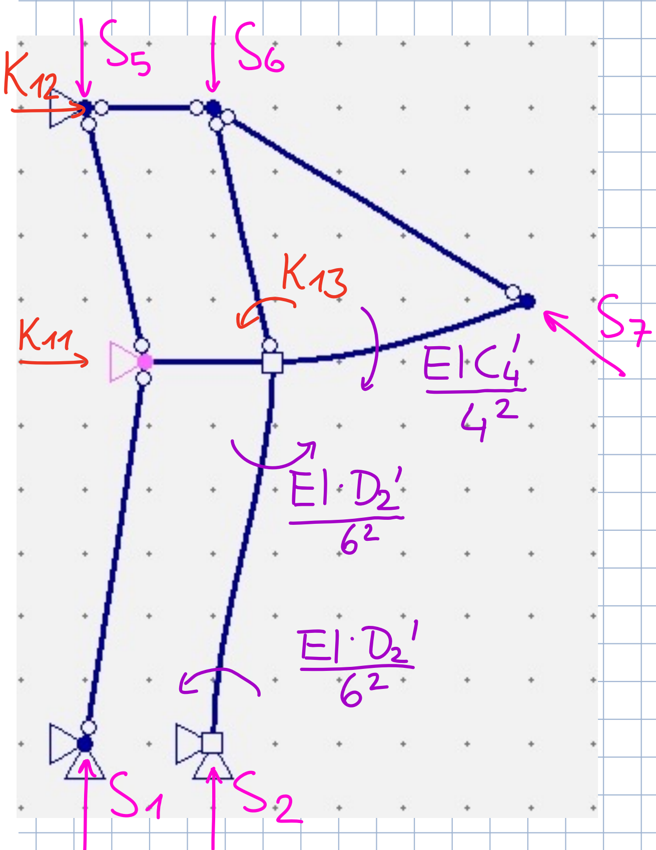


$$K_{10} = - \frac{w \cdot 4}{2} = -4$$

$$K_{20} = - \frac{w \cdot 4}{2} = -4$$

$$K_{30} = 0$$

EZ 1:



PvV (Gelenkfigur 1):

$$K_{11} \cdot 1 - 2 \frac{EI\, D_2'}{6^2} \cdot \frac{1}{6} - \frac{EI\, C_4'}{4^2} \cdot \frac{1}{4}$$

$$+ S_1 \cdot 6 \cdot \left(\frac{1}{6}\right)^2 + S_2 \cdot 6 \cdot \left(\frac{1}{6}\right)^2$$

$$+ S_7 \cdot 4 \cdot \sqrt{2} \left(\frac{1}{4}\right)^2 + S_5 \cdot 1 \cdot \frac{1}{4}$$

$$+ S_6 \cdot 1 \cdot \frac{1}{4} = 0$$

$$K_{11} = 214{,}963 + 187{,}5$$

$$-30{,}333 - 36{,}333 - 2{,}001 - 50{,}0 - 49{,}0$$

$$= 234{,}796$$

$$\left(\underbrace{\underbrace{S \cdot \underbrace{l \cdot \varphi}_{\text{Exzentrizität}}}_{\text{Moment}} \cdot \varphi}_{\text{Arbeit}} \right)$$

PvV (mit Gelenkfigur 2):

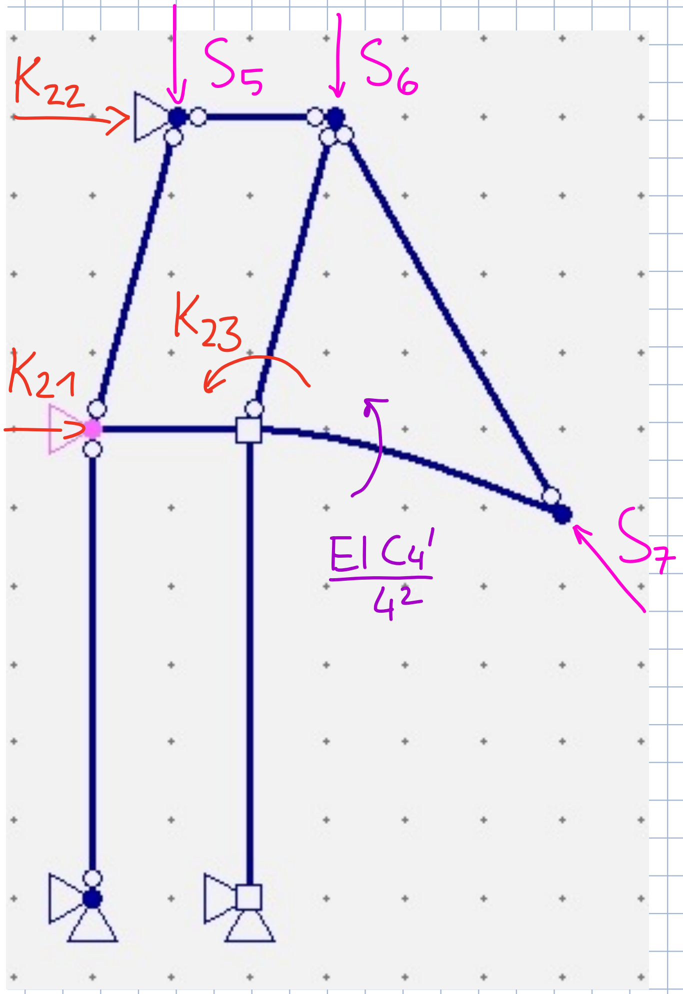
$$K_{12} \cdot 1 + \frac{EI\,C_4'}{4^2} \cdot \frac{1}{4} - S_5 \cdot 1 \cdot \frac{1}{4} - S_6 \cdot 1 \cdot \frac{1}{4} - S_7 \cdot 4\sqrt{2} \cdot \left(\frac{1}{4}\right)^2 = 0$$

$$K_{12} = -187{,}5 + 50{,}0 + 49{,}0 + 2{,}001 = -86{,}499$$

GG:

$$K_{13} = \frac{EI\,D_2'}{6^2} - \frac{EI\,C_4'}{4^2} = 644{,}889 - 750{,}0 = -105{,}111$$

EZ 2:



PvV (mit Gelenkfigur 1):

$$K_{22} \cdot 1 - \frac{EI\,C_4'}{4^2} \cdot \frac{1}{4} + S_5 \cdot 4 \cdot \left(\frac{1}{4}\right)^2 + S_6 \cdot 4 \left(\frac{1}{4}\right)^2 + S_7 \cdot 4\sqrt{2} \left(\frac{1}{4}\right)^2 = 0$$

$$K_{22} = 187{,}5 - 50{,}0 - 49{,}0 - 2{,}001 = 86{,}499$$

PvV (mit Gelenkfigur 2):

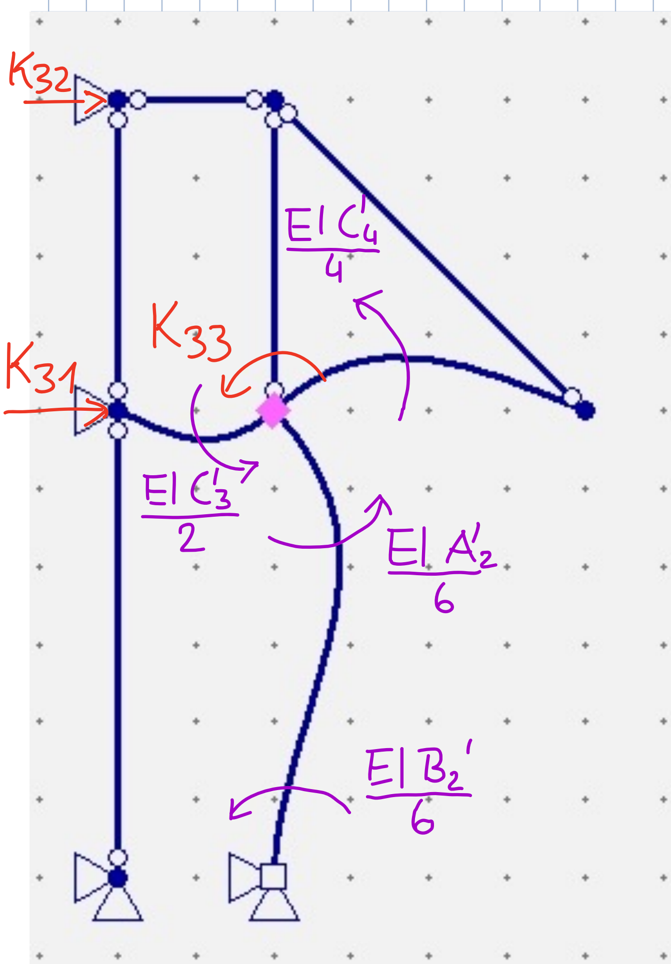
$$K_{21} \cdot 1 + \frac{EI\,C_4'}{4^2} \cdot \frac{1}{4} - S_5 \cdot 1 \cdot \frac{1}{4} - S_6 \cdot 1 \cdot \frac{1}{4} - S_7 \cdot 4 \cdot \sqrt{2} \cdot \left(\frac{1}{4}\right)^2 = 0$$

$$K_{21} = -187{,}5 + 50{,}0 + 49{,}0 + 2{,}001 = -86{,}499$$

GG:

$$K_{23} = \frac{EI \cdot C_4'}{16} = 750$$

EZ 3:



PvV (mit Gelenkfigur 1):

$$K_{31} \cdot 1 + \frac{EI\,C_4'}{4} \cdot \frac{1}{4} - \left(\frac{EI\,A_2'}{6} + \frac{EI\,B_2'}{6}\right)\frac{1}{6} = 0$$

$$K_{31} = -750{,}0 + 644{,}889 = -105{,}111$$

PvV (mit Gelenkfigur 2):

$$K_{32} \cdot 1 - \frac{EI\,C_4'}{4} \cdot \frac{1}{4} = 0$$

$$K_{32} = 750$$

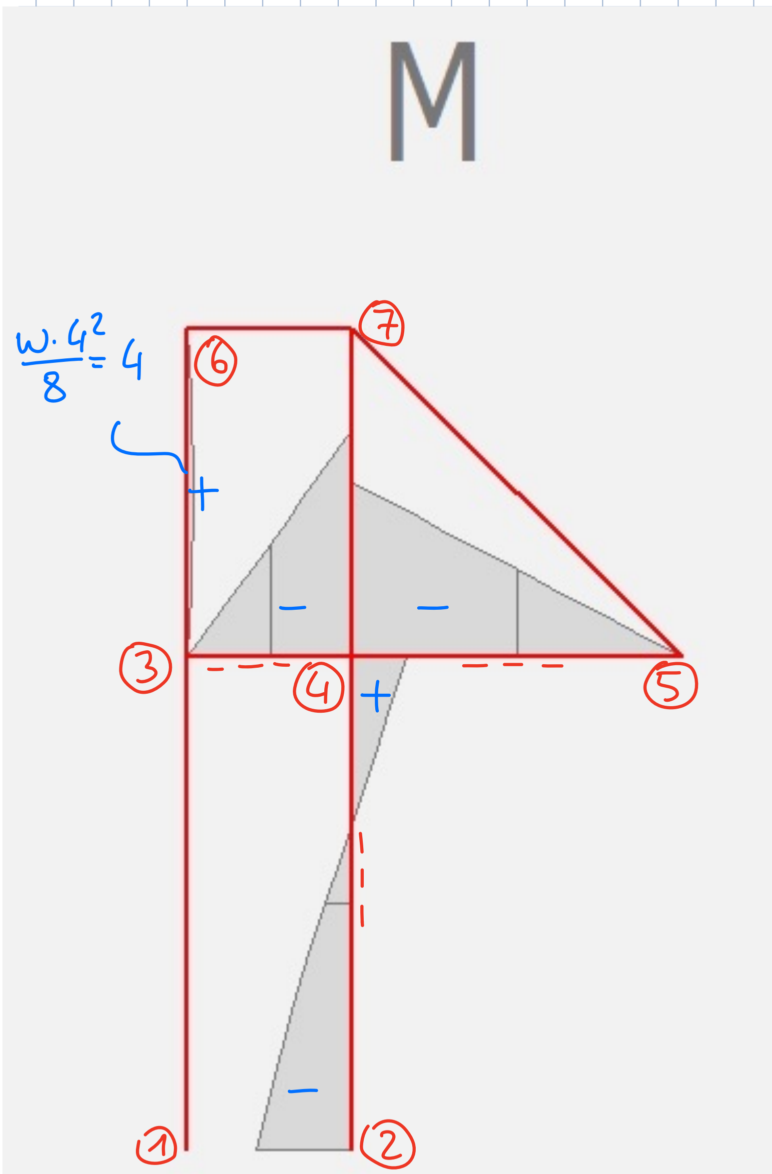
GG:

$$K_{33} = \frac{EI\,C_4'}{4} + \frac{EI\,C_3'}{2} + \frac{EI\,A_2'}{6} = 11492{,}0$$

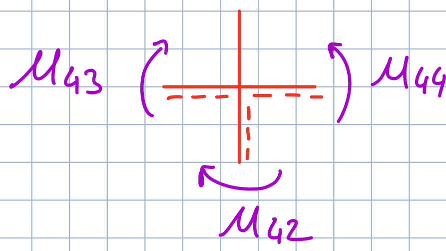
$$\underline{\underline{K}} = \begin{bmatrix} 234{,}796 & -86{,}499 & -105{,}111 \\ -86{,}499 & 86{,}499 & 750 \\ -105{,}111 & 750 & 11492{,}0 \end{bmatrix} \qquad \underline{F} = -\underline{\underline{K}}_0 = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{u} = \underline{F} \longrightarrow \underline{u} = \begin{bmatrix} 0{,}192 \\ 0{,}514 \\ -0{,}032 \end{bmatrix}$$

Momentenverlauf:



($M_{\text{Knoten, Element}}$)



$$M_{43} = \frac{EI\, C_3'}{2} (-0{,}032) = -192$$

$$M_{42} = \frac{EI\, D_2'}{6^2} \cdot 0{,}192 + \frac{EI\, A_2'}{6} (-0{,}032) =$$

$$= 123{,}819 - 79{,}744 = 44{,}075$$

$$M_{44} = M_{23} + M_{42} =$$

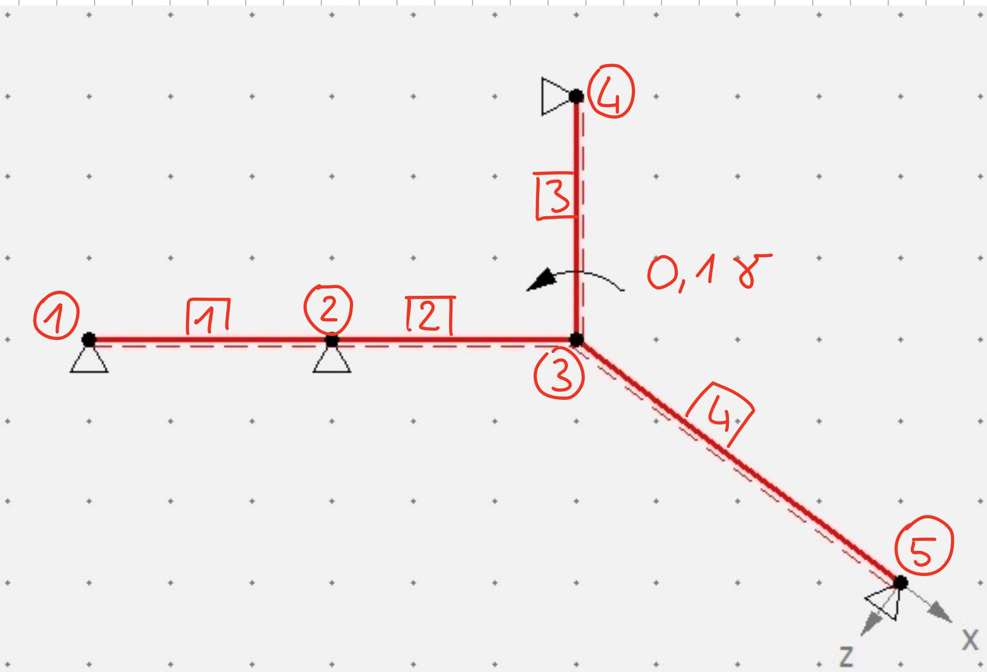
$$= -192 + 44{,}075 = -147{,}925$$

$$M_{22} = -\frac{EI D_2'}{6^2} (0{,}192) - \frac{EI\, B_2'}{6} (-0{,}032)$$

$$= -123{,}819 + 44{,}053 = -79{,}766$$

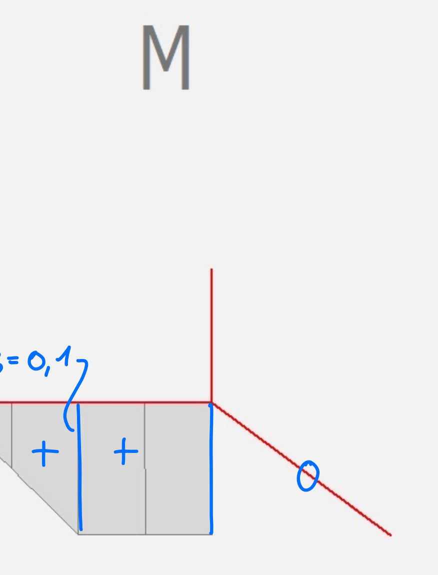
Probeklausur 1 – Aufgabe 2

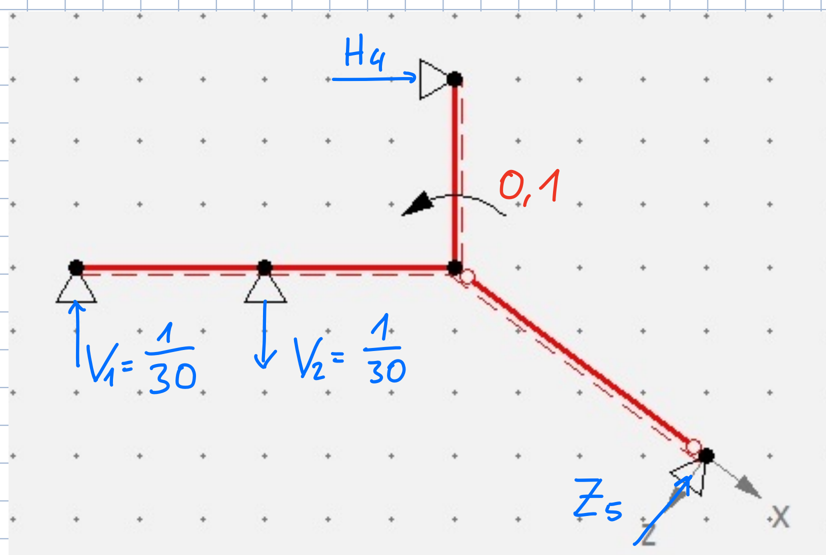
a)



# Stab	1	2	3	4
M_{pl} kNm	1	1,5	1,5	0,5

LZ:





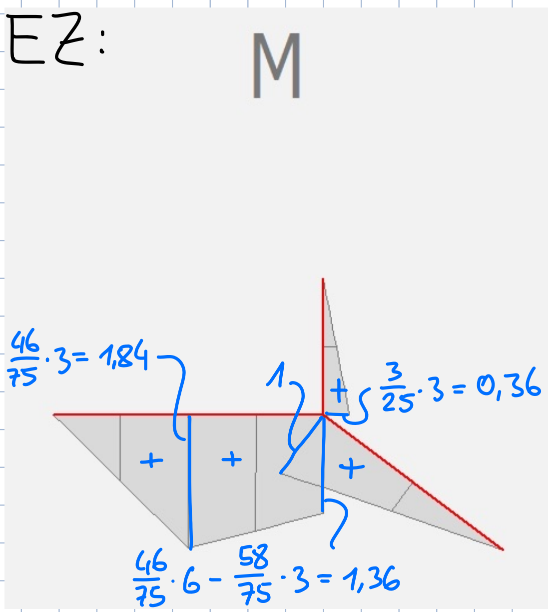
$\Sigma M_3: \quad Z_5 = 0$

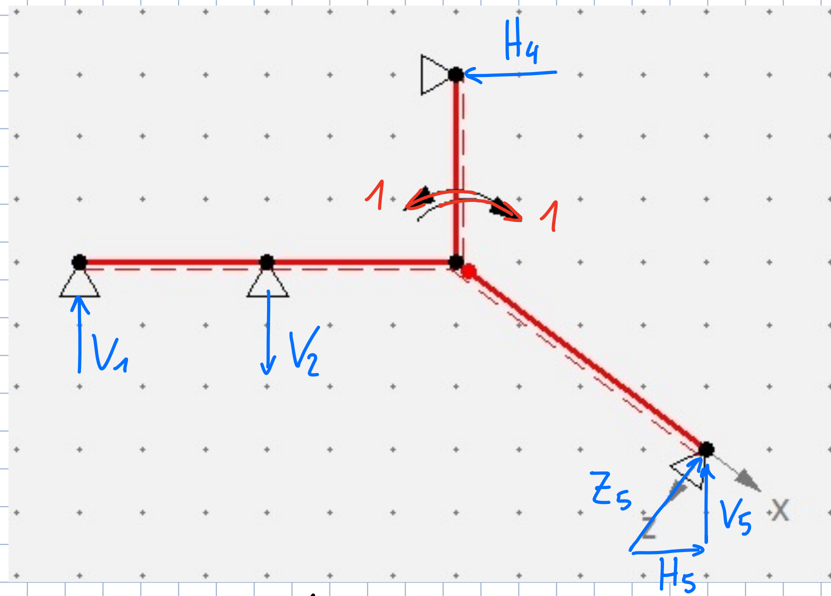
$\Sigma H: \quad H_4 = 0$

$\Sigma M_2: \quad V_1 = \frac{0,1}{3} = \frac{1}{30}$

$\Sigma V: \quad V_2 = V_1$

EZ:





$$\Sigma M_3: \quad Z_5 = \frac{1}{5}$$

$$H_5 = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}$$

$$V_5 = \frac{4}{5} \cdot \frac{1}{5} = \frac{4}{25}$$

$$\Sigma H: \quad H_4 = H_5$$

$$\Sigma M_2: \quad V_1 = \frac{2 \cdot \frac{3}{25} \cdot 3 + \frac{4}{25} \cdot 7}{3} = \frac{46}{75}$$

$$\Sigma V: \quad V_2 = \frac{46}{75} + \frac{4}{25} = \frac{58}{75}$$

$$d_{10} = \frac{1}{3} \cdot 0{,}1 \cdot 1{,}84 \cdot \frac{3}{1000} + \frac{1}{2} \cdot 0{,}1 \cdot (1{,}84 + 1{,}36) \cdot \frac{3}{1000} =$$

$$= 1{,}84 \cdot 10^{-4} + 4{,}8 \cdot 10^{-4} = 6{,}64 \cdot 10^{-4}$$

$$d_{11} = \frac{1}{3} \cdot 1{,}84^2 \cdot \frac{3}{1000} + \frac{1}{6}\left(2 \cdot 1{,}84^2 + 2 \cdot 1{,}84 \cdot 1{,}36 + 2 \cdot 1{,}36^2\right) \frac{3}{1000} + \frac{1}{3} \cdot 0{,}36^2 \cdot \frac{3}{1000} + \frac{1}{3} \cdot 1^2 \cdot \frac{5}{1000}$$

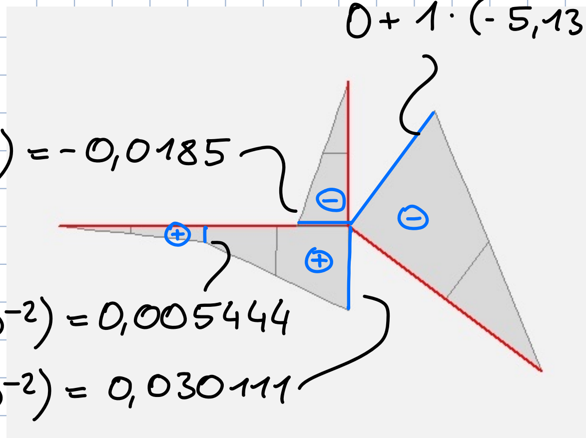
$$= 3{,}386 \cdot 10^{-3} + 7{,}738 \cdot 10^{-3} + 1{,}3 \cdot 10^{-4} + 1{,}667 \cdot 10^{-3} = 1{,}2921 \cdot 10^{-2}$$

$$X_1 = -\frac{d_{10}}{d_{11}} = -\frac{6{,}64 \cdot 10^{-4}}{1{,}2921 \cdot 10^{-2}} = -5{,}1389 \cdot 10^{-2}$$

Momenten-Verlauf für $\gamma = 1$ [kNm]

$0 + 1 \cdot (-5{,}1389 \cdot 10^{-2}) = -5{,}1389 \cdot 10^{-2}$

$0 + 0{,}36 \cdot (-5{,}1389 \cdot 10^{-2}) = -0{,}0185$



$0{,}1 + 1{,}84 \cdot (-5{,}1389 \cdot 10^{-2}) = 0{,}005444$

$0{,}1 + 1{,}36 \cdot (-5{,}1389 \cdot 10^{-2}) = 0{,}030111$

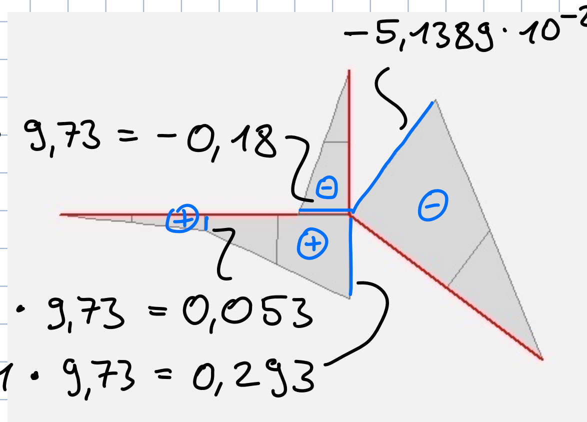
b) Berechnung des 1. Fließgelenk:

$$\gamma_1 = \frac{M_{pl}}{M_{el}} = \frac{0{,}5}{0{,}051389} = 9{,}73$$

Momenten-Verlauf für $\gamma = \gamma_1$:

$-5{,}1389 \cdot 10^{-2} \cdot 9{,}73 = -0{,}5$

$-0{,}0185 \cdot 9{,}73 = -0{,}18$



$0{,}005444 \cdot 9{,}73 = 0{,}053$

$0{,}030111 \cdot 9{,}73 = 0{,}293$

Berechnung des 2. Fließgelenk:

Momentenverlauf wird aus Aufgabe a) Lastzustand übernommen.

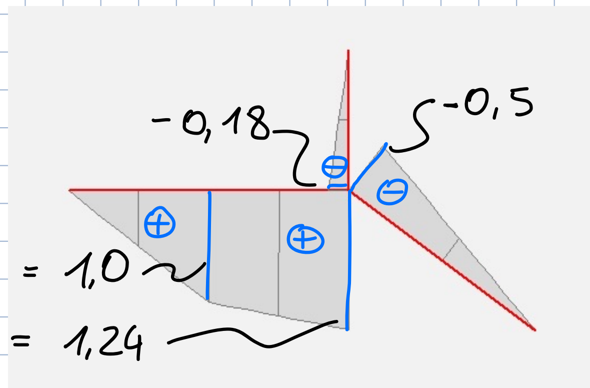
$$\Delta\gamma_2 = \min \begin{cases} 0{,}053 + 0{,}1 \cdot \Delta\gamma_2 = 1 & \rightarrow \Delta\gamma_2 = 9{,}47 \quad \text{maßg.} \\ 0{,}293 + 0{,}1 \cdot \Delta\gamma_2 = 1{,}5 & \rightarrow \Delta\gamma_2 = 12{,}07 \end{cases}$$

Traglastfaktor:

$$\gamma_T = \gamma_1 + \Delta\gamma_2 = 9{,}73 + 9{,}47 = 19{,}2$$

System wird kinematisch, keine weiteren FG vorhanden

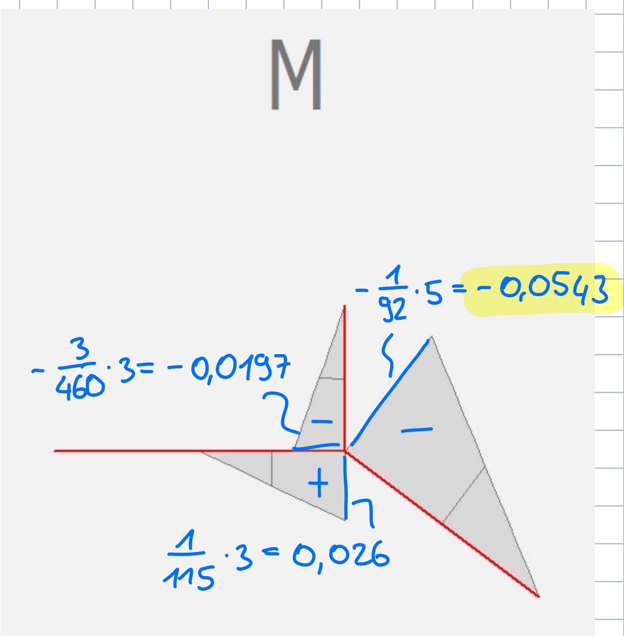
Momenten-Verlauf $\gamma = \gamma_T$:

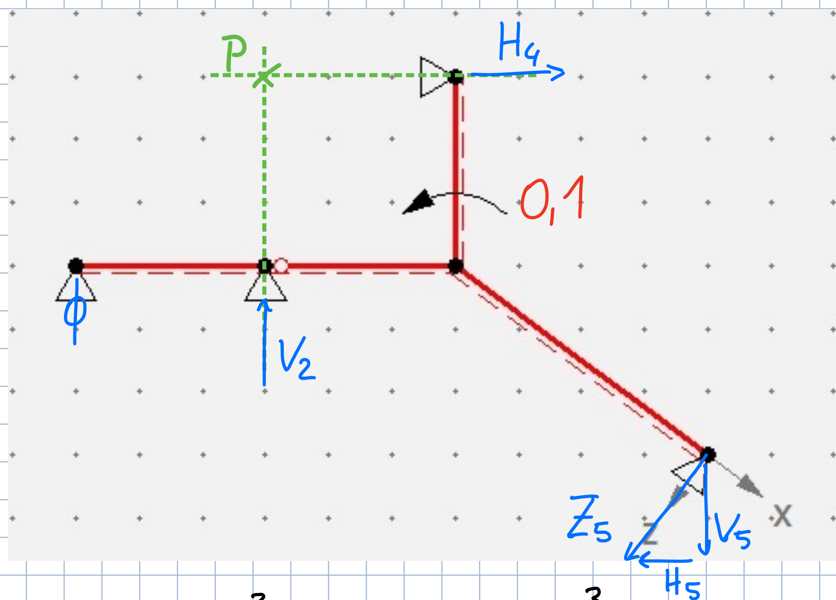


$0{,}053 + 0{,}1 \cdot 9{,}47 = 1{,}0$

$0{,}293 + 0{,}1 \cdot 9{,}47 = 1{,}24$

Prüfen ob sich das 1. FG schließt:





$H_5 = \frac{3}{5} \cdot Z_5 \rightarrow H_5 = \frac{3}{460}$

$V_5 = \frac{4}{5} \cdot Z_5 \rightarrow V_5 = \frac{1}{115}$

$\Sigma M_P: \frac{3}{5} \cdot Z_5 \cdot 6 + \frac{4}{5} \cdot Z_5 \cdot 7 = 0{,}1$

$Z_5 = \frac{1}{92}$

$\Sigma H: H_4 = H_5 = \frac{3}{460}$

$\Sigma V: V_2 = V_5 = \frac{1}{115}$

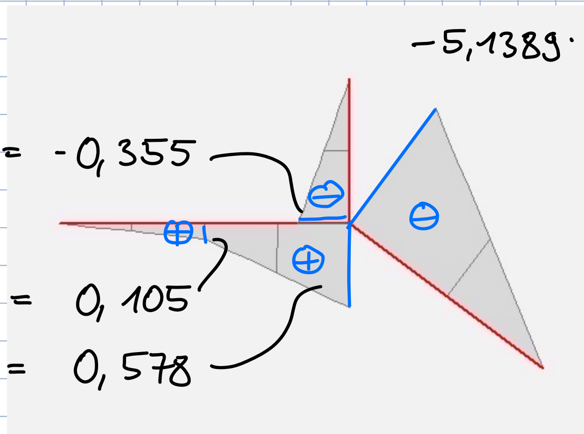
FG 1. schließt sich nicht, da das Vorzeichen von $M_{34} = -0{,}0543$ nicht wechselt.

c)

$M_{el}\ (\gamma = \gamma_T)$:

$-5{,}1389 \cdot 10^{-2} \cdot 19{,}20 = -0{,}987$

$-0{,}0185 \cdot 19{,}20 = -0{,}355$



$0{,}005444 \cdot 19{,}20 = 0{,}105$

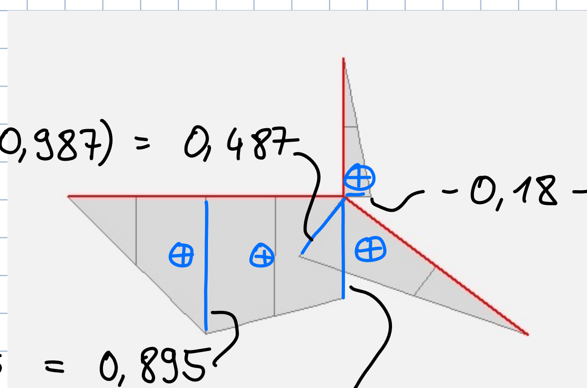
$0{,}030111 \cdot 19{,}20 = 0{,}578$

M - Verlauf nach Entlastung:

$M_{entlastet} = M_T - M_{el}(\gamma_T)$

$-0{,}5 - (-0{,}987) = 0{,}487$

$-0{,}18 - (-0{,}355) = 0{,}175$

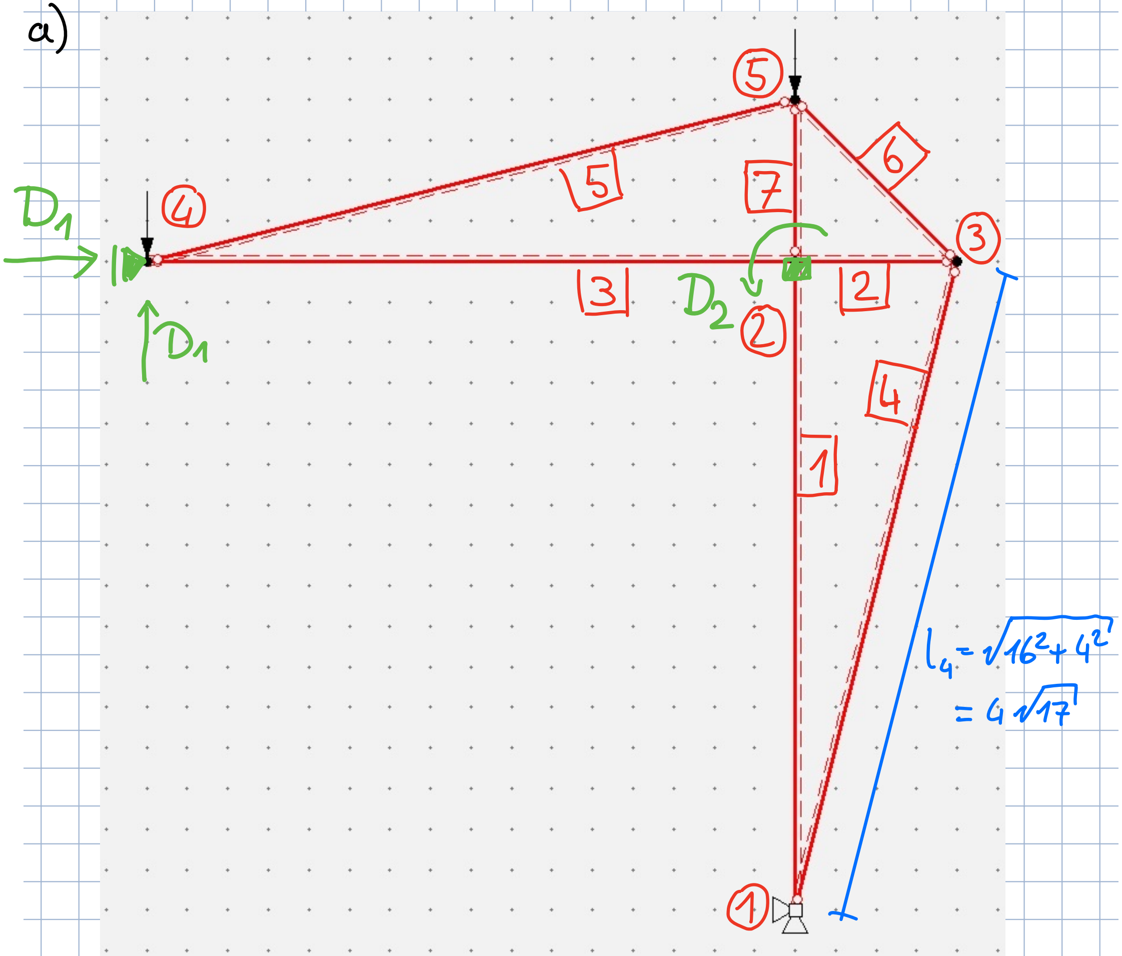


$1 - 0{,}105 = 0{,}895$

$1{,}24 - 0{,}578 = 0{,}662$

Probeklausur 2 Aufgabe 1

a)



EI = 100 000 kNm²
EA → ∞

F = 60 kN

Stab	N [kN]
1	-460,0
2	-345,0
3	-260,0
4	350,5
5	268,0
6	367,7
7	-385,0

relevante α-Werte:

$$\alpha_1 = 16 \cdot \sqrt{\frac{460}{10^5}} = 1{,}09$$

$$A_1' = 4 - \frac{2}{15} \cdot 1{,}09^2 = 3{,}84$$

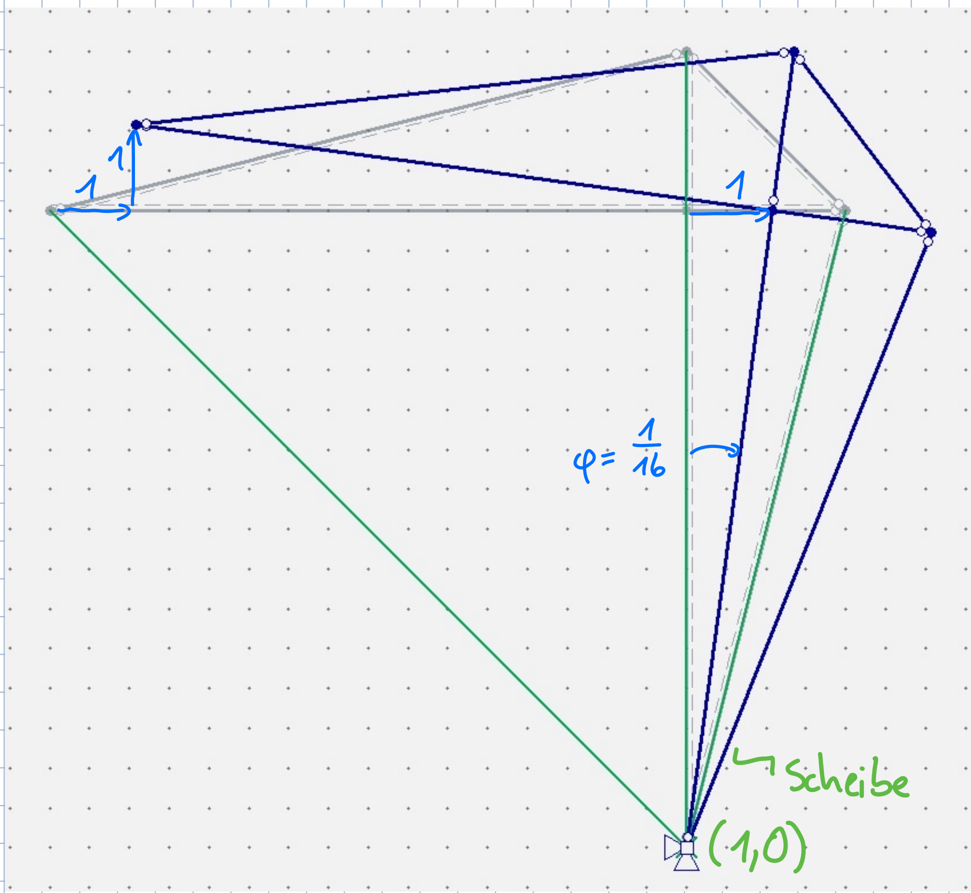
$$B_1' = 2 + \frac{1}{30} \cdot 1{,}09^2 = 2{,}04$$

$$C_1' = 6 - \frac{1}{10} \cdot 1{,}09^2 = 5{,}88$$

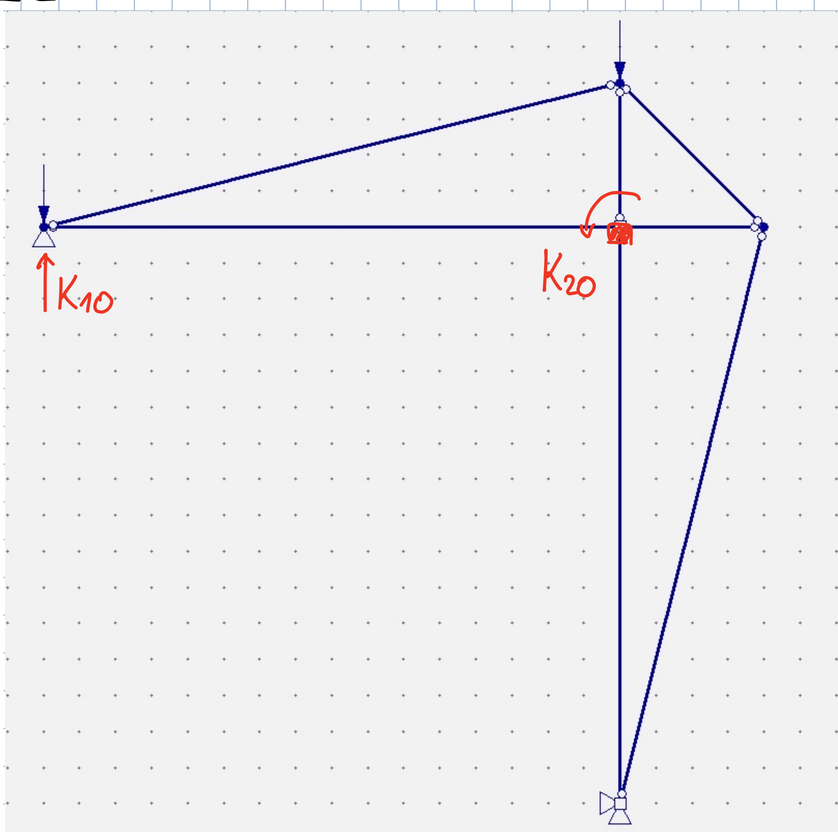
$$\alpha_2 = 4 \cdot \sqrt{\frac{345}{10^5}} = 0{,}23 < 1 \rightarrow C_2' = 3$$

$$\alpha_3 = 16 \cdot \sqrt{\frac{260}{10^5}} = 0{,}82 < 1 \rightarrow C_3' = 3$$

Gelenkfigur / Polplan:



LZ1:

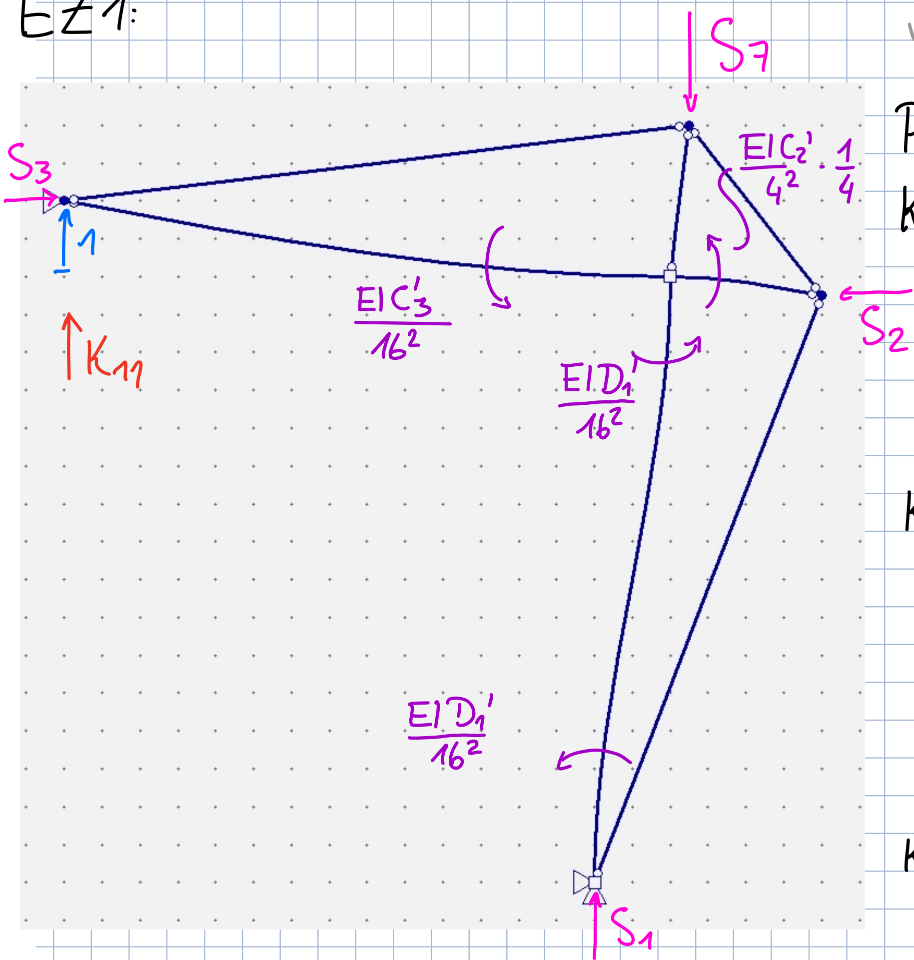


$K_{10} = 60$

$K_{20} = 0$

(Zugstäbe werden in der Aufgabenstellung vernachlässigt)

EZ1:



PvV:

$$K_{11} - \frac{EI\,C_3'}{16^2}\cdot\frac{1}{16} - \frac{2\,EI\,D_1'}{16^2}\cdot\frac{1}{16} - \frac{EI\,C_2'}{4^2}\cdot\frac{1}{4} + S_1\cdot 16\cdot\left(\frac{1}{16}\right)^2 + S_2\cdot 4\cdot\left(\frac{1}{16^2}\right) + S_3\cdot 16\cdot\left(\frac{1}{16}\right)^2 + S_7\cdot 4\cdot\left(\frac{1}{16}\right)^2 = 0$$

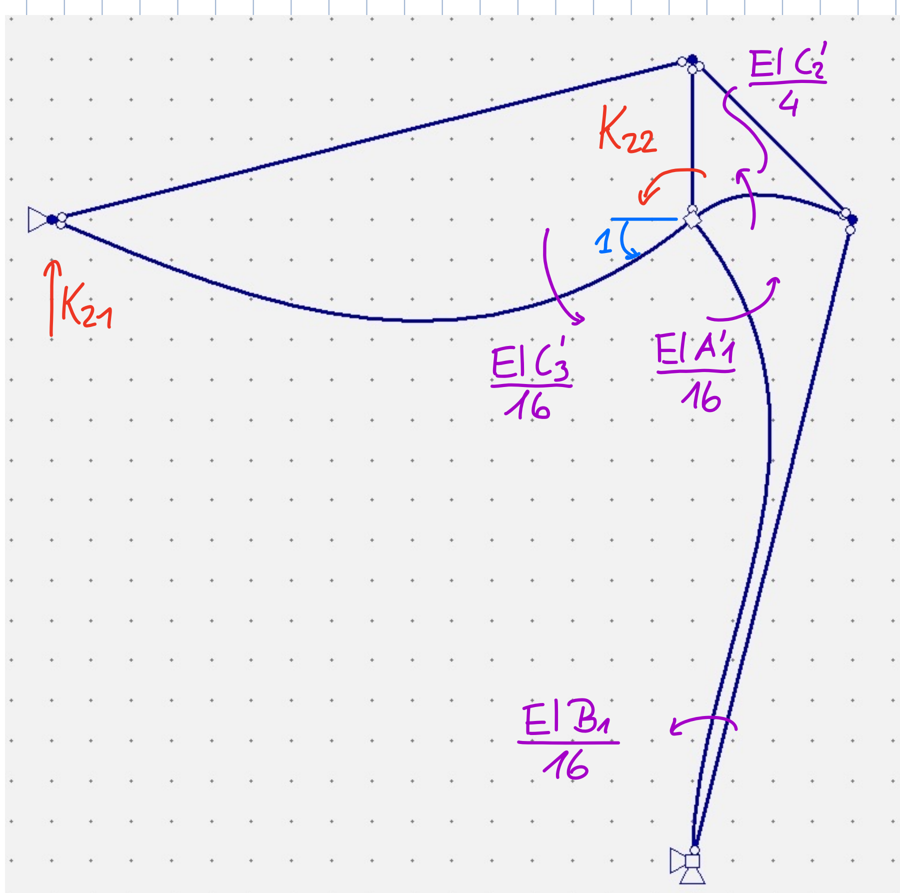
$$K_{11} = 73{,}24 + 287{,}97 + 292{,}97 - 28{,}75 - 5{,}39 - 16{,}25 - 6{,}02 = 596{,}91$$

$$K_{12} = \frac{EI\cdot C_3'}{16^2} + \frac{EI\,D_1'}{16^2} + \frac{EI\,C_2'}{4^2}\cdot\frac{1}{4}$$

$$= 1171{,}88 + 2296{,}88 + 4687{,}5$$

$$= 8156{,}26$$

EZ2:



PvV:

$$K_{21}\cdot 1 = \frac{EI\,C_2'}{4}\,\frac{1}{16} + \frac{EI\,A_1'}{16}\cdot\frac{1}{16} + \frac{EI\,B_1'}{16}\cdot\frac{1}{16} + \frac{EI\,C_3'}{16}\cdot\frac{1}{16}$$

$$= 4687{,}5 + 1500{,}0 + 796{,}88 + 1171{,}88$$

$$= 8156{,}26$$

GG:

$$K_{22} = \frac{EI\,C_2'}{4} + \frac{EI\,A_1'}{16} + \frac{EI\,C_3'}{16} =$$

$$= 75000 + 24000 + 18750$$

$$= 117750{,}0$$

$$\underline{\underline{K}} = \begin{bmatrix} 596{,}91 & 8156{,}26 \\ 8156{,}26 & 117750{,}0 \end{bmatrix} \qquad \underline{\underline{F}} = -\underline{\underline{K_0}} = \begin{bmatrix} -60 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{u} = \underline{\underline{F}} \longrightarrow \underline{u} = \begin{bmatrix} -1{,}88 \\ 0{,}13 \end{bmatrix}$$

Einspannmoment M_{11}:

$$M_{11} = 0 - \frac{EI\, D_1'}{16^2} \cdot (-1{,}88) - \frac{EI\, B_1'}{16} \cdot 0{,}13 =$$

$$= 0 + 4318{,}12 - 1657{,}5 = 2660{,}62 \text{ kNm}$$

b) Berechnung des kritischen Lastfaktors:

$$K_{11,el} = \frac{EI\, C_3'}{16^2} \cdot \frac{1}{16} + \frac{2\, EI\, D_1'}{16^2} \cdot \frac{1}{16} + \frac{EI\, C_2'}{4^2} \cdot \frac{1}{4} \cdot \frac{1}{16}$$

$$= 73{,}24 + 292{,}97 + 292{,}97 = 659{,}18$$

$$K_{11,geo} = K_{11} + \frac{EI \cdot (-\frac{1}{5}\alpha_3^2)}{16^2} \cdot \frac{1}{16} + \frac{EI(-\frac{1}{5}\alpha_2^2)}{4^2} \cdot \frac{1}{4} \cdot \frac{1}{16} - K_{11,el}$$

$$= 596{,}91 - 3{,}17 - 0{,}98 - 659{,}18 = -66{,}42$$

$$K_{12,el} = K_{12} - \frac{EI(-\frac{1}{10}\alpha_1^2)}{16^2} = 8156{,}26 + 46{,}41 = 8202{,}67$$

$$K_{12,geo} = \frac{EI(-\frac{1}{5}\alpha_3^2)}{16^2} + \frac{EI(-\frac{1}{10}\alpha_1^2)}{16^2} + \frac{EI(-\frac{1}{5}\alpha_2^2)}{4^3}$$

$$= -52{,}53 - 46{,}41 - 16{,}53 = -115{,}47$$

$$K_{21,el} = K_{12,el} = 8202{,}67$$

$$K_{21,geo} = K_{12,geo} = -115{,}47$$

$$K_{22,el} = K_{22} - \frac{EI(-\frac{2}{15}\alpha_1^2)}{16} = 117750 + 990{,}08 = 118740{,}08$$

$$K_{22,geo} = \frac{EI(-\frac{1}{5}\alpha_2^2)}{4} + \frac{EI(-\frac{2}{15}\alpha_1^2)}{16} + \frac{EI(-\frac{1}{5}\alpha_3^2)}{16} =$$

$$= -264{,}5 - 990{,}08 - 840{,}5 = -2095{,}08$$

$$\det \begin{vmatrix} 659{,}18 - \gamma \cdot 66{,}42 & 8202{,}67 - \gamma \cdot 115{,}47 \\ 8202{,}67 - \gamma \cdot 115{,}47 & 118740{,}08 - \gamma \cdot 2095{,}08 \end{vmatrix} \overset{!}{=} 0$$

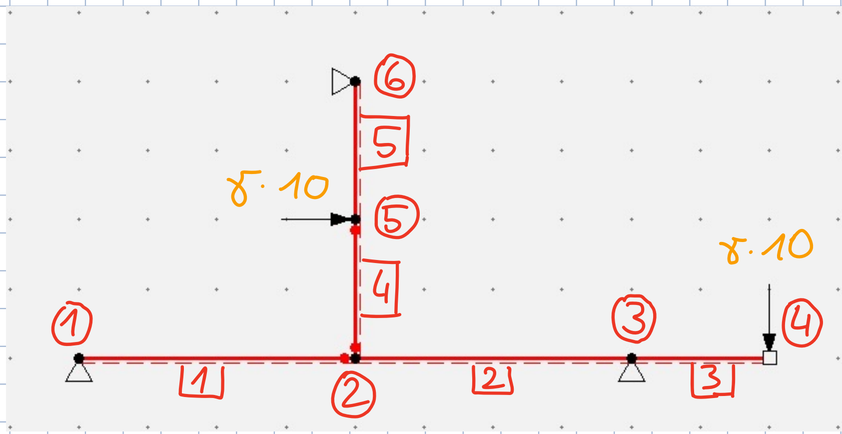
$$(659{,}18 - \gamma \cdot 66{,}42) \cdot (118740{,}08 - \gamma \cdot 2095{,}08) - (8202{,}67 - \gamma \cdot 115{,}47)^2 \overset{!}{=} 0$$

$\longrightarrow \quad \gamma_1 = 1{,}53 \quad$ ~~ma~~ $\quad$ ~~bg.~~

$\gamma_2 = 57{,}07$

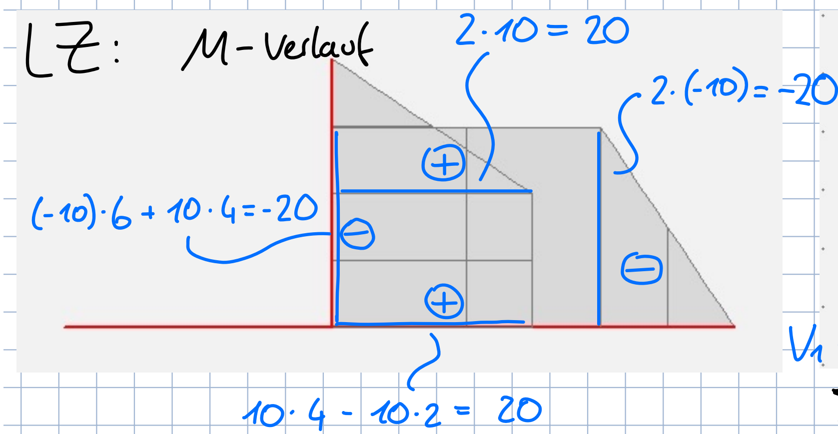
Probeklausur 2 Aufgabe 2

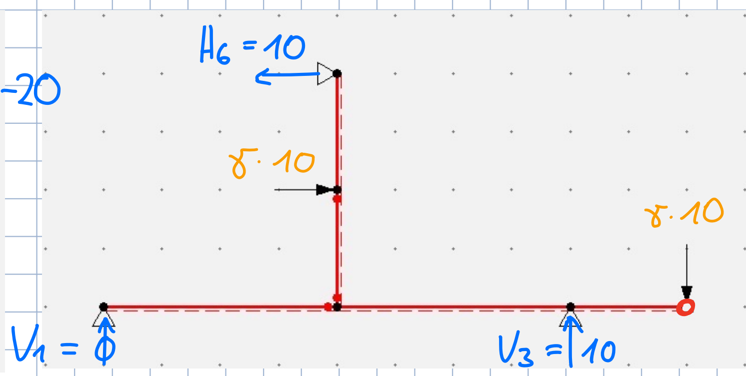
a)



Stab	1,2,3	4
EI [kNm²]	10 000	10 000
EA [kN]	→ ∞	→ ∞
M_{pl} [kNm]	40	100

LZ: M-Verlauf



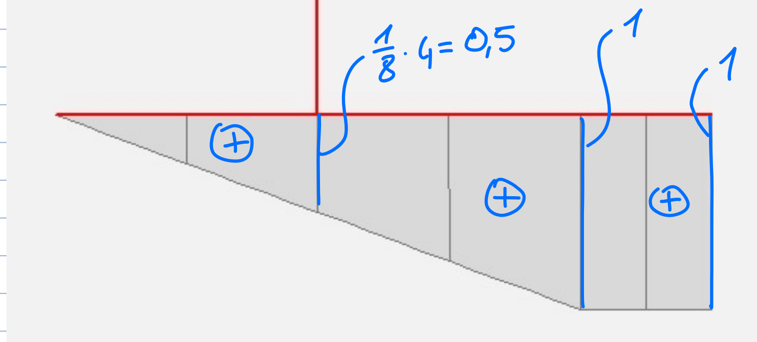


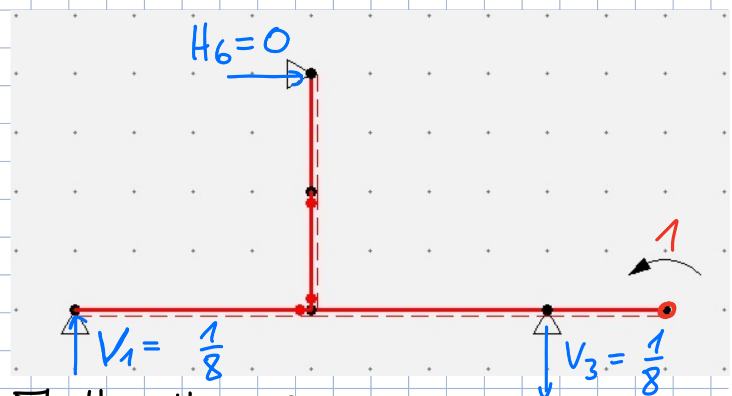
$$\Sigma H: \quad H_6 = 10$$

$$\Sigma M_1: \quad V_3 = \frac{10 \cdot 2 - 10 \cdot 4 + 10 \cdot 10}{8} = 10$$

$$\Sigma V: \quad V_1 = 10 - 10 = 0$$

EZ: M-Verlauf





$$\Sigma H: \quad H_6 = 0$$

$$\Sigma M_1: \quad V_3 = \frac{1}{8}$$

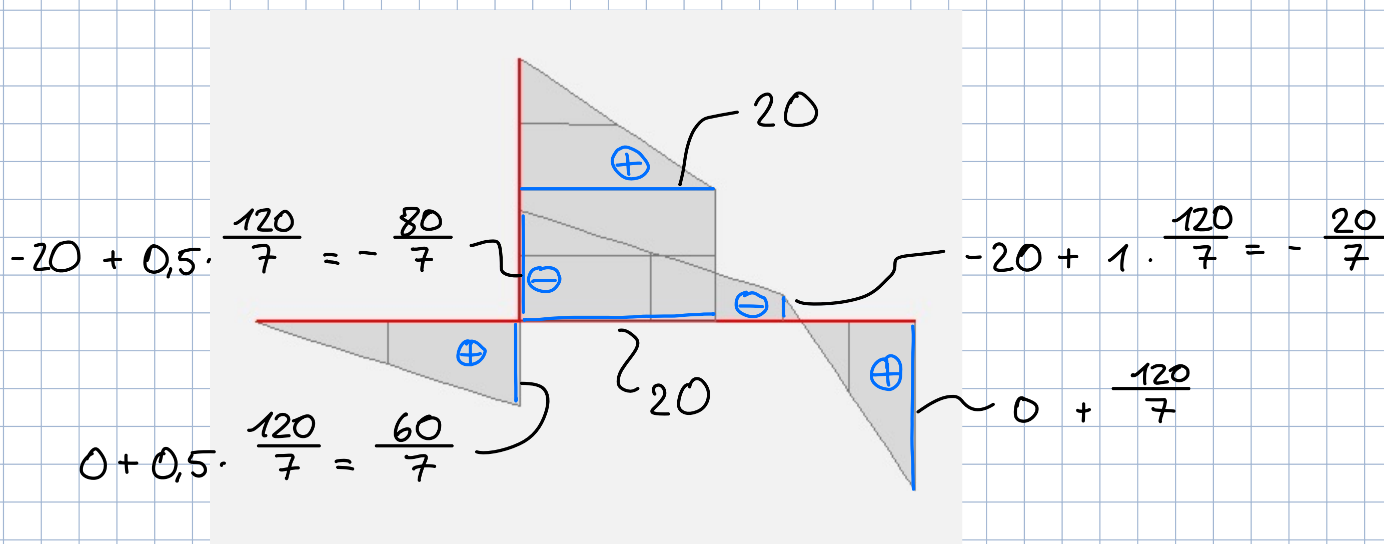
$$\Sigma V: \quad V_1 = V_3 = \frac{1}{8}$$

$$d_{10} \cdot EI = \frac{1}{2} \cdot 4 \cdot (-20) \cdot (0{,}5 + 1) + \frac{1}{2}(-20) \cdot 1 \cdot 2 = -60 - 20 = -80$$

$$d_{11} \cdot EI = \frac{1}{3} \cdot 1^2 \cdot 8 + 1^2 \cdot 2 = \frac{14}{3}$$

$$X_1 = -\frac{d_{10} \cdot \cancel{EI}}{d_{11} \cdot \cancel{EI}} = \frac{80}{\left(\frac{14}{3}\right)} = \frac{120}{7}$$

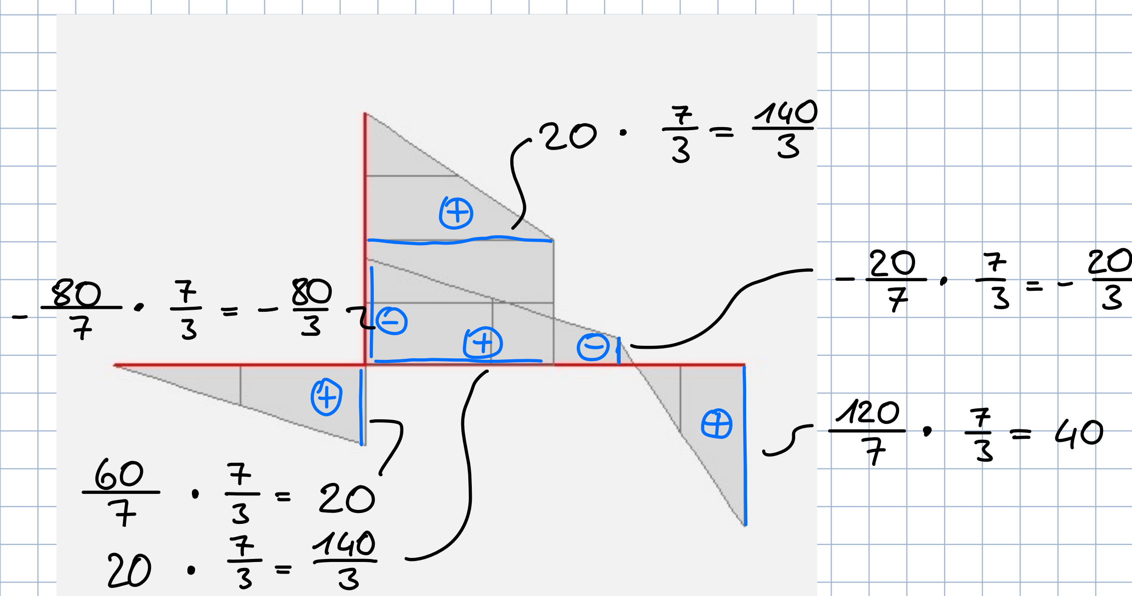
<u>M- Verlauf für $\gamma = 1$ [kNm]</u>



b) Berechnung des 1. FG:

$$\gamma_1 = \min \begin{cases} \frac{M_{pl}}{M_{el}} = \frac{40}{\left(\frac{120}{7}\right)} = \frac{7}{3} \quad \text{maßg.} \\ \qquad = \frac{100}{20} = 5 \end{cases}$$

<u>M- Verlauf für $\gamma = \gamma_1$ [kNm]:</u>



Berechnung des 2. FG:

Der Momenten Verlauf wird aus Aufgabe a) LZ übernommen

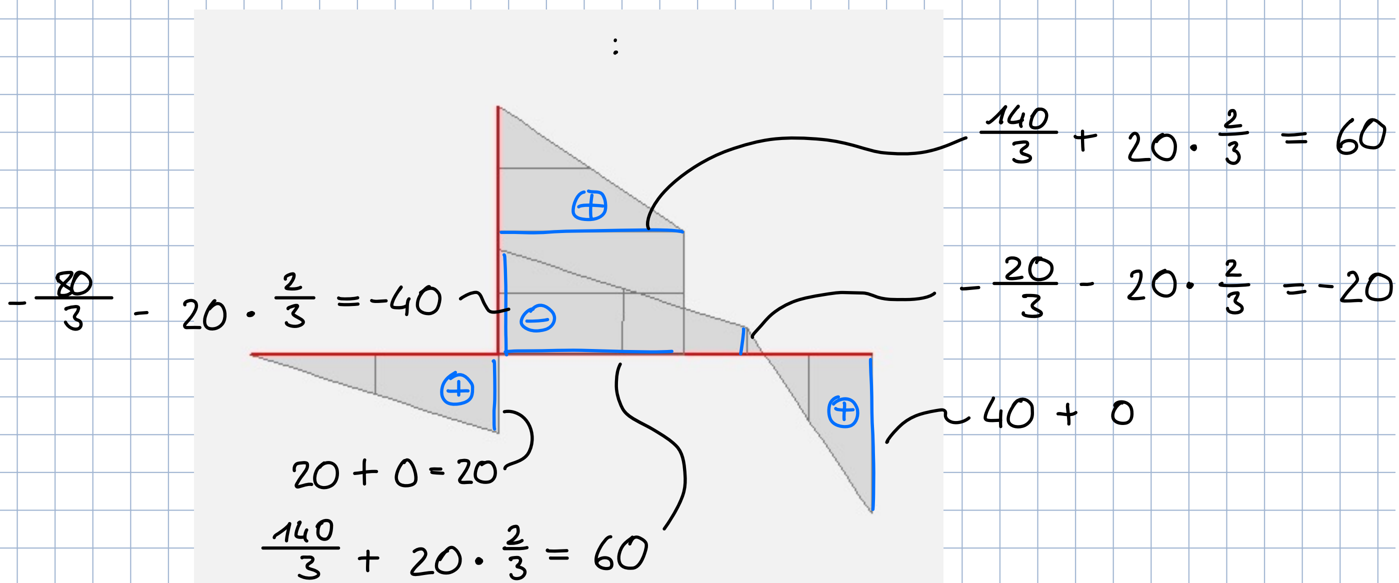
$$\Delta\gamma_2 = \min \begin{cases} \left|-\frac{80}{3}\right| + \left|-20\right| \cdot \Delta\gamma_2 = 40 \rightarrow \Delta\gamma_2 = \frac{2}{3} \text{ maßg.} \\ \frac{140}{3} + 20 \cdot \Delta\gamma_2 = 100 \rightarrow \Delta\gamma_2 = \frac{8}{3} \end{cases}$$

System wird kinematisch → keine weiteren FG vorhanden

Traglastfaktor:

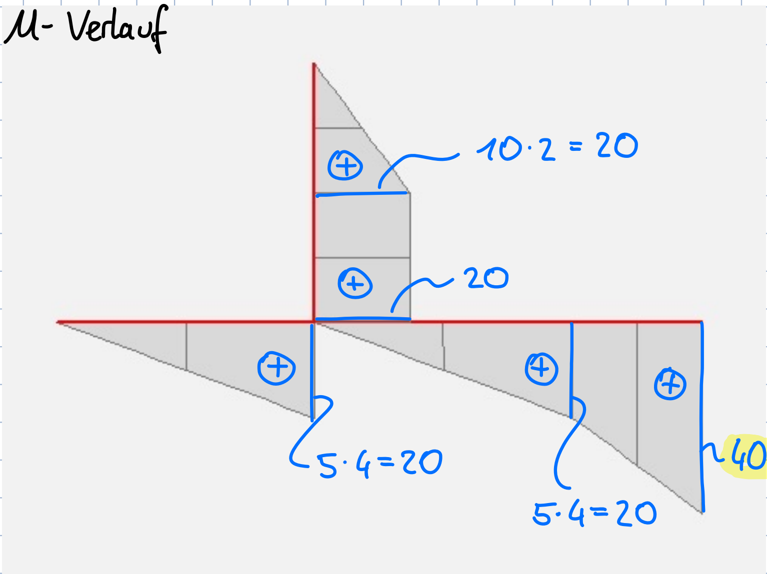
$$\gamma_T = \gamma_1 + \Delta\gamma_2 = \frac{7}{3} + \frac{2}{3} = 3{,}0$$

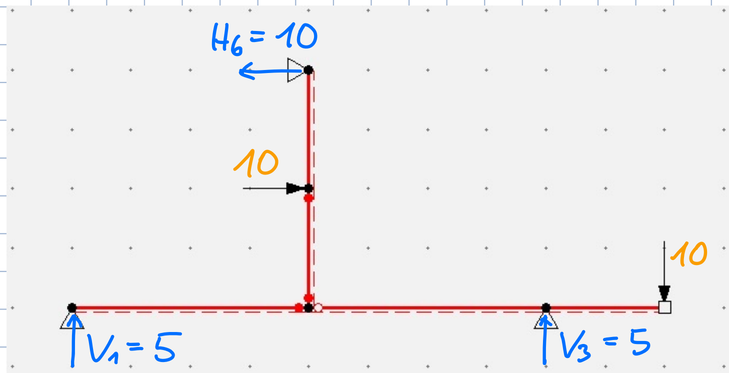
M-Verlauf für $\gamma = \gamma_T$ [kNm]



Prüfen ob sich das 1. FG schließt:

M- Verlauf





$\Sigma H: \quad H_6 = 10$

$\Sigma M_2: \quad V_1 = \frac{10 \cdot 4 - 10 \cdot 2}{4} = 5$

$\Sigma V: \quad V_3 = 10 - 5 = 5$

$\Sigma M_2: \quad M_4 = 10 \cdot 6 - 5 \cdot 4 = 40$

FG 1. schließt sich nicht, da das Vorzeichen von M_{43} nicht wechselt

c) neuer Traglastfaktor wenn $M_{pl,1,2,3} = 100$ kNm

Berechnung des 1. FG:

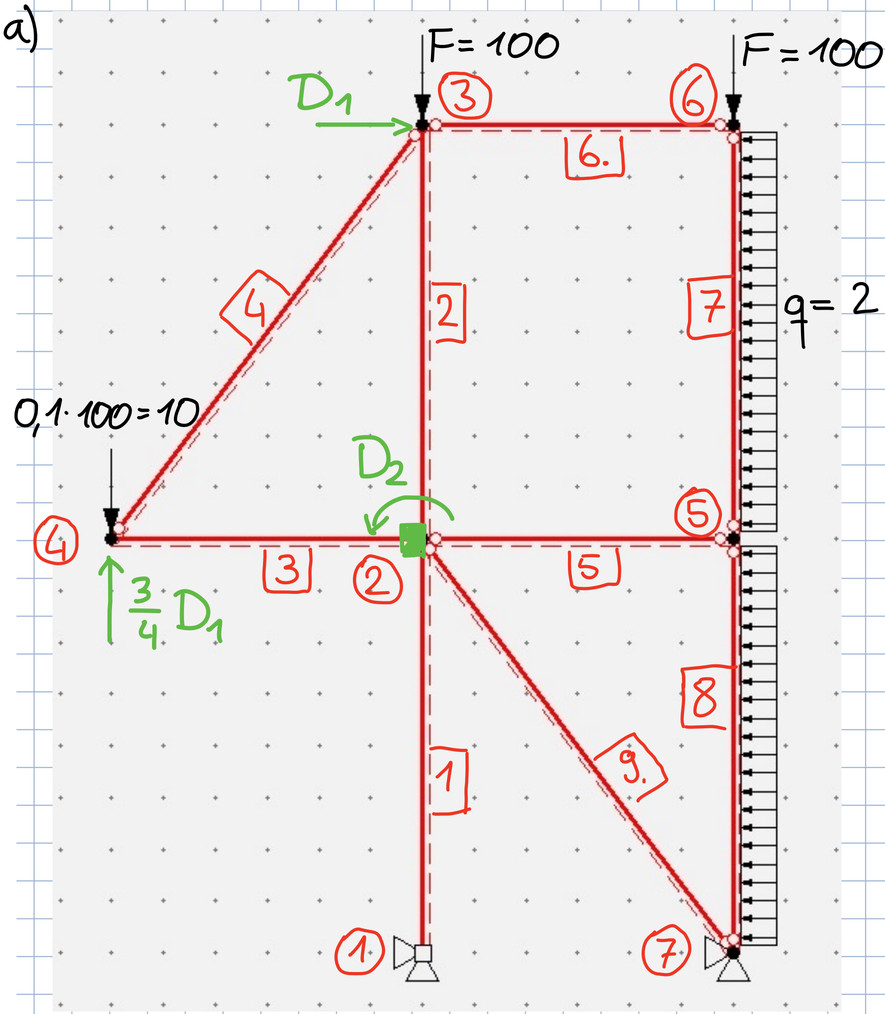
$$\gamma_1 = \min \begin{cases} \frac{M_{pl}}{M_{el}} = \frac{100}{\left(\frac{120}{7}\right)} = \frac{35}{6} \\ \phantom{\frac{M_{pl}}{M_{el}}} = \frac{100}{20} = 5 \quad \text{maßg.} \end{cases}$$

→ Teilsystem wird kinematisch → keine weiteren FG vorhanden

→ Traglast faktor $\gamma_T = \gamma_1 = 5$

Probeklausur 3 Aufgabe 1

a)



EI = 10 000 kNm²
EA → ∞

q = 2 kN/m
F = 100 kN

Stab	N [kN]
1	-173,80
2	-98,19
3	1,36
4	-2,26
5	-16,00
6	-8,00
7	-100,00
8	-100,00
9	78,75

relevante α- Werte:

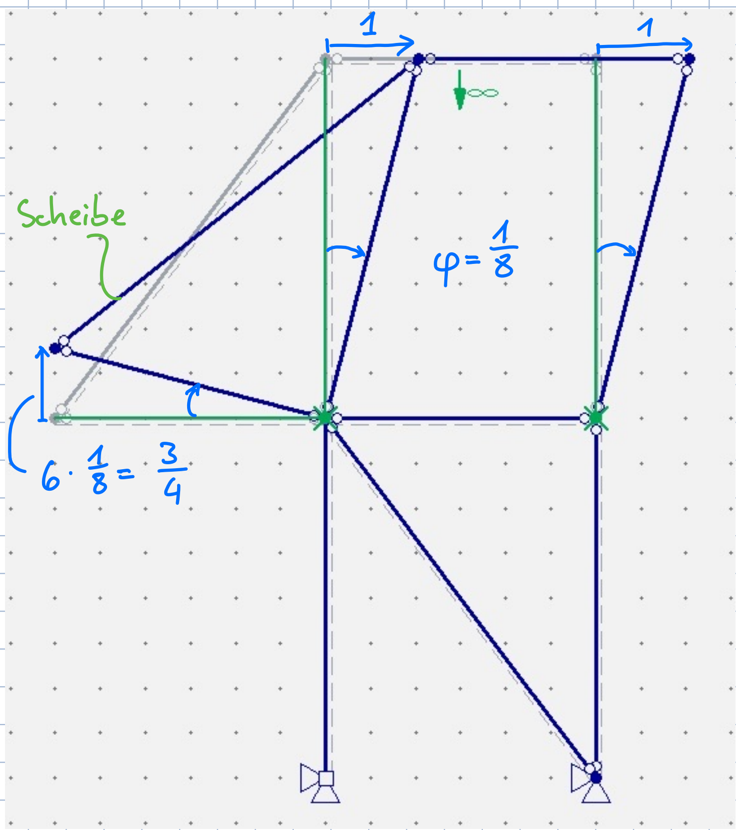
$\alpha_1 = 8 \cdot \sqrt{\frac{173,80}{10^4}} = 1,055 < 2,5$
> 1

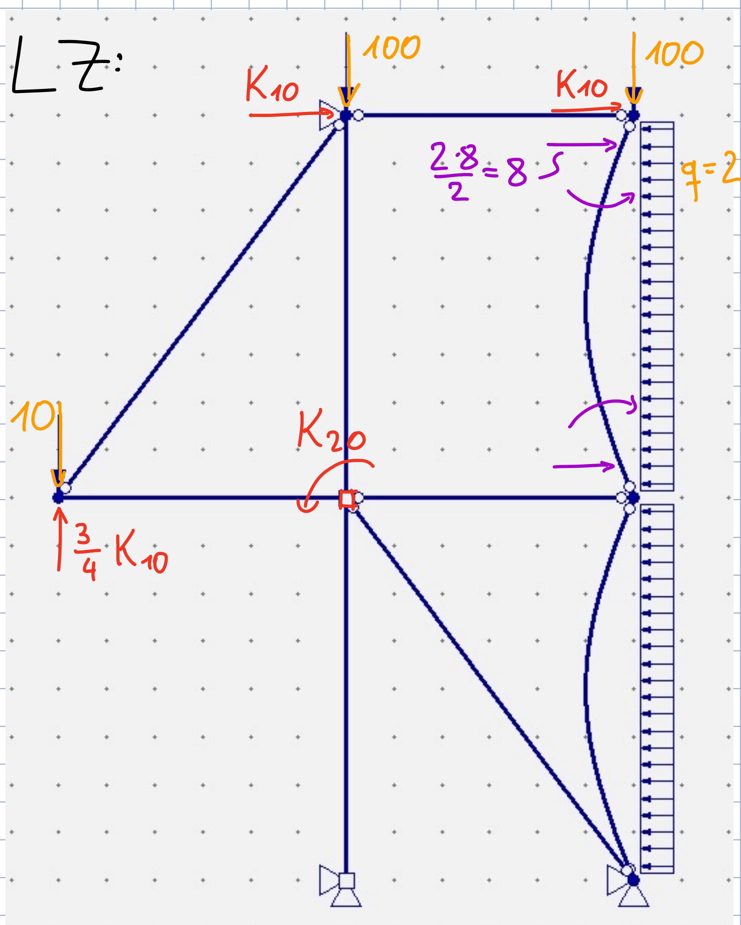
$\alpha_2 = 8 \sqrt{\frac{98,19}{10^4}} = 0,793 < 1$

$A_1' = 4 - \frac{2}{15} \cdot 1,055^2 = 3,852$

$C_2' = 3,0$

Gelenkfigur / Polplan:



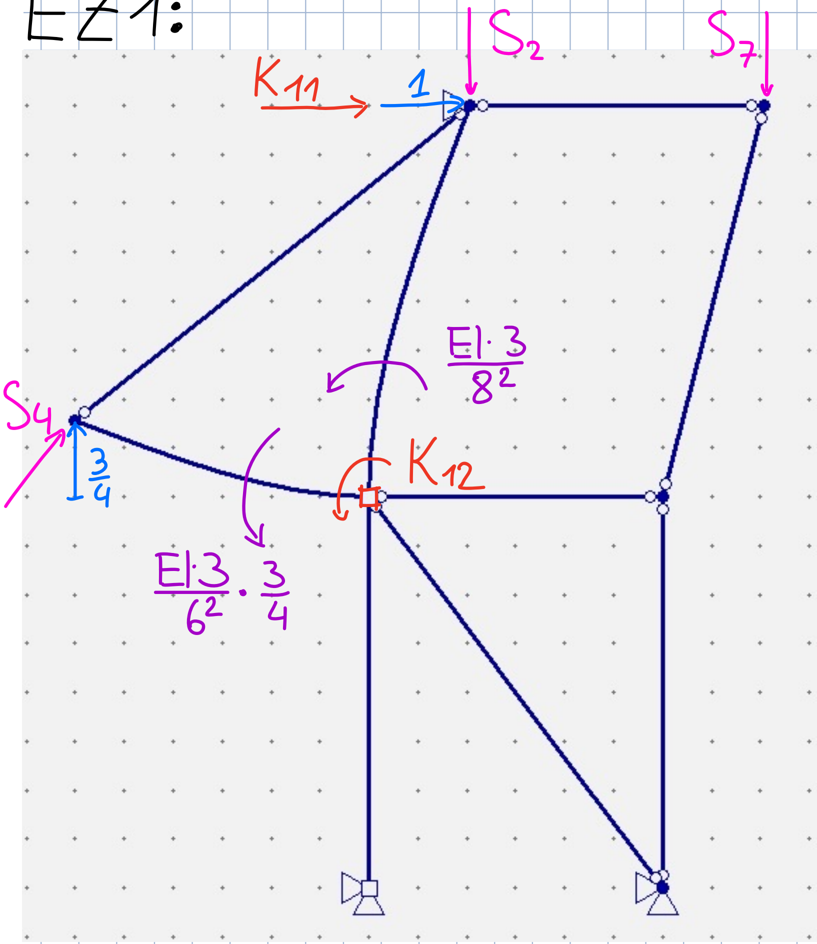


GG:

$K_{10} = 8 + \frac{3}{4} \cdot 10 = 15{,}5$

$K_{20} = 0$

EZ1:



PvV:

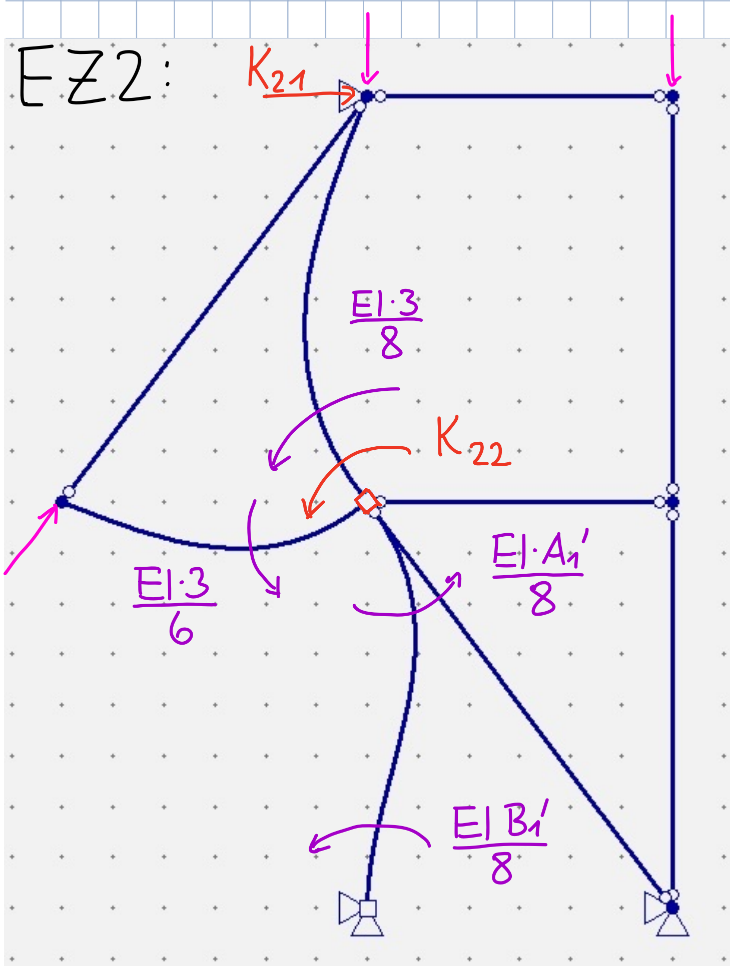
$$K_{11} - \frac{EI\cdot 3}{8^2}\cdot\frac{1}{8} - \frac{EI\cdot 3}{6^2}\cdot\frac{3}{4}\cdot\frac{1}{8} + S_2\cdot 8\cdot\left(\frac{1}{8}\right)^2 + S_7\cdot 8\cdot\left(\frac{1}{8}\right)^2 + S_4\cdot 10\cdot\left(\frac{1}{8}\right)^2 = 0$$

$$K_{11} = 58{,}594 + 78{,}125 - 12{,}274 - 12{,}5 - 0{,}353 = 111{,}592$$

GG:

$$K_{12} = \frac{EI\cdot 3}{8^2} + \frac{EI\cdot 3}{6^2}\cdot\frac{3}{4} = 468{,}75 + 625{,}0 = 1093{,}75$$

EZ2:



PvV:

$$K_{21}\cdot 1 = \frac{EI\cdot 3}{8}\cdot\frac{1}{8} + \frac{EI\cdot 3}{6}\cdot\frac{1}{8} = 468{,}75 + 625{,}0 = 1093{,}75$$

GG:

$$K_{22} = \frac{EI\cdot 3}{8} + \frac{EI\cdot 3}{6} + \frac{EI\, A_1'}{8} = 3750 + 5000 + 4815{,}0 = 13565{,}0$$

$$\underline{\underline{K}} = \begin{bmatrix} 111,592 & 1093,75 \\ 1093,75 & 13565,0 \end{bmatrix} \qquad \underline{\underline{F}} = -\underline{\underline{K_0}} = \begin{bmatrix} -15,5 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{v} = \underline{F} \longrightarrow \underline{v} = \begin{bmatrix} -0,662 \\ 0,052 \end{bmatrix}$$

$$w = \frac{3}{4} \cdot (-0,662) = -0,497$$

b) Berechnung des kritischen Lastfaktors

$$K_{11,el} = \frac{EI \cdot 3}{8^2} \cdot \frac{1}{8} + \frac{EI \cdot 3}{6^2} \cdot \frac{3}{4} = 58,594 + 78,125 = 136,719$$

$$K_{11,geo} = K_{11} + \frac{EI \cdot (-\frac{1}{5}\alpha_2^2)}{8^2} \cdot \frac{1}{8} - K_{11,el} =$$

$$= 111,592 - 2,461 - 136,719 = -27,588$$

$$K_{12,el} = \frac{EI \cdot 3}{8^2} + \frac{EI \cdot 3}{6^2} \cdot \frac{3}{4} = 468,75 + 625,0 = 1093,75$$

$$K_{12,geo} = \frac{EI \cdot (-\frac{1}{5} \cdot \alpha_2^2)}{8^2} = -19,687$$

$$K_{21,el} = K_{12,el}$$

$$K_{21,geo} = K_{12,geo}$$

$$K_{22,el} = \frac{EI \cdot 3}{8} + \frac{EI \cdot 3}{6} + \frac{EI \cdot 4}{8} = 3750 + 5000 + 5000 = 13750,0$$

$$K_{22,geo} = \frac{EI \cdot (-\frac{1}{5}\alpha_2^2)}{8} + \frac{EI(-\frac{2}{15}\alpha_1^2)}{8} = -157,5 - 185,0 = -342,5$$

$$\det \begin{vmatrix} 136,719 - \gamma \cdot 27,588 & 1093,75 - \gamma \cdot 19,687 \\ 1093,75 - \gamma \cdot 19,687 & 13750,0 - \gamma \cdot 342,5 \end{vmatrix} \overset{!}{=} 0$$

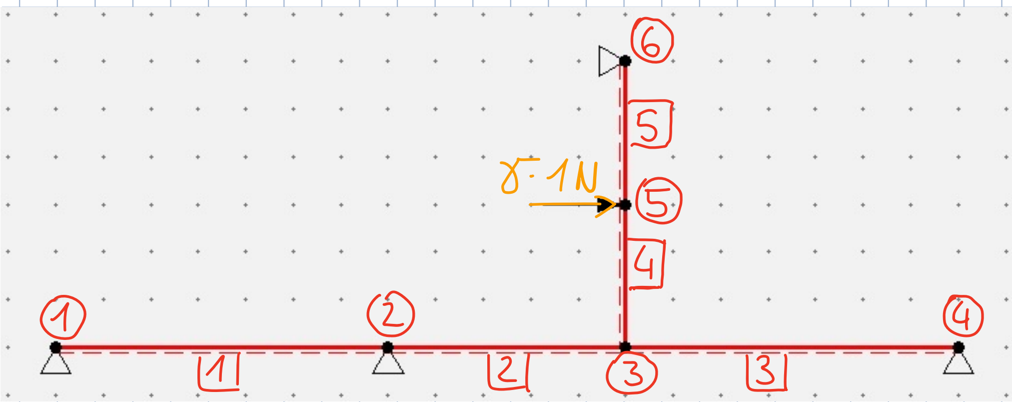
$$(136,719 - \gamma \cdot 27,588) \cdot (13750,0 - \gamma \cdot 342,5) - (1093,75 - \gamma \cdot 19,687)^2 \overset{!}{=} 0$$

Taschenrechner $\longrightarrow$ $\gamma_1 = 1,867$ maßg.

$\gamma_2 = 40,411$

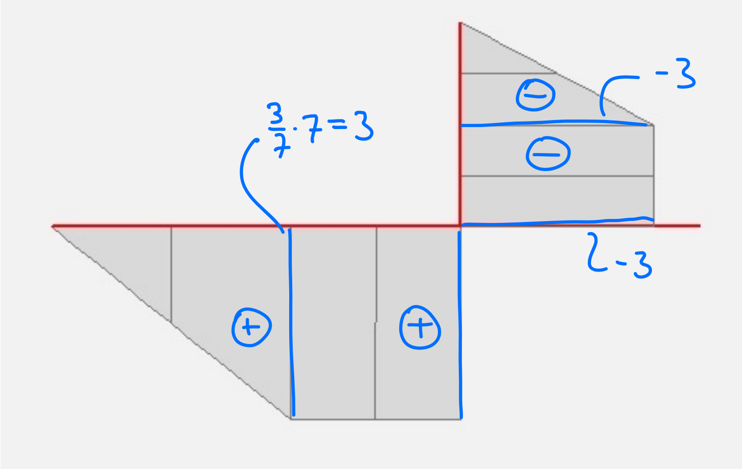
Probeklausur 3 Aufgabe 2

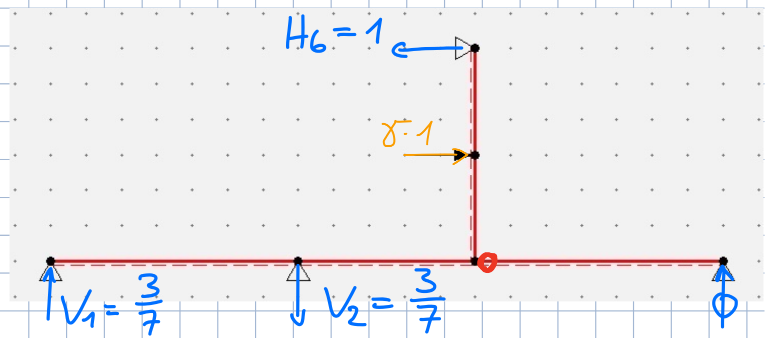
a)



Stab	1,2,3	4,5
EI [Nm²]	10 000	10 000
EA [N]	→ ∞	→ ∞
M_{pl} [Nm]	90	200

LZ: M-Verlauf:



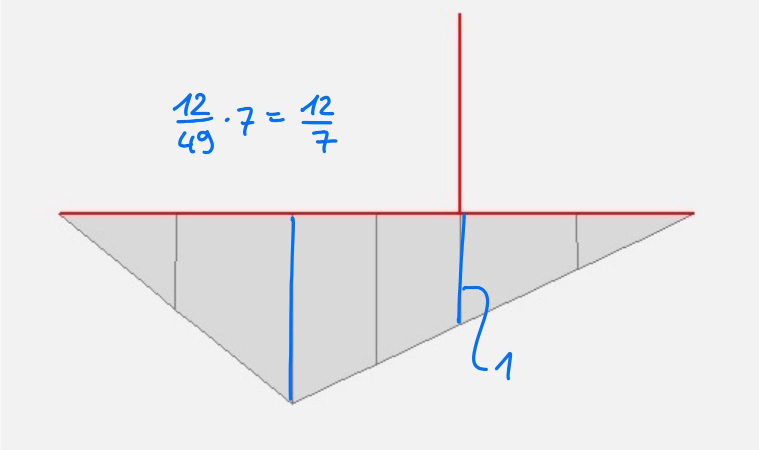


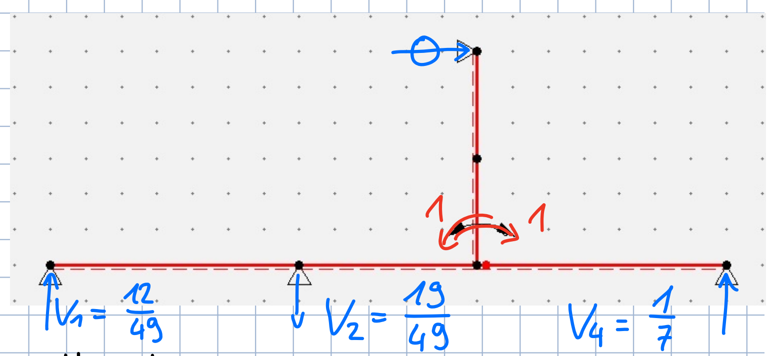
$$\Sigma H: \quad H_6 = 1$$

$$\Sigma M_2: \quad V_1 = \frac{1\cdot 6 - 1\cdot 3}{7} = \frac{3}{7}$$

$$\Sigma V: \quad V_2 = V_1 = \frac{3}{7}$$

EZ: M-Verlauf





$$\Sigma H: \quad H_6 = 0$$

$$\Sigma M_2: \quad V_1 = \frac{\frac{1}{7}\cdot 12}{7} = \frac{12}{49}$$

$$\Sigma M_3: \quad V_4 = \frac{1}{7}$$

$$\Sigma V: \quad V_2 = \frac{12}{49} + \frac{1}{7} = \frac{19}{49}$$

$$d_{10} \cdot EI = \frac{1}{3} \cdot 3 \cdot \frac{12}{7} \cdot 7 + \frac{1}{2} \cdot 3 \cdot \left(\frac{12}{7} + 1\right) \cdot 5 =$$

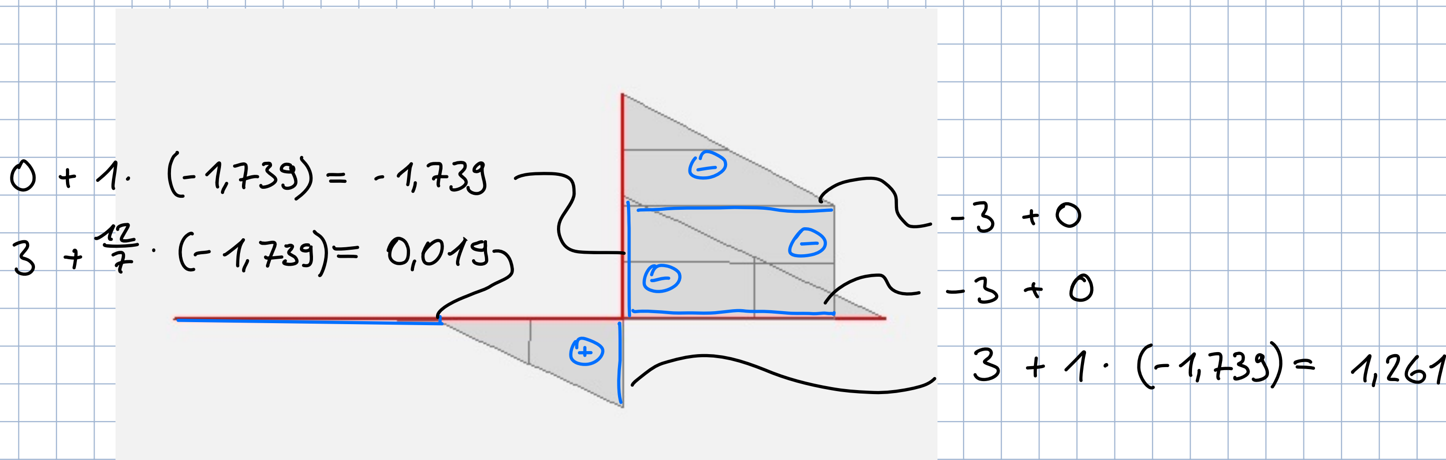
$$= 12{,}0 + 20{,}357 = 32{,}357$$

$$d_{11} \cdot EI = \frac{1}{3} \cdot \left(\frac{12}{7}\right)^2 \cdot 7 + \frac{1}{6} \cdot \left(2 \cdot \left(\frac{12}{7}\right)^2 + 2 \cdot \frac{12}{7} \cdot 1 + 2 \cdot 1^2\right) \cdot 5 + \frac{1}{3} \cdot 1^2 \cdot 7 =$$

$$= 6{,}857 + 9{,}422 + 2{,}333 = 18{,}612$$

$$X_1 = -\frac{d_{10} \cdot \cancel{EI}}{d_{11} \cdot \cancel{EI}} = \frac{-32{,}357}{18{,}612} = -1{,}739$$

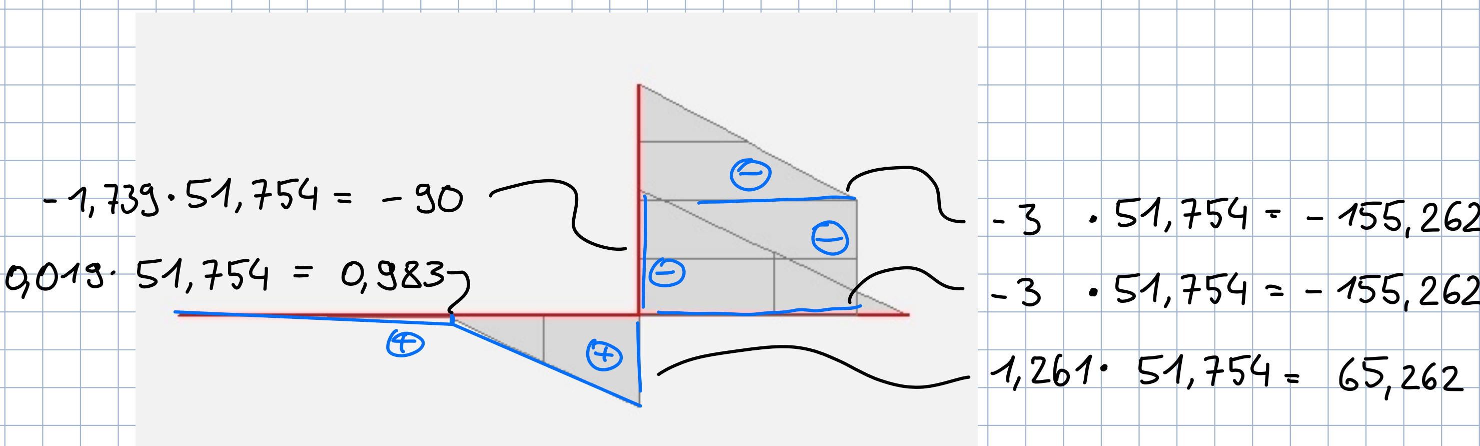
<u>M- Verlauf für $\gamma = 1$: [Nm]</u>



b) Berechnung des 1. FG:

$$\gamma_1 = \min \begin{cases} \frac{M_{pl}}{M_{el}} = \frac{90}{1{,}739} = 51{,}754 & \text{maßg.} \\ \frac{200}{3} = 66{,}667 \end{cases}$$

<u>M- Verlauf für $\gamma = \gamma_1$: [Nm]</u>



Berechnung des 2. Fließgelenk:

$\mathcal{M}$-Verlauf wird aus Aufgabe a) Lastzustand übernommen

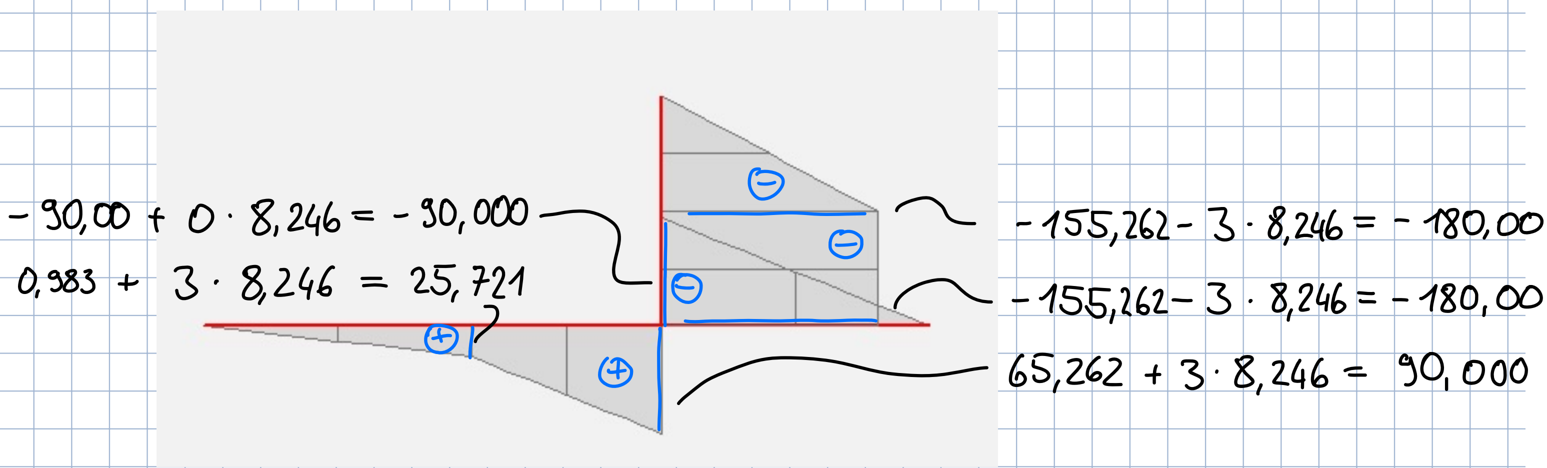
$$\Delta\gamma_2 = \min \begin{cases} 65{,}262 + \Delta\gamma_2 \cdot 3 = 90 & \rightarrow \Delta\gamma_2 = 8{,}246 \text{ maßg.} \\ 155{,}262 + \Delta\gamma_2 \cdot 3 = 200 & \rightarrow \Delta\gamma_2 = 14{,}913 \end{cases}$$

Traglastfaktor:

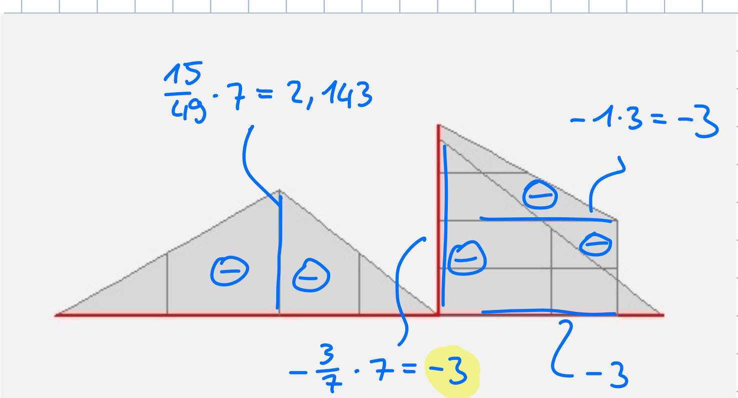
$$\gamma_T = \gamma_1 + \Delta\gamma_2 = 51{,}754 + 8{,}246 = 60{,}0$$

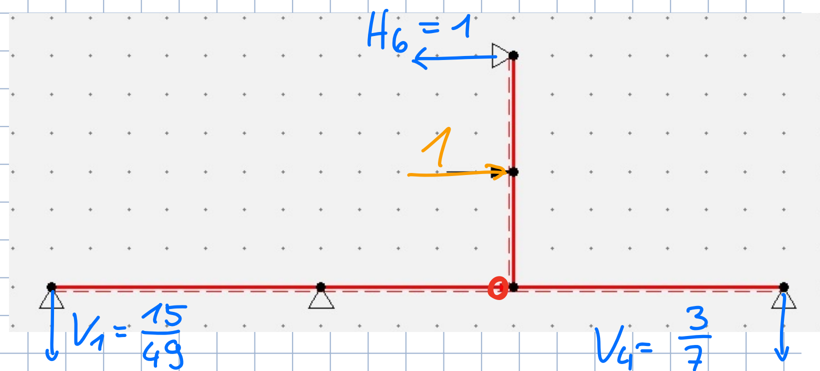
Teilsystem wird kinematisch $\rightarrow$ keine weiteren FG vorhanden

<u>$\mathcal{M}$-Verlauf für $\gamma = \gamma_T$: [Nm]</u>



Prüfen ob sich das 1. FG schließt:





$$\Sigma! \; H: \quad H_6 = 1$$

$$\Sigma! \; \mathcal{M}_3: \quad V_4 = \frac{1 \cdot 6 - 1 \cdot 3}{7} = \frac{3}{7}$$

$$\Sigma! \; \mathcal{M}_2: \quad V_1 = \frac{-1 \cdot 6 + 1 \cdot 3 + \frac{3}{7} \cdot 12}{7} = \frac{15}{49}$$

FG 1 schließt sich nicht, da das Vorzeichen von $\mathcal{M}_{33}$ nicht wechselt

c) neuer Traglastfaktor, wenn $M_{pl,1,2,3} = 200\ Nm$

Berechnung des 1. FG:

$$\gamma_1 = \min \begin{cases} \frac{M_{pl}}{M_{el}} = \frac{200}{1{,}739} = 115{,}009 \\ \qquad = \frac{200}{3} = 66{,}667 \quad \text{maßg.} \end{cases}$$

→ Teilsystem wird kinematisch → keine weiteren FG vorhanden

→ Traglastfaktor $\gamma_T = \gamma_1 = 66{,}667$