SS 2007

Technische Universität München

Lehrstuhl für Statik

Name: ____________________

Vorname: ____________________

Mtr.Nr.: ____________________

Saal Nr.: ____________________

Platz Nr.: ____________________

Diplomhauptprüfung für Bauingenieure

Statik-Vertiefung Teil II: **Theory of Shells**
11.9.2007

Baustatik Vertiefer
Bearbeitungszeit 45 Minuten

Rechenteil
45 Punkte sind erreichbar

	Punkte	
Aufgabe	max.	
Allgemeine Fragen	15	
1	45	
Σ	**60**	

Aufgabe 1: (45 points)

A conical gas storage has the foundation at the sea bed. The top of the storage is closed by a circular plate. The junction B is a simple support, transmitting only vertical forces. Self-weight does not have to be considered.

Please determine:

a) vertical und horizontal reaction force at support A (global direction)

b) ring extension (Δr) and meridian rotation (β) of the conical shell at support A due to the given load condition.

Ein kegelförmiger Gastank ist im Meeresboden verankert. Der Kegelstumpf ist an der oberen Kante durch eine kreisförmige Platte abgeschlossen. Die Verbindungsstelle B überträgt ausschließlich vertikale Lasten. Das Eigengewicht der Konstruktion muss nicht berücksichtigt werden.

Berechnen Sie:

a) vertikale und horizontale Auflagerkraft (in globaler Richtung) am Auflager A.

b) Ringaufweitung (Δr) und Meridianverdrehung (β) der kegelförmigen Schale am Auflager A unter der gegebenen Belastung.

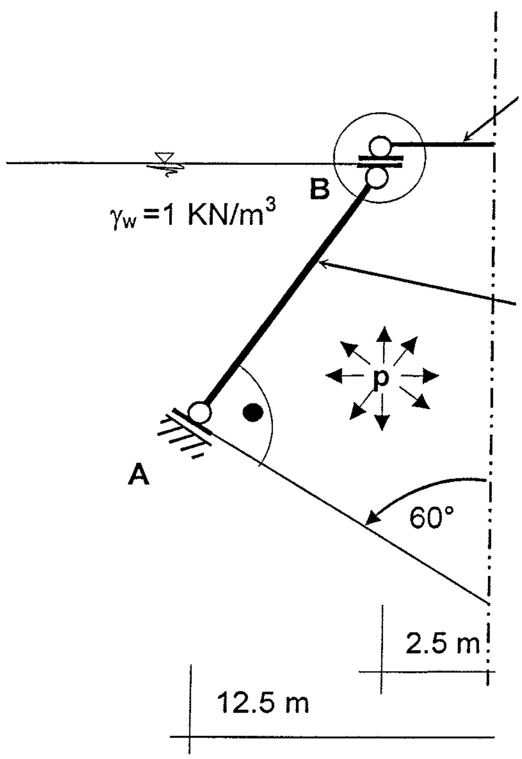


Plate / *Platte*

Schale / *shell*

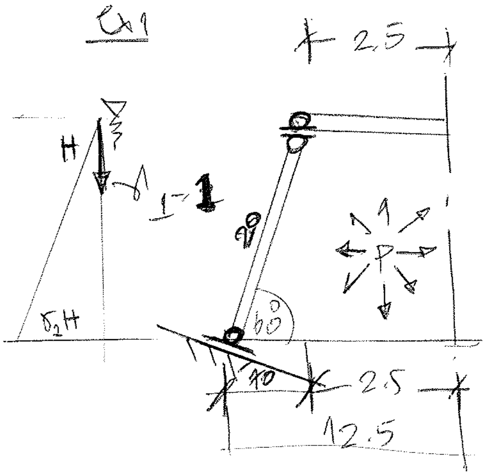
Dicke / *Thickness*: $t = 0.1$ m

E-Modul / *Young's Modulus*: $E = 200\,000$ KN/m²

Querdehnzahl / *Poisson's ratio*: $\nu = 0.0$

Gasdruck / *gas pressure* $p = 10$ KN/m²

Q1



cone (weighless)

$P = 10$ kN/m²

$\gamma = 0$

$\gamma_1 = 1$

cone

thickness $= 0.1$ m.

$\nu = 0$

$E = 200{,}000$ kN/m².

weightless

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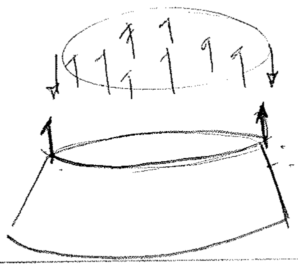
top deck



$A = \pi R^2 = \pi(2.5)^2$ m².

Peri $= 2\pi R$ m.

Ring load $= \dfrac{pA}{\text{peri}} = \dfrac{10(\pi)(2.5)^2}{2\pi(2.5)}$

$= 12.5$ kN/m. ↑

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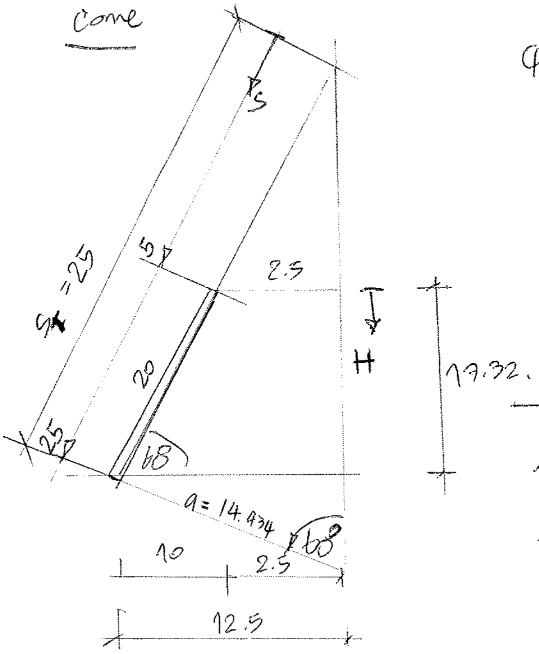
cone



$\varphi = 60°$ $\quad a = 14.434$.

$\lambda = \sqrt{\dfrac{a}{t}\sqrt{3(1-0)}} = \sqrt{\dfrac{14.434}{0.1}\sqrt{3}} = 15.811$

$s_L = \dfrac{2\pi a}{\lambda} = \dfrac{2\pi\, 14.434}{15.811} = 5.736$.

$2s_L = 11.47 < 20$ ∴ No far end distorbance.

$p_\varphi = p_{\varphi_{gas}} + p_{\varphi_{hydro}} = 0 + 0 = 0$.

$p_z = p_{z_{gas}} + p_{z_{hydro}} = 10 - \gamma H$

$= 10 - H$

$s_T \cos 60° = 12.5$

$s_T = 25$

$a \sin 60° = 12.5$

$a = 14.434$

$H = (s-5)\sin 60°$

$P_r = 2\pi \sin\varphi \cos\varphi \int_{s_0}^{s} s(p_s - p_z \cot\varphi)\, ds + C$.

$= C = -1.25$ kN/m.

$n_s = \dfrac{-P_r}{2\pi s \sin\varphi \cos\varphi} = \dfrac{+12.5}{2\pi s \sin 60° \cos 60°}$

$= \dfrac{12.5}{2\pi(25)\sin 60 \cos 60°}$ $\Rightarrow n_{s(send)} = 0.18378$.

$n_\theta = p_z s \cot\varphi = (10 - 2H)s\cot\varphi$

$n_{\theta\, at\, send} = (10 - 2(17.32))\, 25 \cot 60° = -355.65$

$$P_r = 2\pi \sin\varphi \cos\varphi \int_{s_u}^{s} s\,(\cancel{p_s}^{\,0} - p_z \cot\varphi\,)\,ds + C$$

$$= 2\pi \sin\varphi \cos\varphi \int_{5}^{25} s\left(-(10 - H)\cot\varphi\right) ds + C.$$

$$= 2\pi \cancel{\sin 60} \cos 60 \int_5^{25} -\frac{\cos\varphi}{\cancel{\sin\varphi}}\left(10s - s(s-5)\sin\varphi\right) ds + C.$$

$$= -2\pi \cos^2\varphi \int_5^{25} \left(10s - s^2\sin\varphi + 5\,s\sin\varphi\right) ds + C.$$

$$= -2\pi \cos^2\varphi \left(\frac{10s^2}{2} - \frac{s^3}{3}\sin\varphi + \frac{5\sin\varphi\, s^2}{2}\right)\Big[_5^{25} + C.$$

$$= -2\pi \cos^2\varphi \left(\left(5 + \frac{5}{2}\sin\varphi\right) s^2 \Big[_5^{25} - \frac{\sin\varphi}{3} s^3 \Big[_5^{25} \right) + C$$

$$= -2\pi \cos^2 60 \left(\left(5 + \frac{5}{2}\sin 60\right)(25^2 - 5^2) - \frac{2\sin 60}{3}(25^3 - 5^3) \right) + -62.5\pi$$

$$= -2\pi \cos^2 60 \left(4{,}299 - 4{,}474.46 \right) - 62.5\pi$$

$$= \qquad 87.73\pi - 62.5\pi = 25.21324\pi$$

$$n_s = \frac{-P_r}{2\pi s \sin\varphi\cos\varphi} = \frac{-25.213\pi}{2\pi s \sin 60 \cos 60}$$

$$n_{s\,\text{at send}} = \frac{-25.213\pi}{2\pi\, 25 \sin 60 \cos 60} = -1.16455$$

-1.1646 -1.00853 0.5823

$$n_\theta = p_z\, s\cot\varphi$$

$$= (10 - H)\, s\cot\varphi$$

$$n_{\theta\,\text{at send}} = \left(10 - (25-5)\sin 60^\circ\right) 25\cot 60^\circ$$

$$= -105.66243.$$

$$Et\,\Delta_r^{\,om} = s\cos\varphi\left(n_\theta - \cancel{\nu}^{\,0} n_s\right)$$

$$= 25\cos 60^\circ\,(-105.66) = -1320.78$$

$$Et\,\beta^{om} = \cot\varphi\left(s\frac{d}{ds}\left(n_\theta - \cancel{\nu} n_s\right) + (1+\cancel{\nu}^{\,0})(n_\theta - n_s) \right)$$

$$= \cot 60\left(s\,\frac{d}{ds}\left(\left(10 - (s-5)\sin 60^\circ\right) s\cot\varphi\right) + -105.66 + 1.166 \right)$$

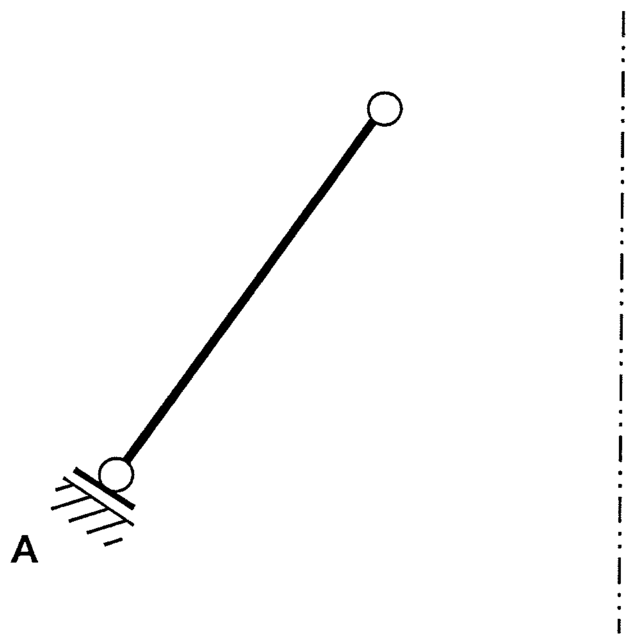
$$= \cot 60\left(s\,\frac{d}{ds}\left(10 s\cot\varphi - \sin\varphi\cot\varphi\,(s^2 - 5s)\right) - 104.5 \right)$$

$$= \cot 60^\circ\left(s\left(10\cot\varphi - \cos\varphi\,(2s - 5)\right) - 104.5 \right)$$

$$\text{at send} = \cot 60^\circ\left(25\left(10\cot 60^\circ - \cos 60\,(50 - 5)\right) - 104.5 \right).$$

c) Sketch the deformed shape. Especially show the direction of the ring extension (Δr) and the meridian rotation (β) at support A as calculated in 1.b.) .

Skizzieren Sie die Verschiebungsfigur. Markieren Sie insbesondere die in Aufgabe 1.b) berechnete Richtung der Ringaufweitung (Δr) und der Meridianverdrehung (β) am Auflager A.

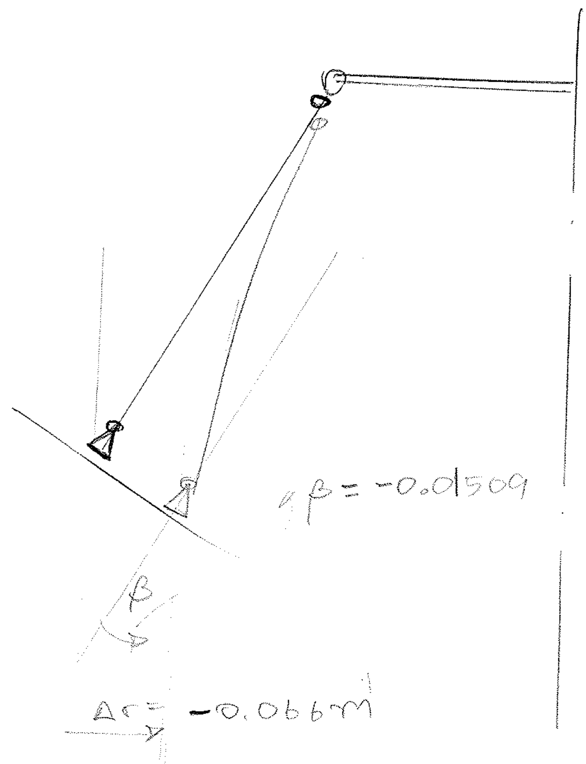


$$\bar{\beta}^{om}_{at\ send} = \cot 60^\circ \left(25(5.9735 - \cancel{22}.5) - 104.5 \right)$$
$$= -301.758$$

$$\Delta \bar{r}^{om} = \frac{-1,320.78}{200e3(0.1)} = -0.066 \text{ m}$$

$$\bar{\beta}^{om} = \frac{-301.758}{200e3(0.1)} = -0.01509$$

plot



edge disturbance

recheck

$$V = \frac{25.2139}{2\pi(12.5)} = 11.0085$$

0.5823 o.K.

$H \leftarrow = \oplus\ 0.5823$

$$Et\,\Delta\bar{r}^{oR} = 0.5823(342.324) = 199.328$$
$$Et\,\bar{\beta}^{oR} = 0.5823(432.99) = 252.132$$

$$Et\,\Delta\bar{r}^{o} = -1320.78 + 199.328 = -1121.45$$
$$Et\,\bar{\beta}^{o} = -301.758 + 252.132 = -49.626$$

unit state 1 $\alpha = 60^\circ$

$$Et\,\Delta\bar{r}^{1}_{cone} = 2a\lambda\sin^2\alpha = 2(14.434)\,15.811\sin^2 60$$
$$= 342.324.$$

$$Et\,\bar{\beta}^{1}_{cone} = 2\lambda^2\sin\alpha = 2(15.811)^2\sin 60$$
$$= 432.99$$

unit state 2

$$Et\,\Delta\bar{r}^{2}_{cone} = 2\lambda^2\sin\alpha = 432.99$$

$$Et\,\bar{\beta}^{2}_{cone} = \frac{4\lambda^3}{a} = \frac{4(15.811)^3}{14.434}$$
$$= 1095.346$$

A larger type of gas storage, consisting of the conical shell (as in task 1.a.) and a spherical shell, is fixed in the sea bed. The storage is filled with gas of pressure **p** and supports a circular plate deck at the upper edge. The junction B is a simple support, transmitting only vertical forces. Self-weight does not have to be considered.

Ein größerer Gastank, der aus einer konischen (wie in Aufgabe 1.a.) und einer kugelförmigen Schale besteht ist im Meeresboden eingespannt. Der Gastank ist mit dem Druck p gefüllt und wird durch eine kreisförmige Platte nach oben hin abgeschlossen. Die Verbindungsstelle B überträgt ausschließlich vertikale Lasten. Das Eigengewicht der Konstruktion muss nicht berücksichtigt werden.

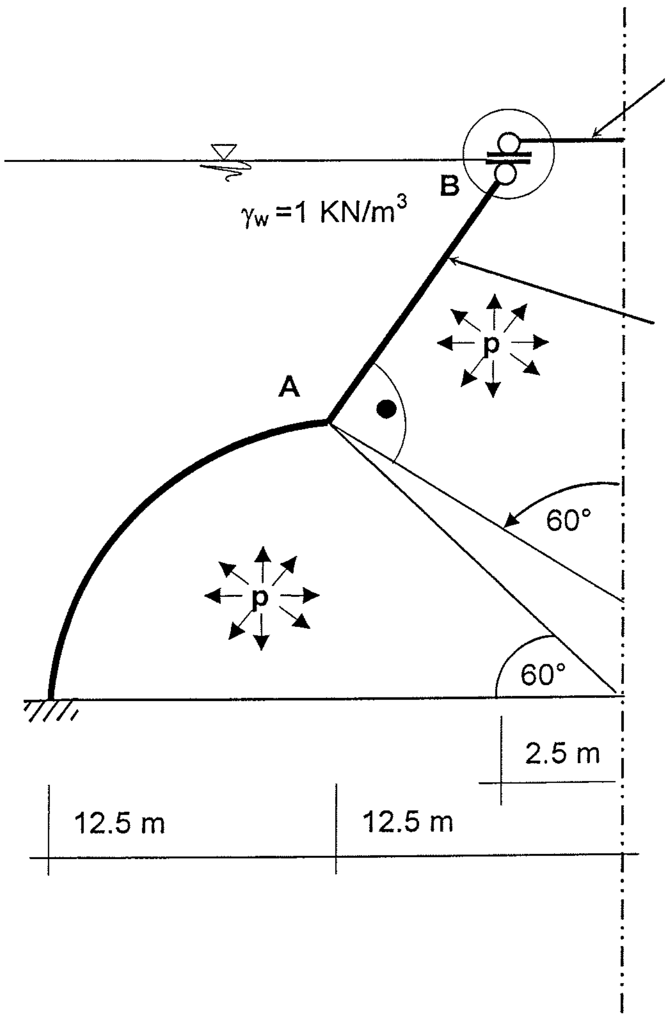


Plate / *Platte*

Schale / *shell*

Dicke / *Thickness*: $t = 0.1$ m

E-Modul / *Young's Modulus*: $E = 200\,000$ KN/m²

Querdehnzahl / *Poisson's ratio*: $\nu = 0.0$

Gasdruck / *gas pressure* **p** = 10 KN/m² d.) Determine the ring extension (Δr) and the meridian rotation (β) of cone and sphere at junction A by the force method due to the given load condition. Use the deformations given in the box below!

Berechnen Sie die Ringaufweitung (Δr) und die Meridianverdrehung (β) des Kegels und der kugelförmigen Schale an der Verbindungsstelle A infolge der angegebenen Belastung mit dem Kraftgrößenverfahren. Benutzen Sie dazu die in dem folgenden Kasten gegebenen Verformungen!

The meridian rotation and the ring extension at the top edge of the spherical shell in the membrane state are given by:

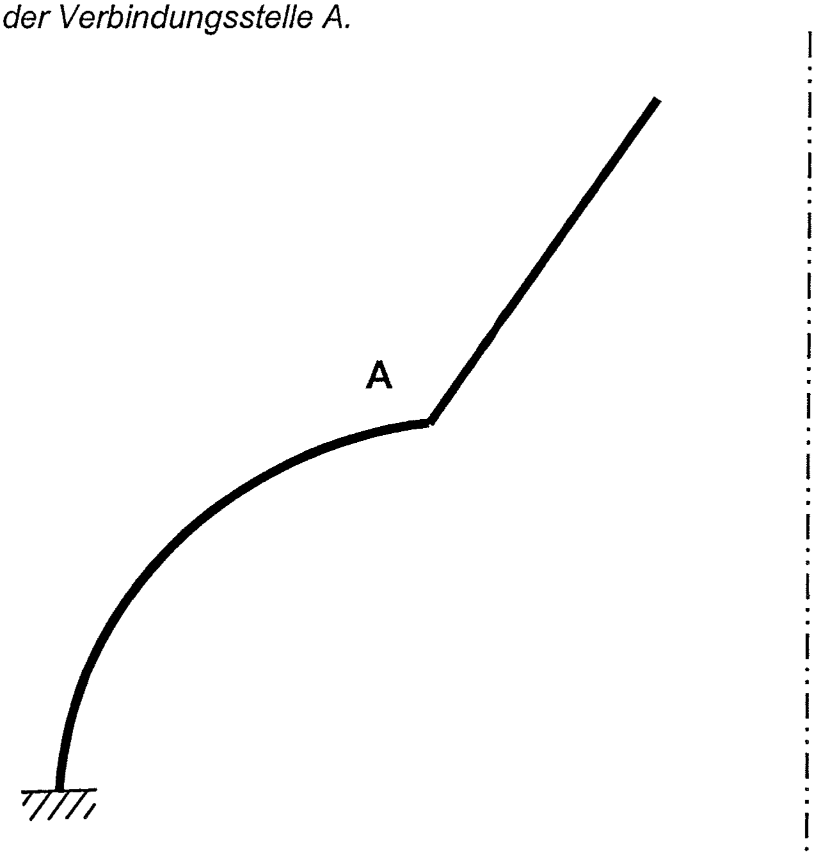
Die Meridianverdrehung und die Ringaufweitung an der Oberkante der kugelförmigen Schale für den Membranzustand ergeben sich zu:

$$Et\,\beta_{sphere}^{OM} = -312.5$$

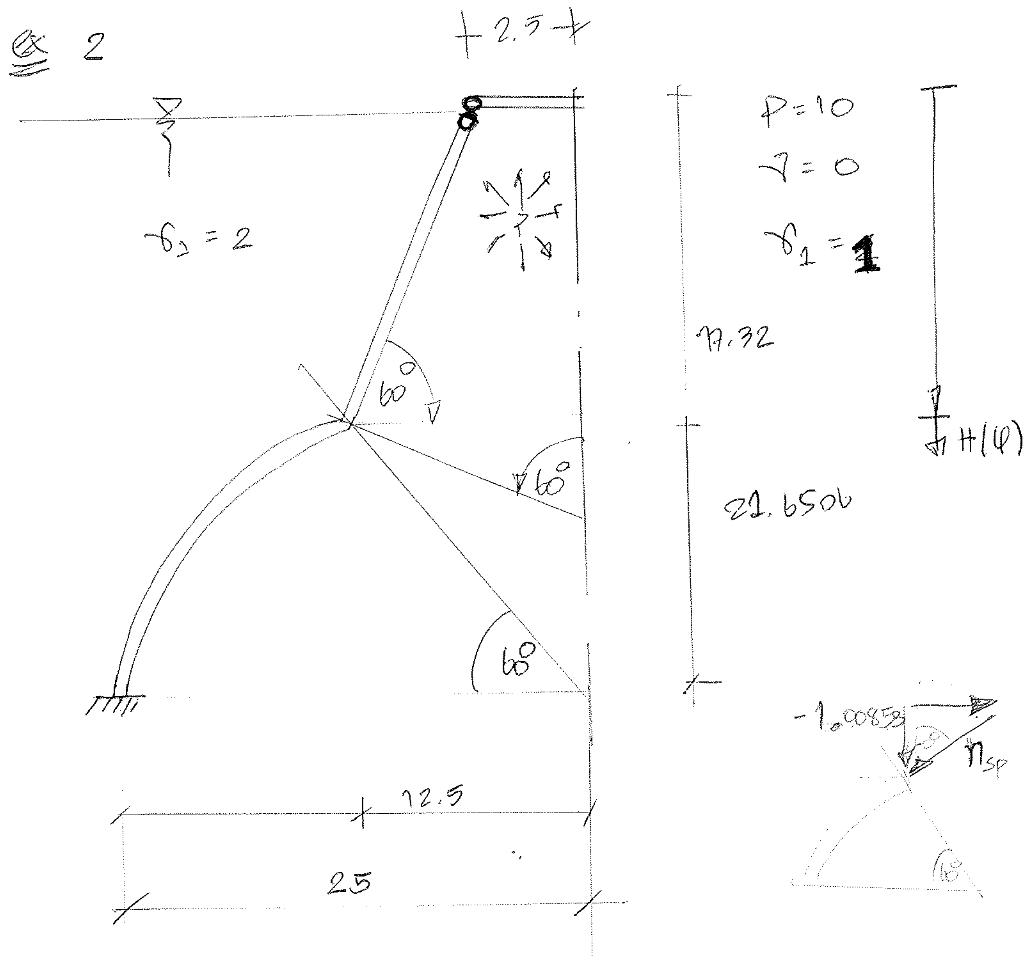
$$Et\,\Delta r_{sphere}^{OM} = -2262.4$$

e.) Sketch the deformed shape. Especially show the direction of the ring extension (Δr) and the meridian rotation (β) at junction A as calculated in question d.).

Zeichnen Sie die Verschiebungsfigur. Zeigen Sie insbesondere die in Aufgabe d.) berechnete Richtung der Ringaufweitung (Δr) und die Meridianverdrehung (β) an der Verbindungsstelle A.



Q 2



$H(\varphi) = 17.32 + 25\sin 60° - 25\cos\varphi$
$= 38.9706 - 25\cos\varphi$

$n_{sp}\cos 60° : 1.00853$

$n_{sp}\sin 60° = 1.00853 \tan 60°$
$= 1.747$

(+) →

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sphere $R = 25 = a$

$$\lambda = \sqrt{\frac{25}{0.1}\sqrt{3(1-0)}} = 20.809$$

$$s_L = \frac{2\pi a}{\lambda} = \frac{2\pi(25)}{20.809} = 7.548 = 2.4028\pi$$

$2s_L = 4.8056\pi < 8.33\pi$ no far end disturbance.

unit state 1 $\quad \alpha = 30° \quad a = 25$

$$Et\,\Delta r^1_{sph} = -2a\lambda\sin^2\alpha = -2(25)(20.809)\sin^2 30 = -260.11$$

$$Et\,\beta^1_{sph} = 2\lambda^2\sin\alpha = 2(20.809)^2\sin 30 = 433$$

unit state 2

$$Et\,\Delta r^2_{sph} = 2\lambda^2\sin\alpha = 433$$

$$Et\,\beta^2_{sph} = -\frac{4\lambda^3}{a} = -\frac{4(20.809)^3}{25} = -1441.7$$

edge disturbance.

$$Et\,\Delta r^{OR} = (-260.11)\,1.747 = -454.37$$

$$Et\,\beta^{OR} = (433)\,1.747 = 756.397$$

membrane state

$p_\varphi = 0$

$p_z = p_{z\,gas} + p_{z\,hydro}$

$= 10 - H(\varphi)$

$= 10 - (38.9706 - 25\cos\varphi) = 10 - 38.971 + 25\cos\varphi$

$= -28.971 + 25\cos\varphi$

$P_v = 2\pi R^2 \int_{\varphi_0}^{\varphi} \sin\varphi(\cancel{p_\varphi \sin\varphi}^{=0} - p_z\cos\varphi)\,d\varphi + C.$

$= -2\pi R^2 \int_{\varphi_0}^{\varphi} \sin\varphi(-28.971 + 25\cos\varphi)\cos\varphi\,d\varphi + C$

$= -2\pi R^2 \int_{\varphi_0}^{\varphi} -28.971 \sin\varphi\cos\varphi\,d\varphi + 25\cos^2\varphi\sin\varphi\,d\varphi + C.$

$= -2\pi R^2 \left(\int_{\prime\prime} -28.971 \sin\varphi\, d\sin\varphi + \int_{\prime\prime} -25\cos^2\varphi\, d\cos\varphi\right) + C.$

$= -2\pi R^2 \left(\frac{-28.971}{2}\sin^2\varphi \Big[_{\varphi_0}^{\varphi} + \frac{-25}{3}\cos^3\varphi\Big[_{\varphi_0}^{\varphi}\right) + C.$

$= -1250\pi\left(-14.49(\sin^2\varphi - \sin^2\varphi_0) - \frac{25}{3}(\cos^3\varphi - \cos^3\varphi_0)\right) + C.$

at the upper edge $\varphi = \varphi_0$

$\therefore P_v = C = 25.213\pi$

$n_\varphi = \frac{-P_v}{2\pi R\sin^2\varphi} = \frac{-\left(-1250\pi\left(-14.49(\sin^2\varphi - \sin^2\varphi_0) - \frac{25}{3}(\cos^3\varphi - \cos^3\varphi_0)\right) + 25.213\pi\right)}{2\pi R\sin^2\varphi}$

n_φ at $\varphi=\varphi_0$ $= \frac{-25.213\pi}{2\pi\, 25\sin^2 30} = -2.017$

$n_\theta = Rp_z - n_\varphi = 25(-28.971 + 25\cos\varphi) - n_\varphi$

n_θ at $\varphi=\varphi_0$ $= 25(-7.32) + 2.017 = -180.996$

$Et\,\Delta r^{0M} = R\sin\varphi\,(n_\theta - \cancel{\nu}^{0} n_\varphi)$

$= 25\sin 30\,(-180.99) = -2{,}262.37$

$Et\,\beta^{0M} = \frac{\partial}{\partial\varphi}(n_\theta - 0) - (1 + \cancel{\nu}^{0})\cot\varphi\,(n_\varphi - n_\theta))$

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second term

$-\cot 30(-2.071 + 180.996) = \quad 310$

$\frac{\partial}{\partial\varphi}\left(25(-28.971 + 25\cos\varphi) - n_\varphi \right) = 25(-25\sin\varphi) - \frac{\partial n_\varphi}{\partial\varphi}$

$$\frac{\partial n_\varphi}{\partial\varphi} = \frac{\left(\begin{array}{l} 2\pi R\sin^2\varphi\left(-\left(-1250\pi\left(-14.49\,(2\sin\varphi\cos\varphi) - \frac{25}{3}(3\cos^2\varphi(-\sin\varphi))\right)\right)\right) \\ -\ (\text{upper}\ldots)\ 2\pi R(2\sin\varphi\cos\varphi) \end{array} \right)}{(2\pi R\sin^2\varphi)^2}$$

$$\left.\frac{\partial n_\varphi}{\partial\varphi}\right|_{\varphi=\varphi_0=30^\circ} = \frac{\left(\begin{array}{l} -2\pi\, 25\sin^2 30\left(-1250\pi\,(-12.55 + 9.375)\right) \\ +\ 25.213\pi\,(43.3\pi) \end{array} \right)}{\pi^2(156.25)}$$

$$= \frac{\pi^2\left((-12.5)(3{,}968.75) + 1091.7\right)}{\pi^2(156.25)}$$

$$= \frac{-48{,}517.675}{156.25} = -310$$

$\therefore\ Et\beta^{om} = 25(-25\sin 30) + \quad -310 + 310$

$= -312.5$

$$\boxed{\begin{array}{l} \therefore\ Et\Delta r^0 = -454.37 - 2{,}262.37 = -2{,}716.74 \\ Et\beta^0 = 756.397 - 312.5 = 443.9 \end{array}}$$

$\delta_1^{\,0} = Et\Delta\bar{r}^0_{cone} - Et\Delta r^0_{sph} = -1121.45 + 2716.74 = 1595.363$

$\delta_2^{\,0} = Et\bar{\beta}^0_{cone} - Et\beta^0_{sph} = -49.626 - 443.9 = -493.53$

$\delta_1^{\,1} = Et\Delta\bar{r}^1_{cone} - Et\Delta r^1_{sph} = 342.324 + 260.11 = 602.43$

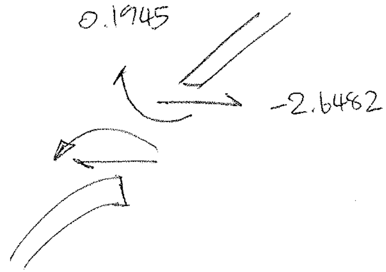
$\delta_2^{\,1} = Et\bar{\beta}^1_{cone} - Et\beta^1_{sph} = 432.99 - 433 = 0$

$\delta_1^{\,2} = Et\Delta\bar{r}^2_{cone} - Et\Delta r^2_{sph} = 432.99 - 433 = 0$

$\delta_2^{\,2} = Et\bar{\beta}^2_{cone} - Et\beta^2_{sph} = 1095.346 + 1441.7 = 2{,}537$

$$\begin{bmatrix} 602.43 & 0 \\ 0 & 3537 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -1595.363 \\ 493.53 \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -2.6482 \\ 0.1945 \end{bmatrix}$$



Cone

$$Et\,\Delta\bar{r}_{cone} = Et\,\Delta\bar{r}^0_{cone} + X_1\,Et\,\Delta\bar{r}^1_{cone} + X_2\,Et\,\Delta\bar{r}^2_{cone}$$

$$= -1121.45 + (-2.6482)(342.324) + 0.1945(432.99)$$

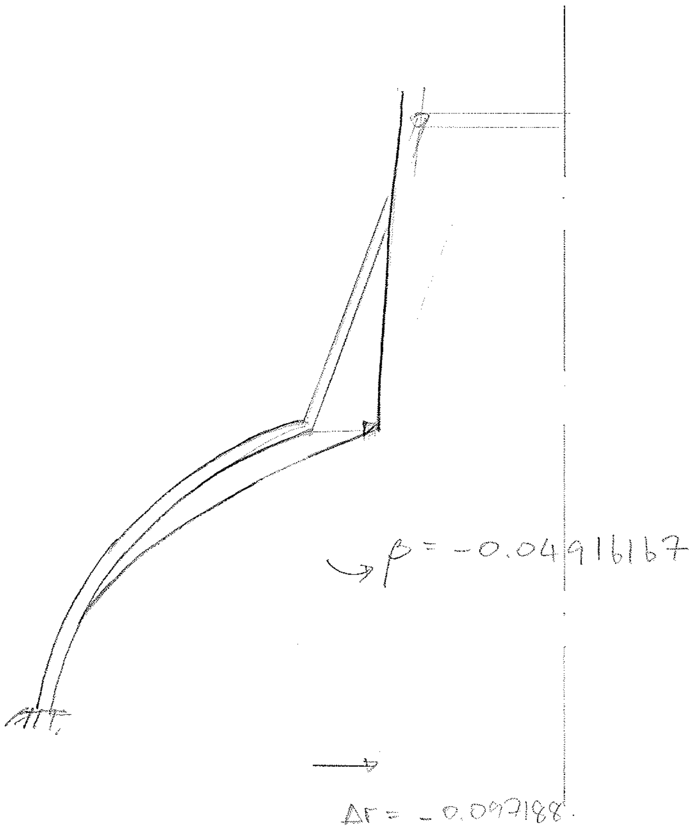
$$= -1943.743 \qquad \Delta\bar{r}_{cone} = -0.0972\ m.$$

$$Et\,\bar{\beta}_{cone} = Et\,\bar{\beta}^0_{cone} + X_1\,Et\,\bar{\beta}^1_{cone} + X_2\,Et\,\bar{\beta}^2_{cone}$$

$$= -49.626 \quad (-2.6482)(432.99) + 0.1945(1095.346)$$

$$= -983.233$$

$$\bar{\beta}_{cone} = -0.0491167.$$



→) compare the result in ex1 and ex2 and explain the cause of different.