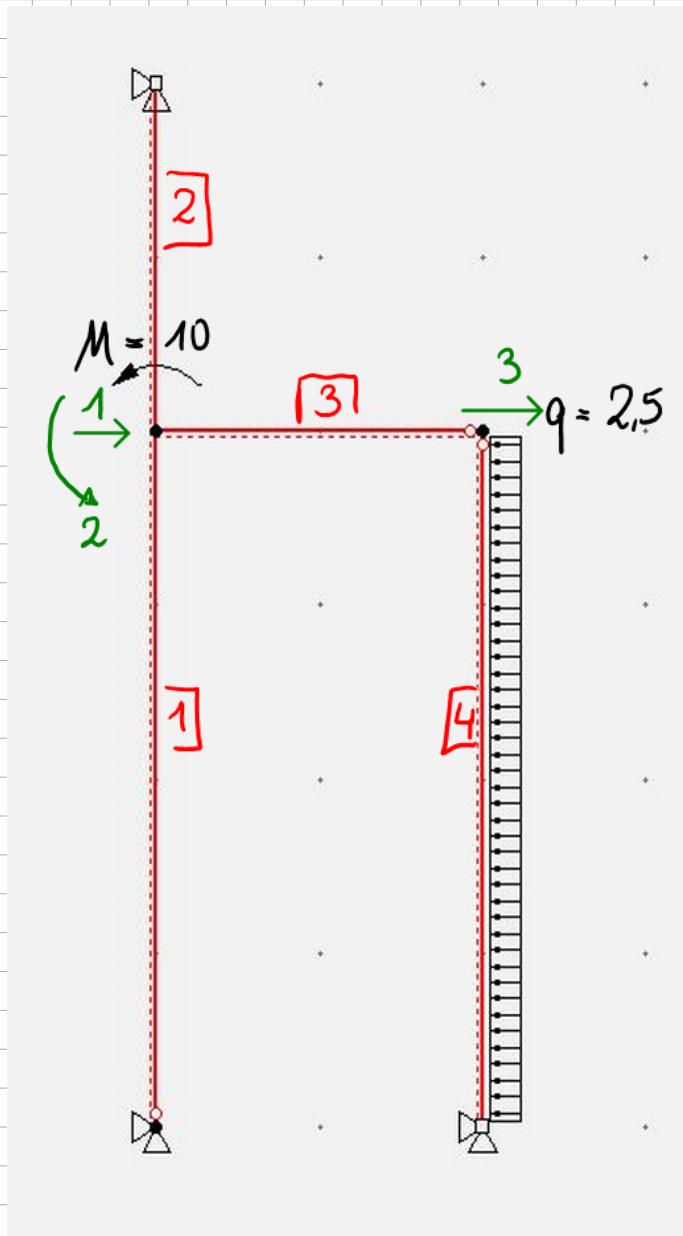


Statik Musterlösungen

Probeklausur 1 - Aufgabe 1



$$q = 2,5 \text{ kN/m}$$

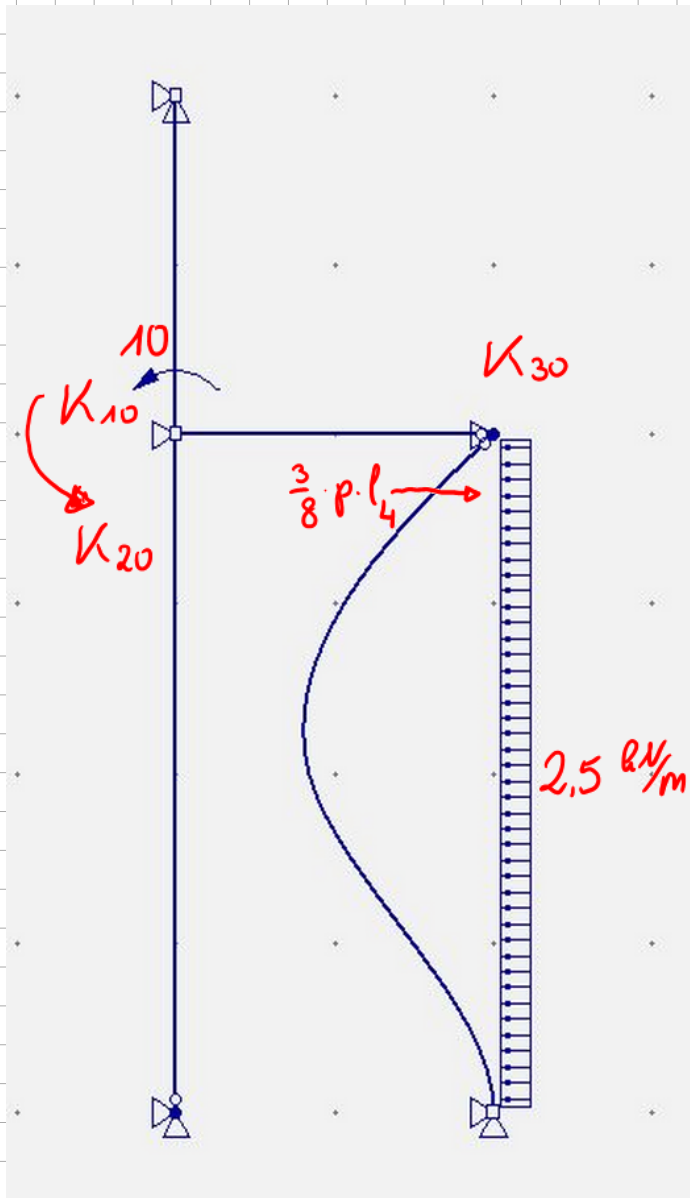
$$M = 10 \text{ kNm}$$

$$EI = 5000 \text{ kNm}^2$$

$$EA_3 = 5000 \text{ kN}$$

$$EA_{1,2,4} \rightarrow \infty$$

L2

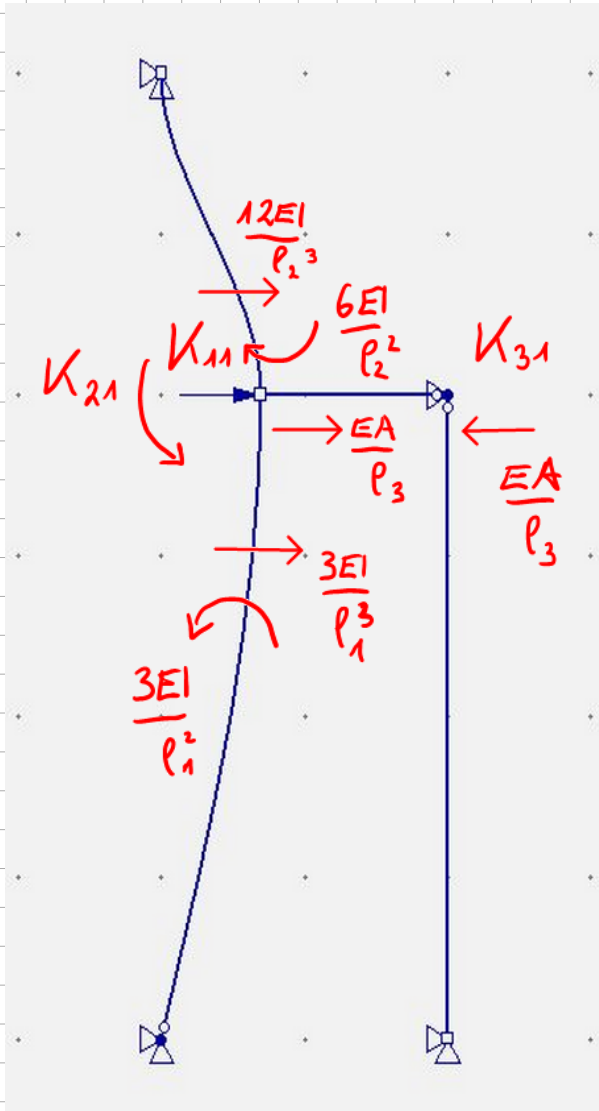


$$K_{10} = 0$$

$$K_{20} = -10$$

$$K_{30} = \frac{3}{8} \cdot p \cdot l = 3,75$$

E2 1



$$K_{11} = \frac{12EI}{l_2^3} + \frac{3EI}{l_1^3} + \frac{EA}{l_3} =$$

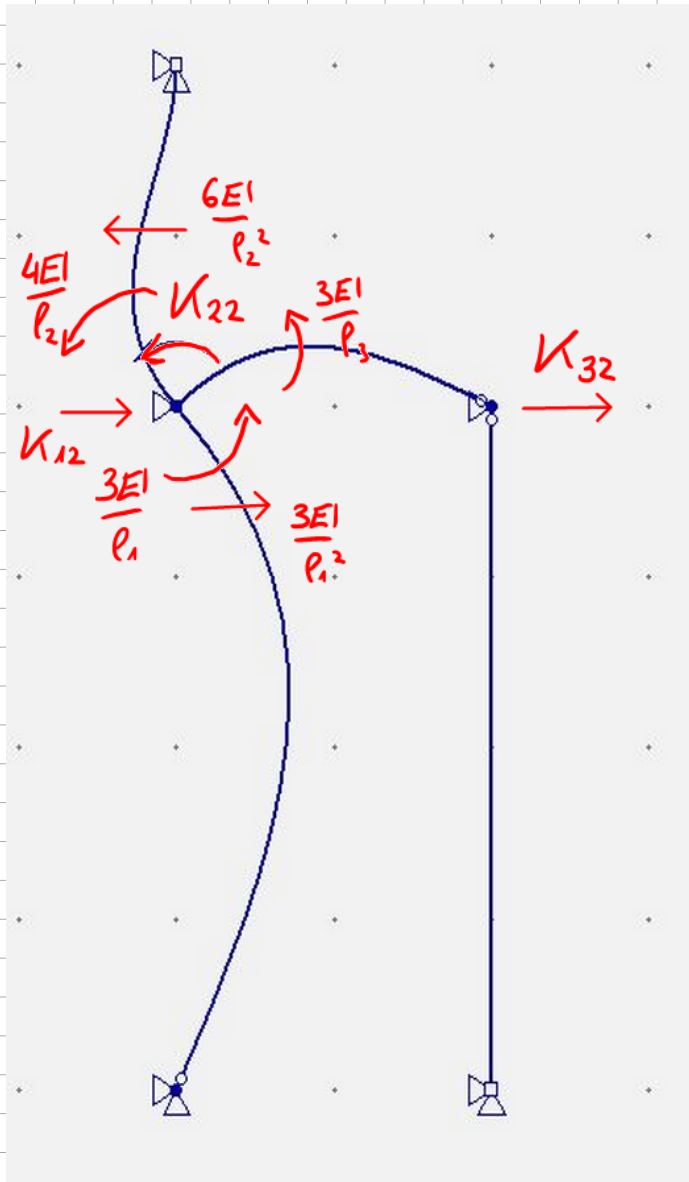
$$= 7500 + 234,375 + 2500 = 10\,234,375$$

$$K_{21} = -\frac{6EI}{l_2^2} + \frac{3EI}{l_1^2} = -7500 + 937,5 =$$

$$= -6562,5$$

$$K_{31} = -\frac{EA}{l_3} = -2500$$

E2 2



$$K_{12} = -\frac{6EI}{l_2^2} + \frac{3EI}{l_1^2} = -7500 + 937,5$$

$$= -6562,5 = K_{21}$$

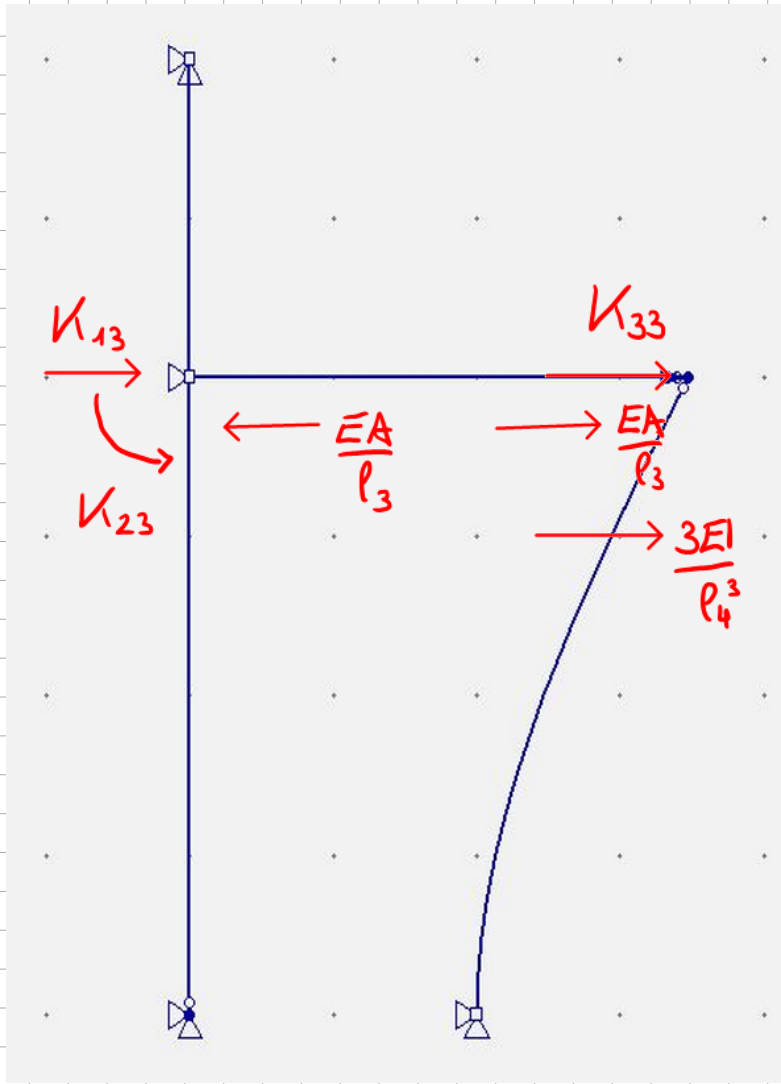
$$K_{22} = \frac{4EI}{l_2} + \frac{3EI}{l_1} + \frac{3EI}{l_3} =$$

$$= 10\,000 + 3750 + 7500 =$$

$$= 21\,250$$

$$K_{32} = 0$$

EZ 3



$$K_{13} = -\frac{EA}{l_3} = -2500 = K_{31}$$

$$K_{23} = 0 = K_{32}$$

$$K_{33} = \frac{EA}{l_3} + \frac{3EI}{l_4^3} = 2734,375$$

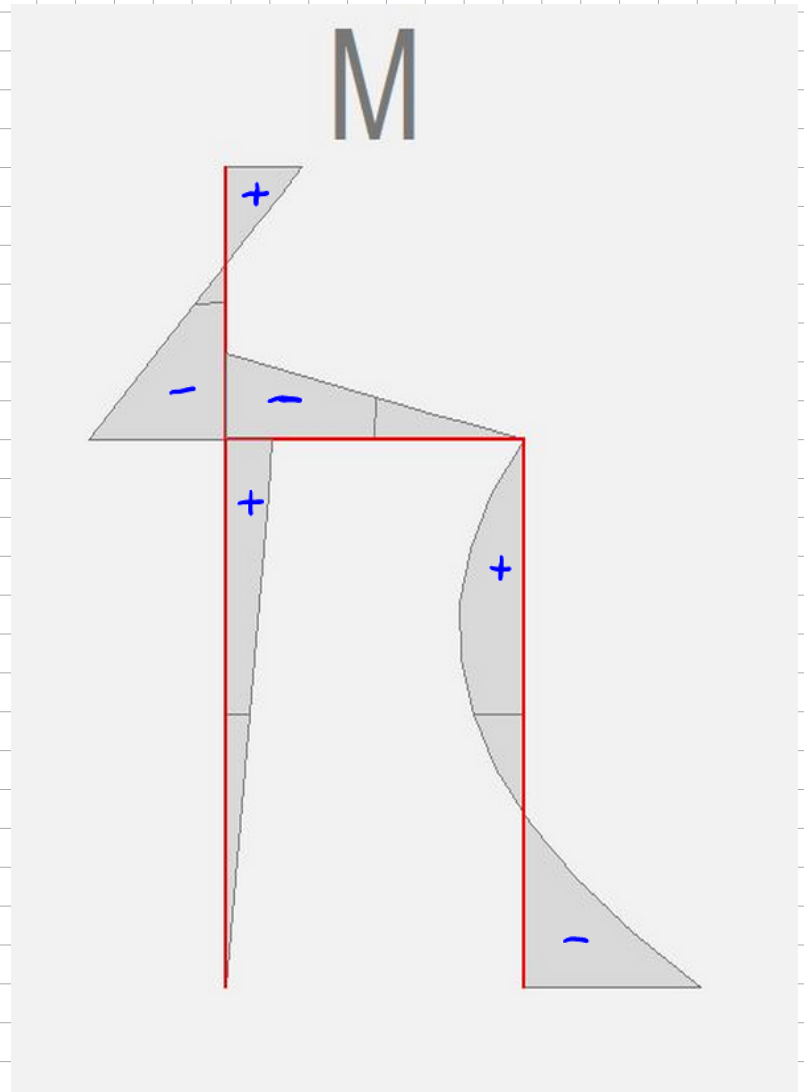
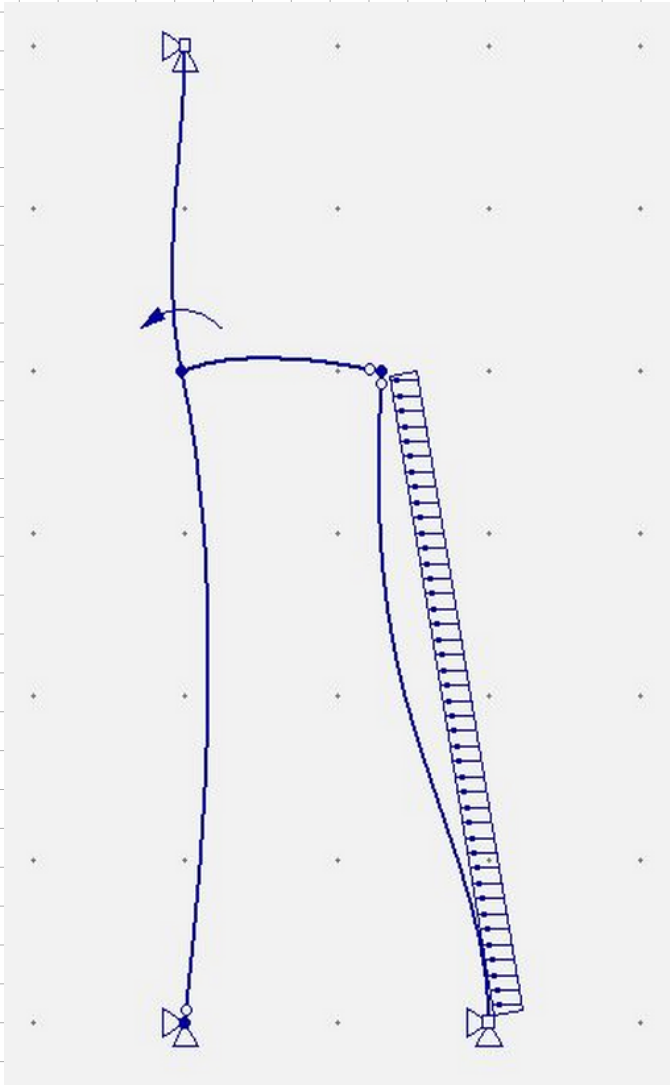
$$\underline{\underline{K}} = \begin{bmatrix} 10234,375 & -6562,5 & -2500 \\ -6562,5 & 21250 & 0 \\ -2500 & 0 & 2734,375 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 0 \\ 10 \\ -3,75 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F}$$

$$\Rightarrow \underline{u} = \begin{bmatrix} -5,74 \cdot 10^{-5} & 4,53 \cdot 10^{-4} & -1,42 \cdot 10^{-3} \end{bmatrix}^T$$

b)



$$c) \quad EA_3 \rightarrow \infty \rightarrow D_1 = D_3$$

$$\underline{L2}: K_{10} = 3,75 ; \quad K_{20} = -10$$

$$\underline{E2\ 1}: K_{11} = 7500 + 234,375 + 234,375 = 7968,75$$

K_{21} unverändert

$$\underline{E2\ 2}: \text{unverändert}$$

$$\underline{\underline{K}} = \begin{bmatrix} 7968,75 & -6562,5 \\ -6562,5 & 21250 \end{bmatrix}$$

$$\underline{F} = -\underline{K}_0 = \begin{bmatrix} -3,75 \\ 10 \end{bmatrix}$$

$$\Rightarrow D_1 = -1,11 \cdot 10^{-4} - D_3 ; D_2 = 4,36 \cdot 10^{-4}$$

d)

Normalkraft Fall 1 ($EA = 5000 \text{ kN}$):

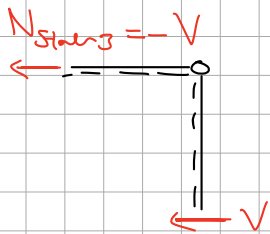
↳ Normalkraft aus Einsitzzuständen entnehmen (siehe a)!)

$$N_{\text{Stab 3}} = \frac{EA}{l_3} \cdot D_3 - \frac{EA}{l_3} \cdot D_1 = \underline{\underline{-3,41 \text{ kN}}}$$

Normalkraft Fall 2 ($EA \rightarrow \infty$):

↳ Normalkraft aus Gleichgewicht ermitteln!

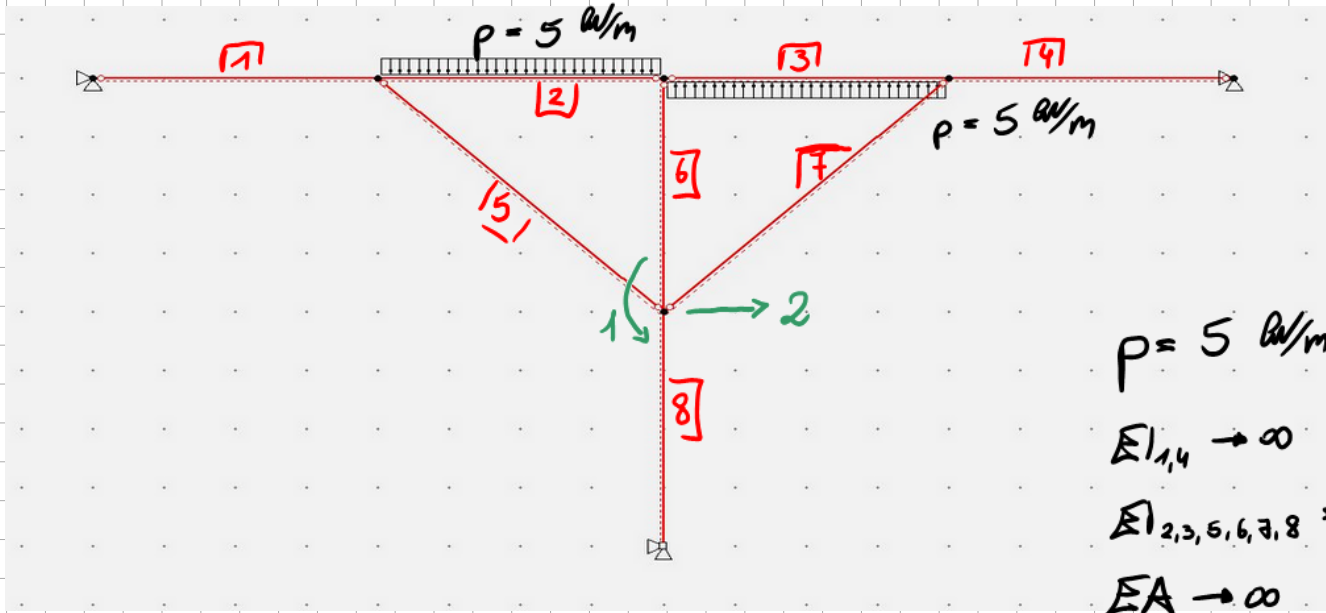
$$N_{\text{Stab 3}} = -V = -\left(\frac{3EI}{l_4^3} \cdot D_3 + \frac{3}{8} \cdot q \cdot l_4\right) = \underline{\underline{-3,74 \text{ kN}}}$$



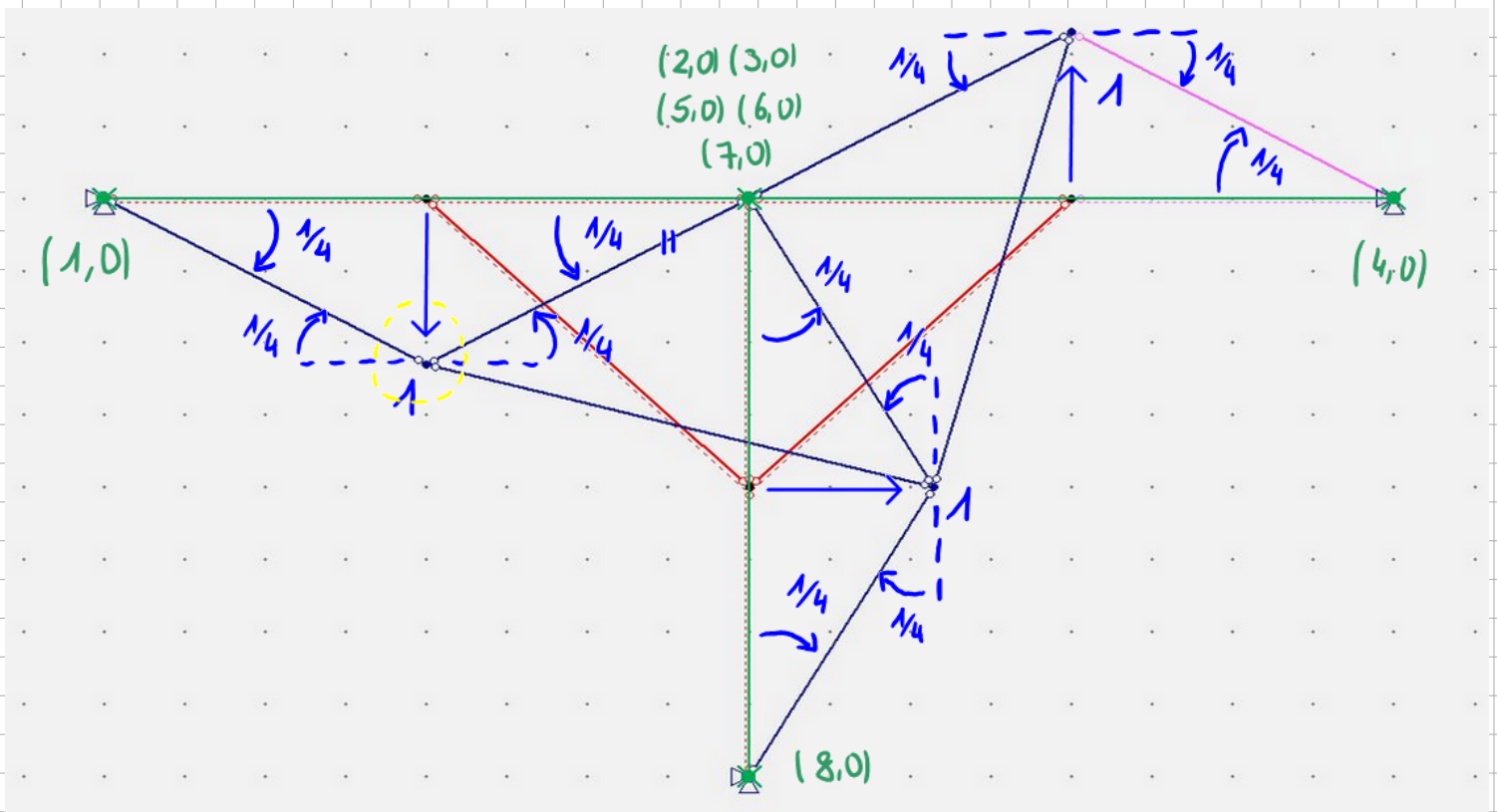
Aufgrund höherer Steifigkeiten im 2. Fall, ist wie zu erwarten eine höhere Normalkraft aufgetreten!

Je höher Steifigkeiten, desto höher werden die Schnittkräfte bei statisch unbestimmten Systemen ausfallen!

Probeklausur 1 - Aufgabe 2



Gelenkfigur / Polplan

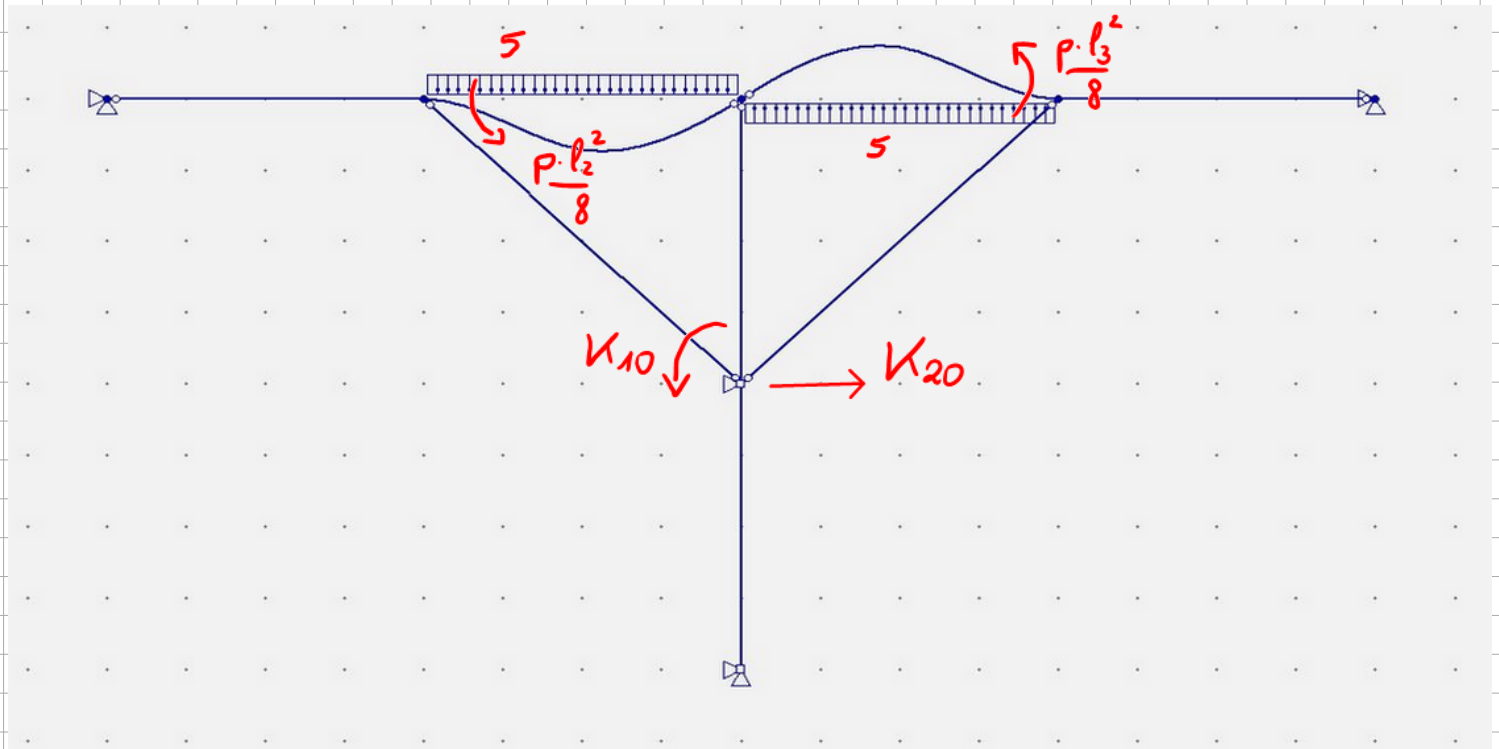


Detail:



$$\varphi = \frac{1}{2}$$

L2

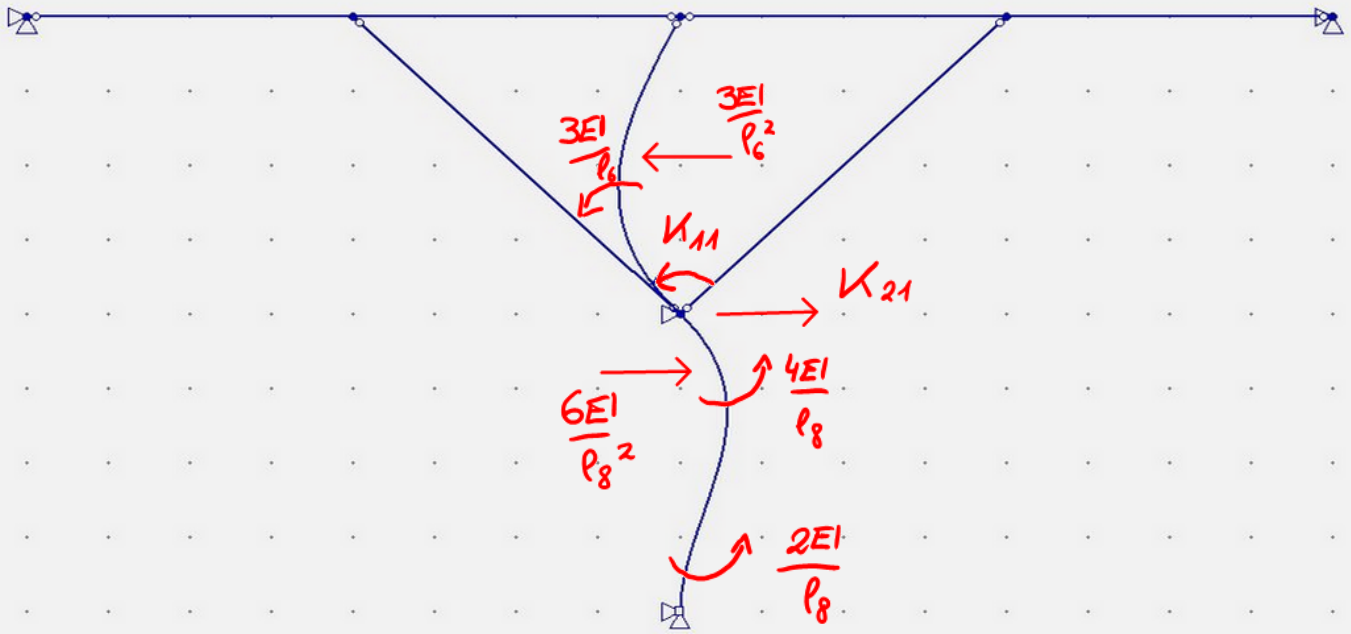


GG: $K_{10} = 0$

PvV: $K_{20} \cdot \bar{1} + \frac{\bar{1}}{2} \cdot p \cdot l_2 + \frac{\bar{1}}{2} \cdot p \cdot l_3 + \frac{\bar{1}}{2} \cdot \frac{p \cdot l_2^2}{8} + \frac{\bar{1}}{2} \cdot \frac{p \cdot l_3^2}{8} = 0$

$$K_{20} = -10 - 10 - 5 - 5 = -30$$

E2 1

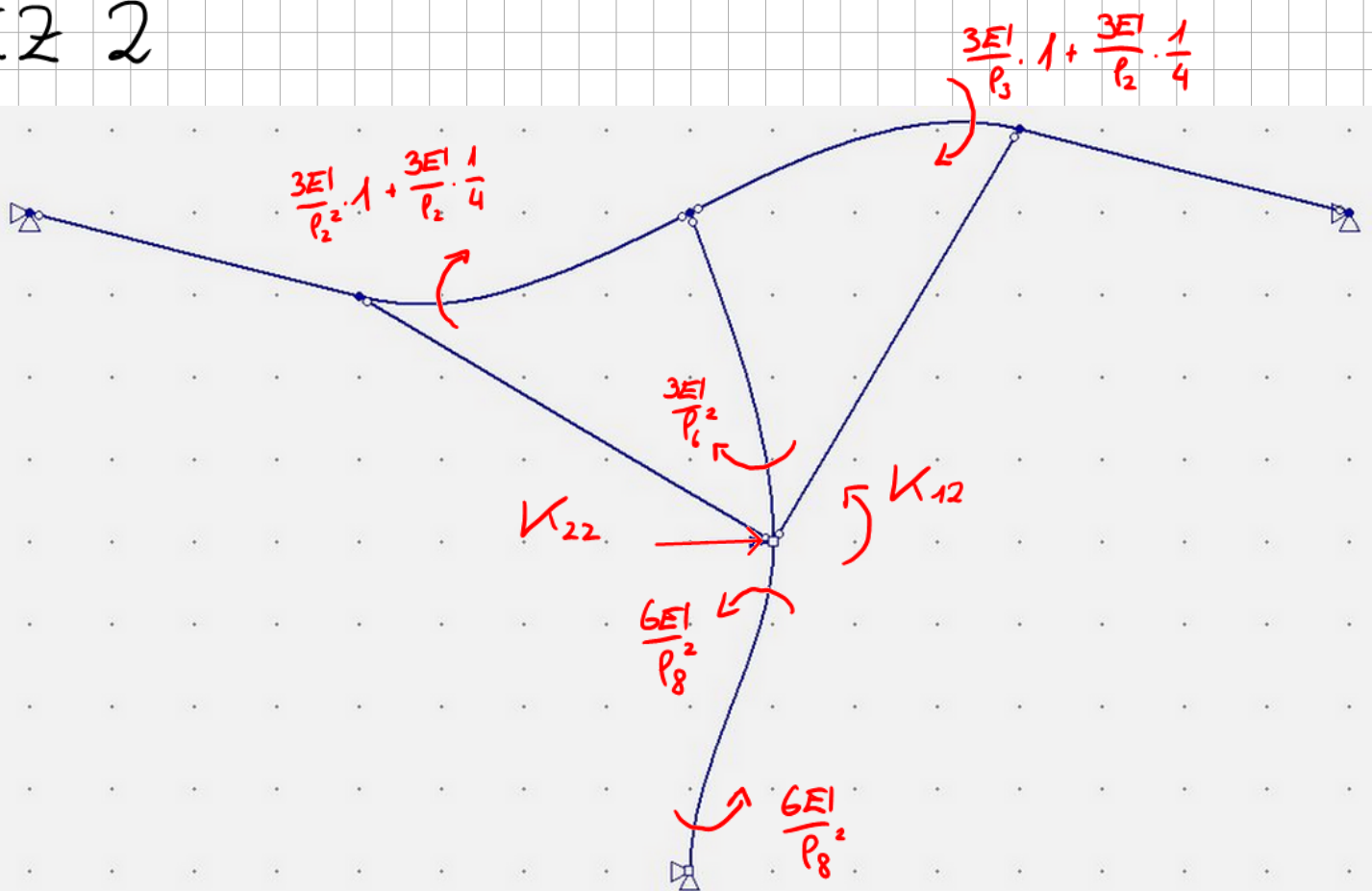


$$GG: K_{11} = \frac{3EI}{l_6} + \frac{4EI}{l_8} = 750 + 1000 = 1750$$

$$P_{rV}: K_{21} \cdot \bar{1} + \frac{3EI}{l_6} \cdot \frac{\bar{1}}{4} - \frac{4EI}{l_8} \cdot \frac{\bar{1}}{4} - \frac{2EI}{l_8} \cdot \frac{\bar{1}}{4}$$

$$K_{21} = -187,5 + 250 + 125 = 187,5$$

E2 2



$$GG: K_{12} = \frac{GEI}{l_8^2} - \frac{3EI}{l_6^2} = 375 - 187,5 = 187,5 = K_{21}$$

$$P_v V: K_{22} \cdot 1 - \frac{GEI}{l_8^2} \cdot \frac{1}{4} \cdot 2 - \left(\frac{3EI}{l_2^2} \cdot 1 + \frac{3EI}{l_2} \cdot \frac{1}{4} \right) \cdot \frac{1}{2} \cdot 2 - \frac{3EI}{l_6^2} \cdot \frac{1}{4} = 0$$

$$K_{22} = 187,5 + 375 + 46,875 = 609,375$$

$$\underline{K} = \begin{bmatrix} 1750 & 187,5 \\ 187,5 & 609,375 \end{bmatrix}$$

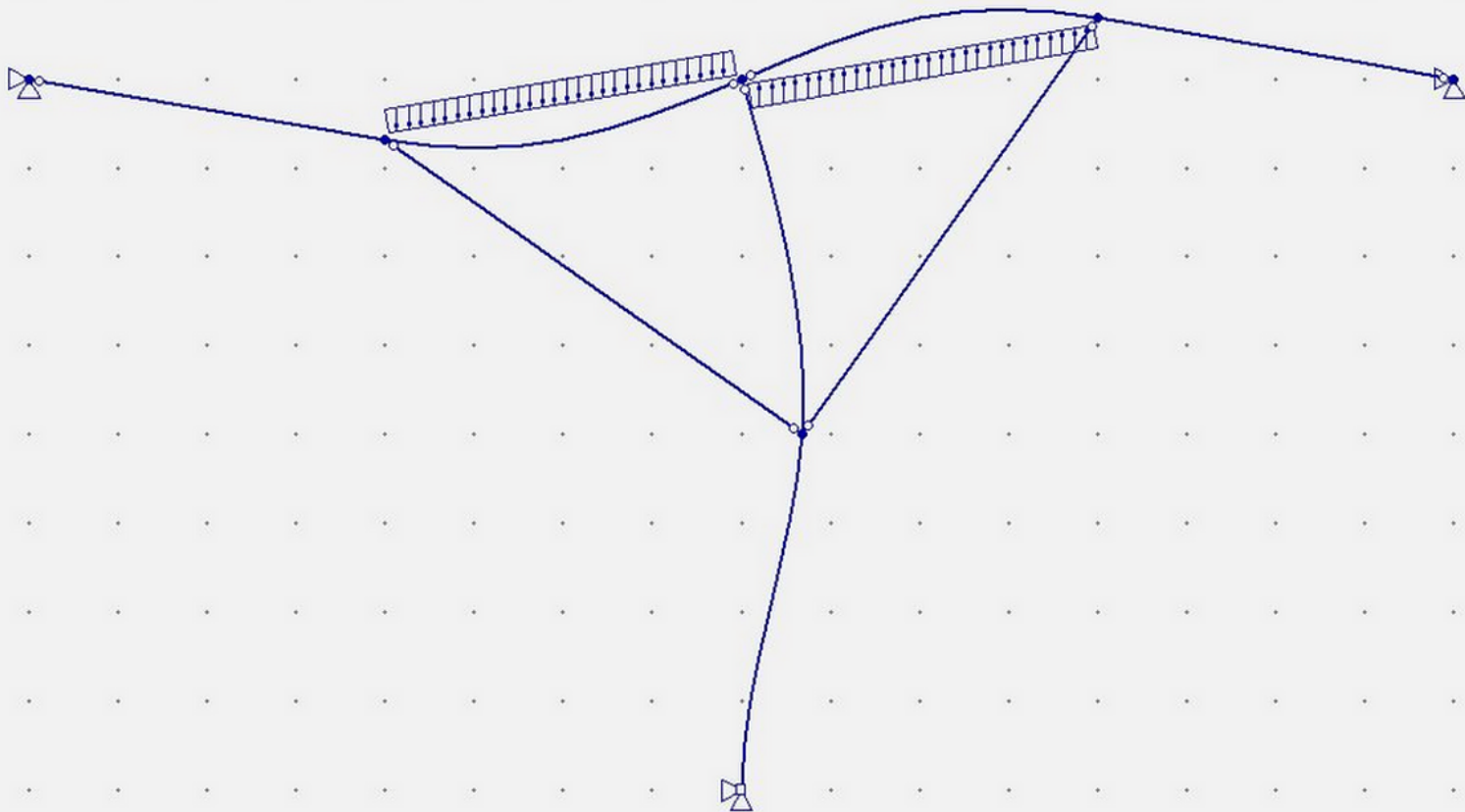
$$\underline{F} = -\underline{K}_0 = \begin{bmatrix} 0 \\ 30 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F}$$

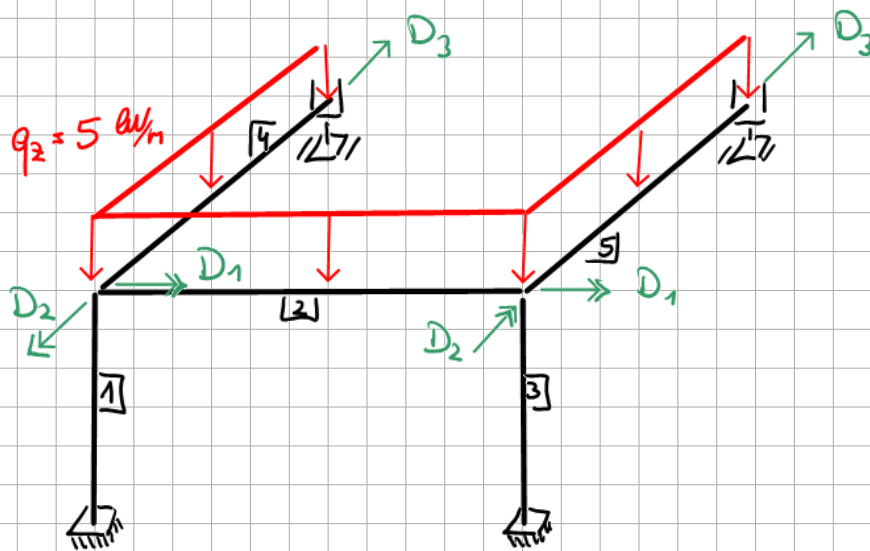
$$\underline{u} = \begin{bmatrix} -5,45 \cdot 10^{-3} \text{ rad} \\ 5,09 \cdot 10^{-2} \text{ m} \end{bmatrix}$$

$$M_k = \underbrace{0}_{L2} + \underbrace{(-5,45 \cdot 10^{-3}) \cdot \left(-\frac{3EI}{l_6}\right)}_{E21} + \underbrace{5,09 \cdot 10^{-2} \cdot \frac{3EI}{l_6^2}}_{E22} = 13,64 \text{ kNm}$$

b) Biegelinie



Probeklausur 1 - Aufgabe 3



$$EI = 1500 \text{ kNm}^2$$

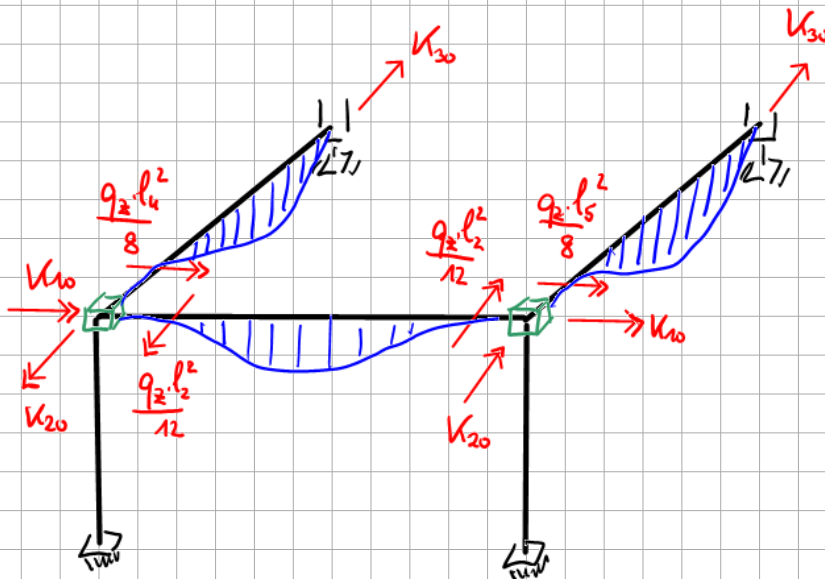
$$GI_{T,1,3} \rightarrow \infty$$

$$GI_{T, \text{rest}} = 20000$$

$$EA \rightarrow \infty$$

$$q_2 = 5 \text{ kN/m}$$

L2

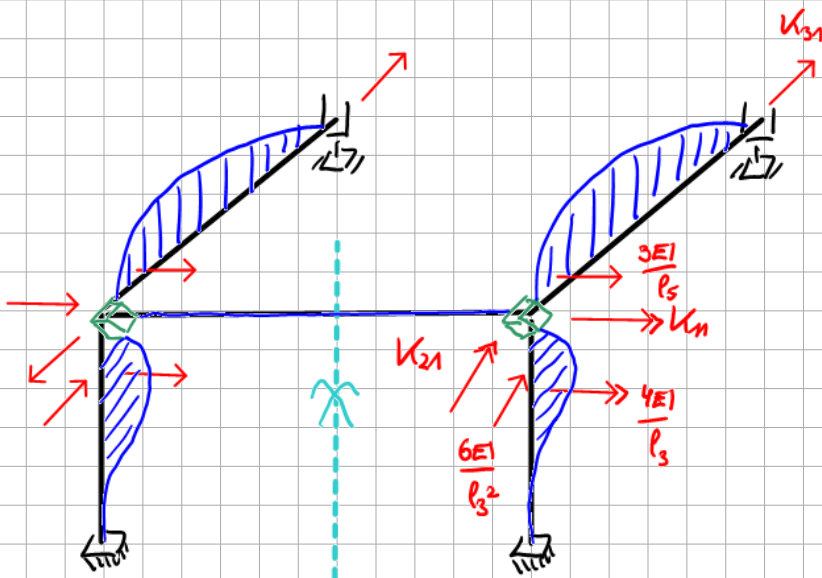


$$V_{10} = \frac{q_2 l_1^2}{8} = 15,625$$

$$V_{20} = \frac{q_2 l_2^2}{12} = 15$$

$$V_{30} = 0$$

Er 1

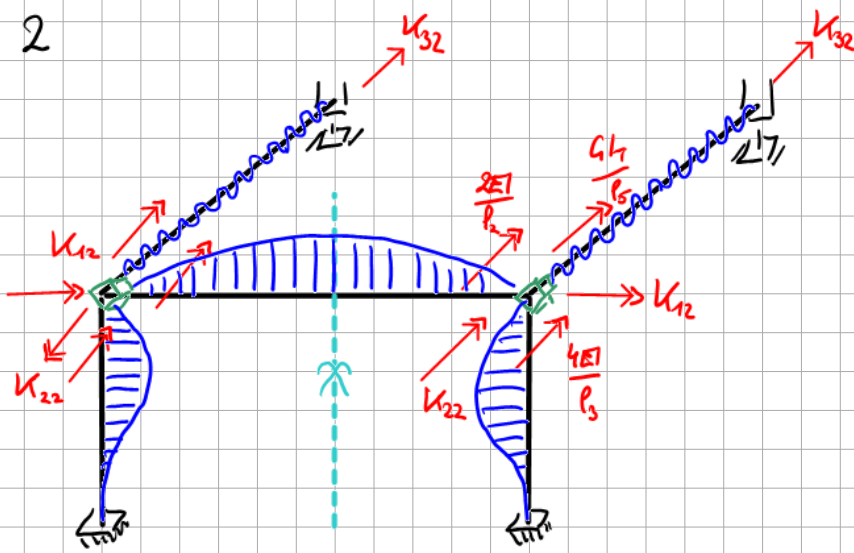


$$M_{11} = \frac{4EI}{l_1} + \frac{3EI}{l_1} = 1500 + 900 = 2400$$

$$M_{21} = 0$$

$$M_{31} = \frac{6EI}{l_1^2} = 562,5$$

E2 2

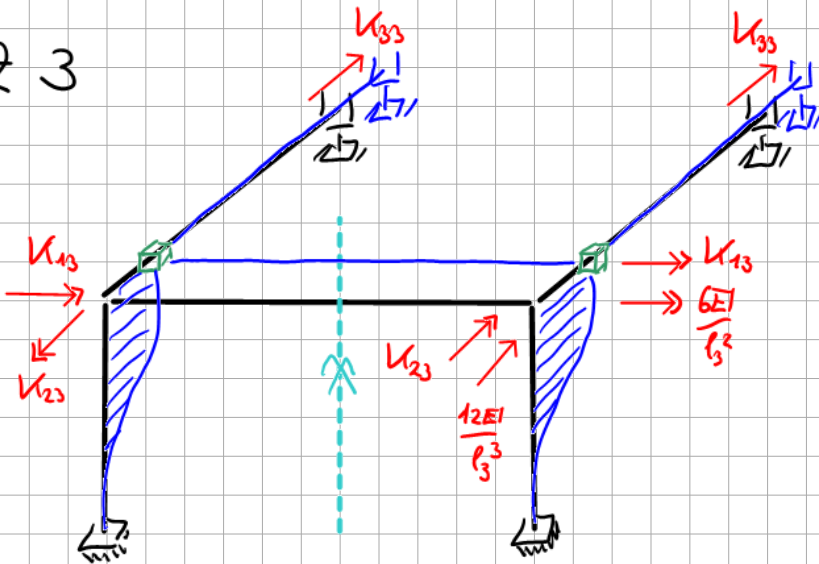


$$K_{12} = 0$$

$$K_{22} = \frac{4EI}{l_3} + \frac{2EI}{l_2} + \frac{GI}{l_3} = 1500 + 500 + 4000 = 6000$$

$$K_{32} = 0$$

E2 3



$$K_{13} = \frac{6EI}{l_3^2} = 562,5$$

$$K_{23} = 0$$

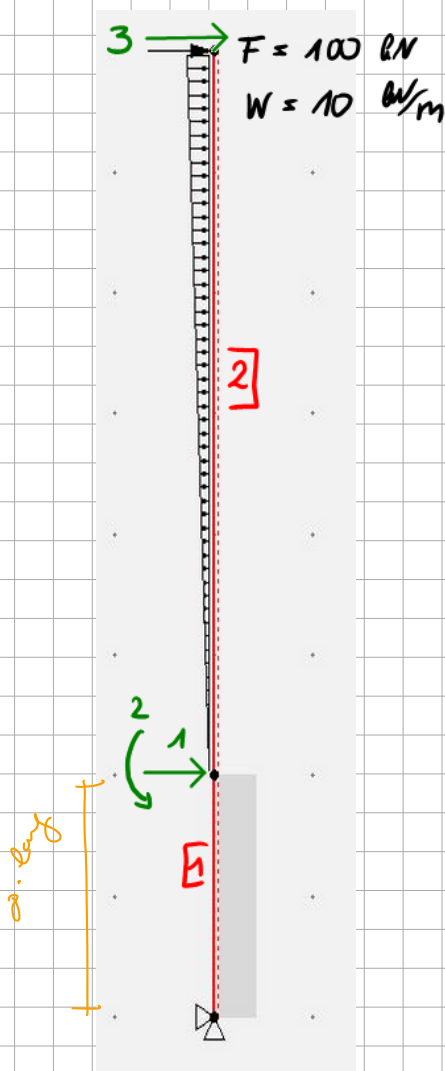
$$K_{33} = \frac{12EI}{l_3^3} = 281,25$$

$$\underline{K} = \begin{bmatrix} 2400 & 0 & 562,5 \\ 0 & 6000 & 0 \\ 562,5 & 0 & 281,25 \end{bmatrix}$$

$$\underline{F} = -\underline{K}_0 \begin{bmatrix} -15,625 \\ -15 \\ 0 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \Rightarrow \underline{u} = \begin{bmatrix} -0,0122 \text{ rad} \\ -2,5 \cdot 10^{-3} \text{ rad} \\ 0,0245 \text{ m} \end{bmatrix}$$

Probeklausur 1 - Aufgabe 4



$$EI_{Kragarm} = 10\,000 \text{ kNm}^2$$

$$EI_{Bettung} = 80\,000 \text{ kNm}^2$$

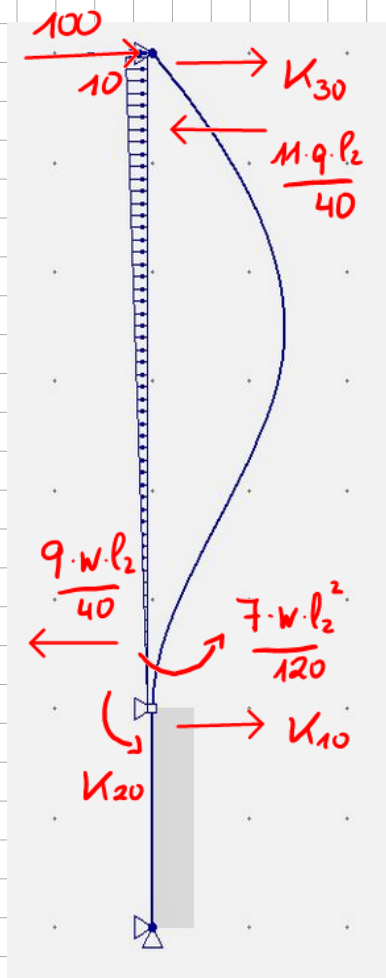
$$k = 8000 \text{ kNm}^2$$

$$\lambda = \sqrt[4]{\frac{R}{4EI_B}} = \sqrt[4]{\frac{8000}{4 \cdot 80\,000}} = 0,398$$

$$\lambda \cdot l_1 = 0,398 \cdot 20 = 7,95 > \pi$$

→ ∞ langer Balken

L2

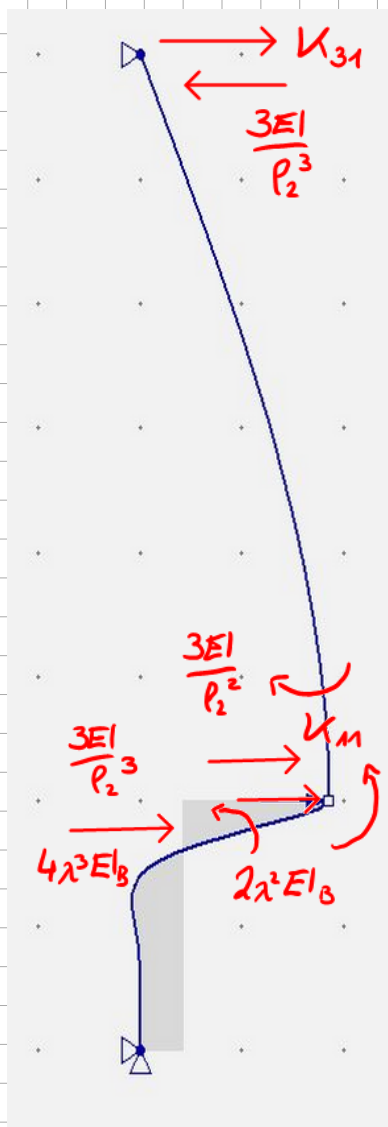


$$K_{10} = -\frac{9 \cdot q \cdot l_2}{40} = -135$$

$$M_{20} = \frac{7 \cdot q \cdot l_2^2}{120} = 2100$$

$$K_{30} = -\frac{11 \cdot q \cdot l_2}{40} - F = -165 - 100 = -265$$

E2 1

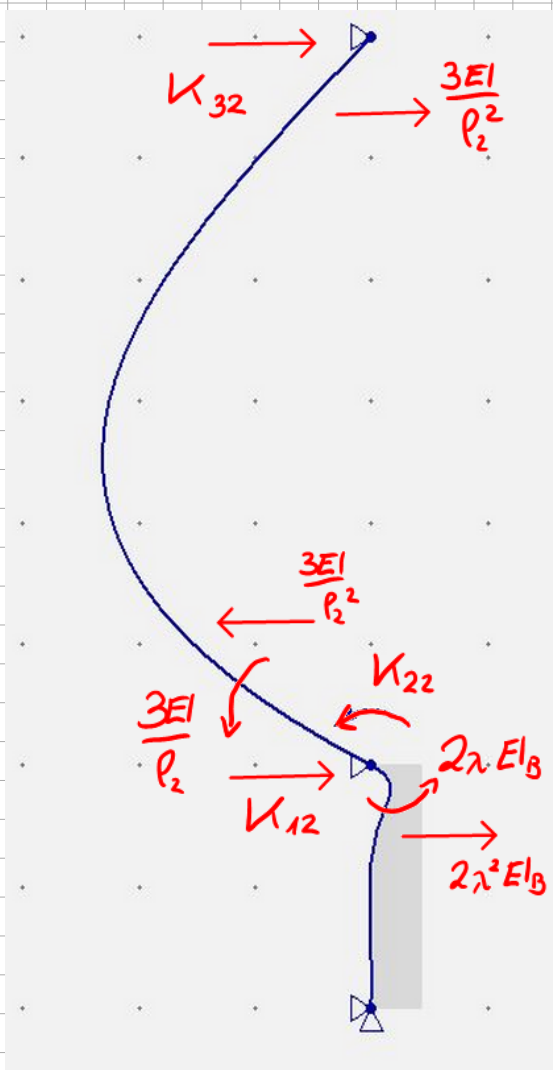


$$K_{11} = \frac{3EI}{l_2^3} + 4\lambda^3 EI_B = 0,138 + 20\,118,93 = 20\,119,074$$

$$K_{21} = -\frac{3EI}{l_2^2} + 2\lambda^2 EI_B = -8,3 + 25\,298,221 = 25\,289,888$$

$$K_{31} = -\frac{3EI}{l_2^3} = -0,138$$

E2 2



$$K_{12} = -\frac{3EI}{l_2^2} + 2\lambda^2 EI_B = -8,3 + 25\,298,221 = 25\,289,888$$

$$K_{22} = \frac{3EI}{l_2} + 2\lambda EI_B = 500 + 63\,621,658 = 64\,121,658$$

$$K_{32} = \frac{3EI}{60^2} = 8,3$$

E2 3



$$K_{13} = -\frac{3EI}{l_2^3} = -0,138$$

$$K_{23} = \frac{3EI}{l_2^2} = 8,3$$

$$K_{33} = \frac{3EI}{l_2^3} = 0,138$$

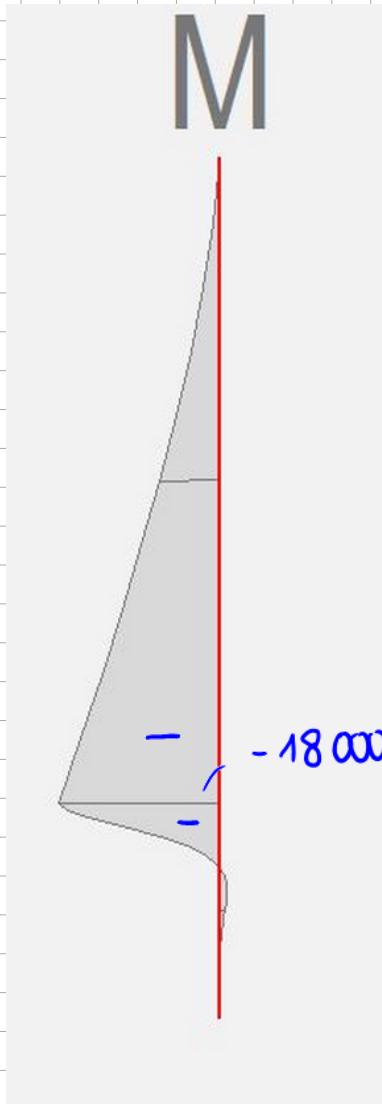
$$\underline{\underline{K}} = \begin{bmatrix} 20\,119,074 & 25\,289,888 & -0,138 \\ 25\,289,888 & 64\,121,658 & 8,3 \\ -0,138 & 8,3 & 0,138 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_0 \underline{\underline{u}} = \begin{bmatrix} 135 \\ -2100 \\ 265 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}}$$

$$\Rightarrow \underline{\underline{u}} = \begin{bmatrix} 0,751 \text{ m} \\ -0,582 \text{ rad} \\ 1943,65 \text{ m} \end{bmatrix}$$

$$\begin{aligned}
 M_2 &= \underbrace{-2100}_{L_2} + \underbrace{0,751 \cdot \frac{3EI}{60^2}}_{E21} + \underbrace{(-0,582) \cdot \left(-\frac{3EI}{60}\right)}_{E22} + \underbrace{1943,65 \cdot \left(-\frac{3EI}{60^2}\right)}_{E23} \\
 &= -18000 \text{ kNm}
 \end{aligned}$$



b) Grenzwert der Fußverschiebung = $0,06 \text{ m} \neq 0,751 \text{ m} = \text{D}_1$

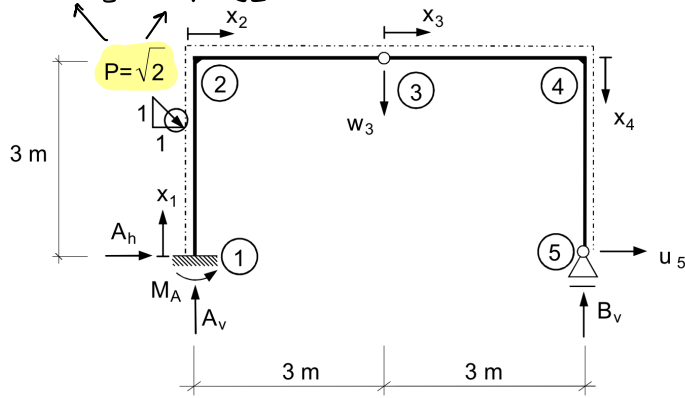
⇒ Verbesserung erforderlich

$$\lambda = \sqrt[4]{\frac{k}{4EI_B}}$$

$\rightarrow k$ durch Verbesserung des Baugrundes erhöhen
 $\rightarrow EI_B$ durch steifern Fundamentbalken erhöhen
 (wirkt sich in den Steifigkeiten positiv aus)

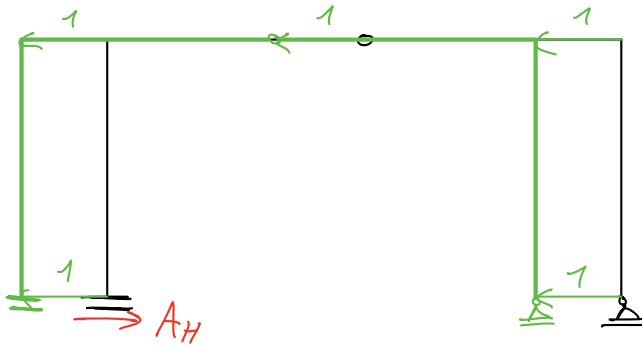
5. Aufgabe

a) $P_H = \frac{1}{\sqrt{2}} \cdot P$ $P_V = \frac{1}{\sqrt{2}} \cdot P$

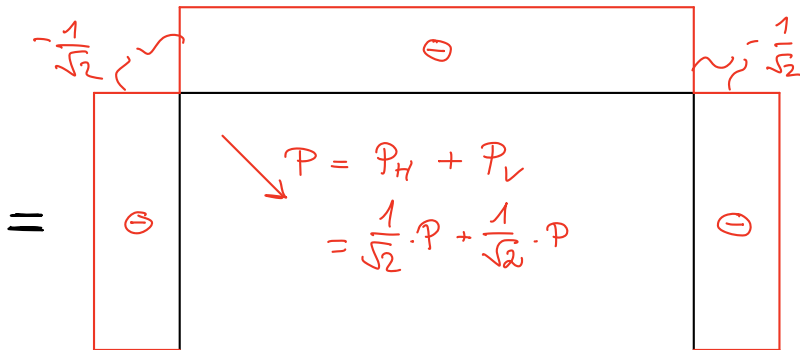
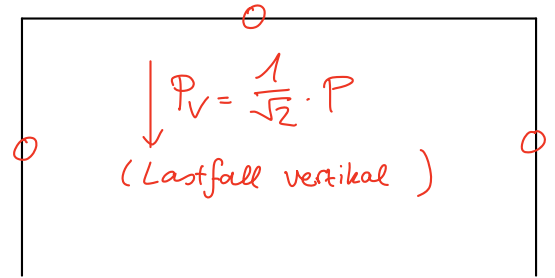
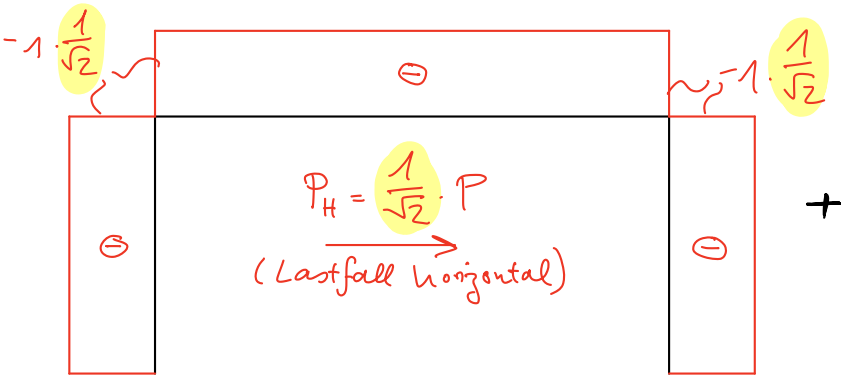


$EI = 800 \text{ kNm}^2$
 $EA \rightarrow \infty$

Kinematik:



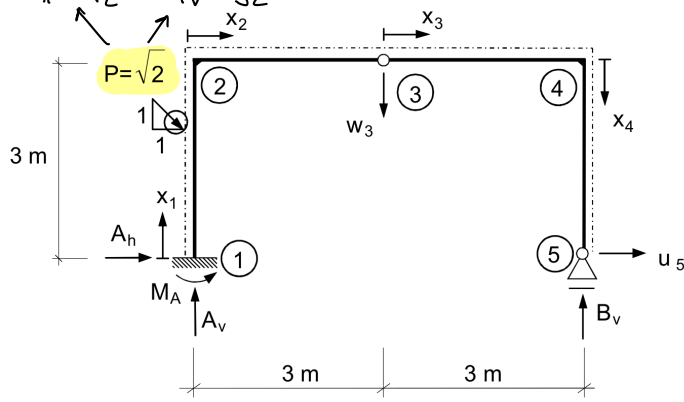
Einflusslinien:



$\eta(x_1) = -\frac{1}{\sqrt{2}}$
 $\eta(x_2) = -\frac{1}{\sqrt{2}}$

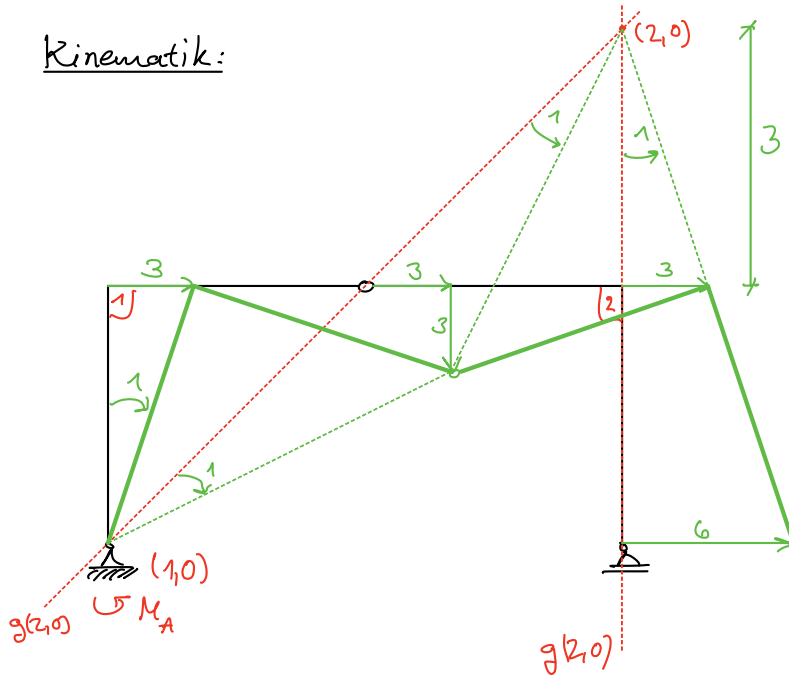
$\eta(x_3) = -\frac{1}{\sqrt{2}}$
 $\eta(x_4) = -\frac{1}{\sqrt{2}}$

b) $P_H = \frac{1}{\sqrt{2}} \cdot P$ $P_V = \frac{1}{\sqrt{2}} \cdot P$

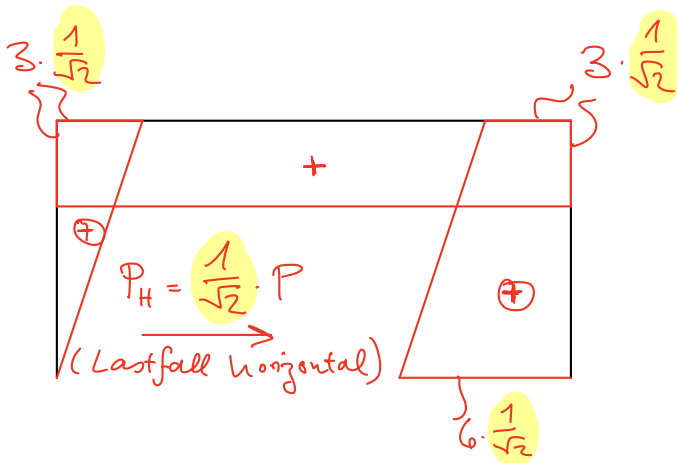


$EI = 800 \text{ kNm}^2$
 $EA \rightarrow \infty$

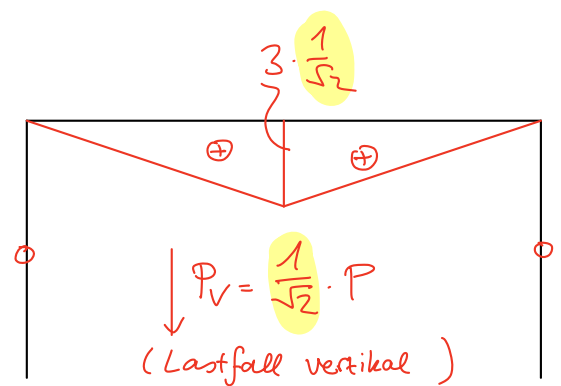
Kinematik:



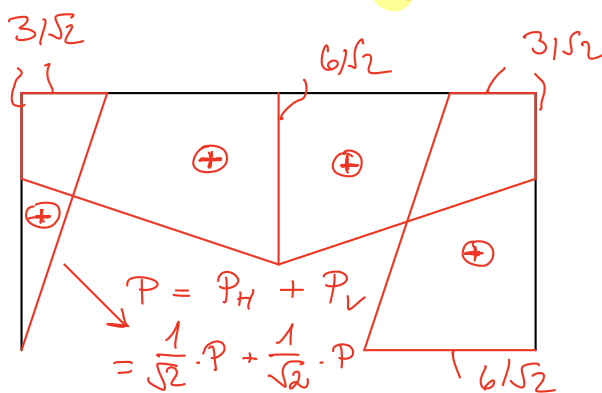
Einflusslinien:



+



=



$$\eta(x_1) = \frac{1}{\sqrt{2}} \cdot x_1$$

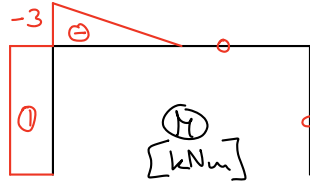
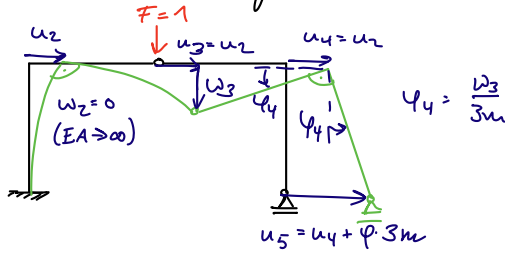
$$\eta(x_2) = \frac{1}{\sqrt{2}} \cdot x_2 + \frac{3}{\sqrt{2}}$$

$$\eta(x_3) = -\frac{1}{\sqrt{2}} \cdot x_3 + \frac{6}{\sqrt{2}}$$

$$\eta(x_4) = \frac{1}{\sqrt{2}} \cdot x_4 + \frac{3}{\sqrt{2}}$$

c) Alle Knotenverschiebungen mit PIV bestimmen

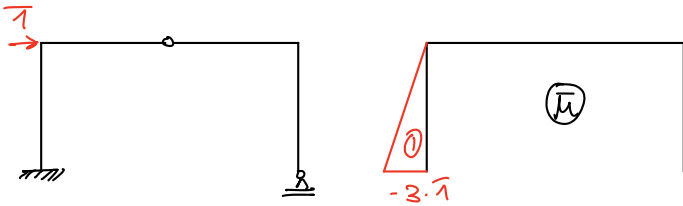
1. Realer Lastfall:



2. $w_3 \rightarrow$ virtuelle Lastfälle stimmen mit realen überein

$$w_3 = \bar{3} \cdot 3 \cdot \frac{3}{EI} + \frac{1}{3} \cdot \bar{3} \cdot 3 \cdot \frac{3}{EI} = \underline{\underline{0,045m}}$$

3. $u_2 = u_3 = u_4$ (da $EA \rightarrow \infty$? keine Dehnung)



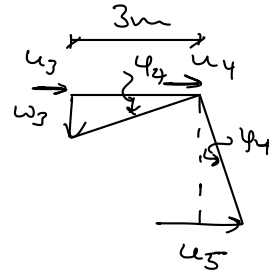
$$u_2 = \frac{1}{2} \cdot \bar{3} \cdot 3 \cdot \frac{3}{EI} = 0,017m$$

4. u_5 (Über kinematik bestimmbar)

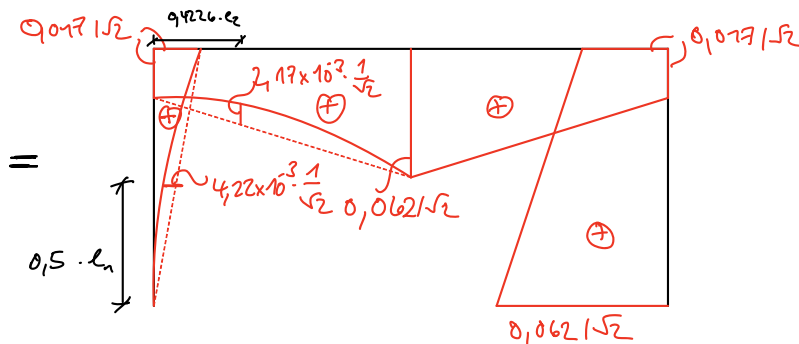
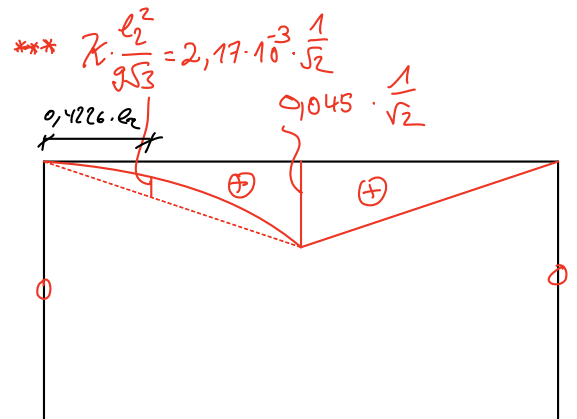
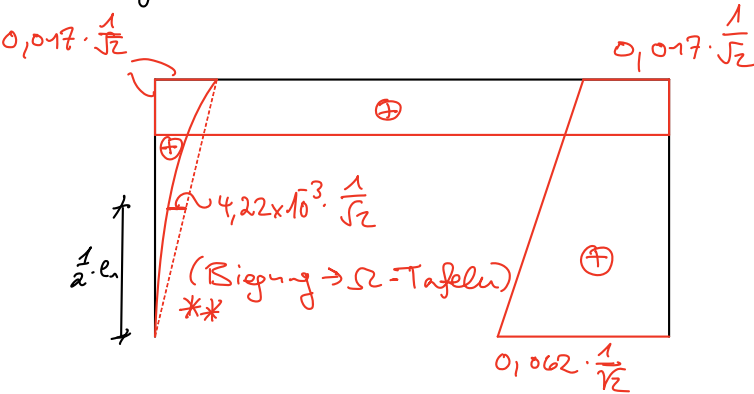
$$\Rightarrow \varphi_4 = \frac{w_3}{3m} = \frac{0,045}{3} = 0,015$$

$$u_5 = 3m \cdot \varphi_4 + u_4$$

$$= 3m \cdot 0,015 + 0,017m = \underline{\underline{0,062m}}$$



Einflusslinien:



$$w_{max} = \kappa \cdot \frac{l^2}{8}$$

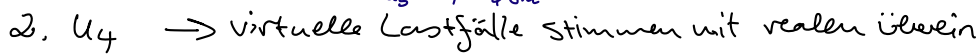
$$x_{max} = \frac{l}{2}$$

$$\kappa = -\frac{3}{EI}$$

$$w_{max} = \kappa \cdot \frac{l^2}{8}$$

$$x_{max} = 0,4226 \cdot l$$

1. Realer Castfall:



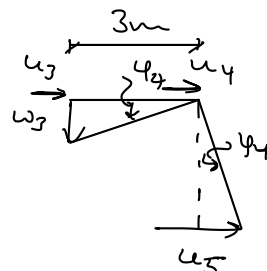
3. ω_3

$$\omega_3 = \frac{1}{2} \cdot \frac{3 \cdot 3 \cdot 3}{EI} = \underline{0,017 \text{ m}}$$

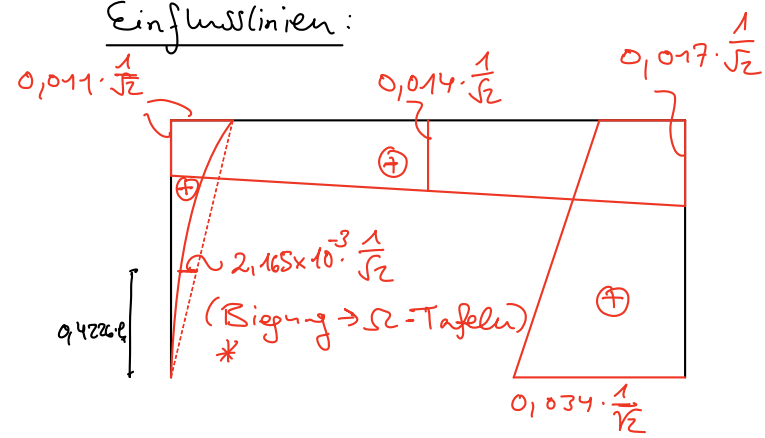
4. u_3

$$u_3 = \frac{1}{3} \cdot \frac{3 \cdot \bar{3} \cdot 3}{EI} + \frac{1 \cdot \bar{1} \cdot 3}{EA} = \underline{\underline{0,014m}}$$
$$u_2 = \frac{1}{3} \cdot \frac{3 \cdot \bar{3} \cdot 3}{EI} = \underline{\underline{0,011m}}$$
$$\Rightarrow \varphi_4 = \frac{\omega_3}{3m} = \frac{0,017}{3} = 0,0056$$

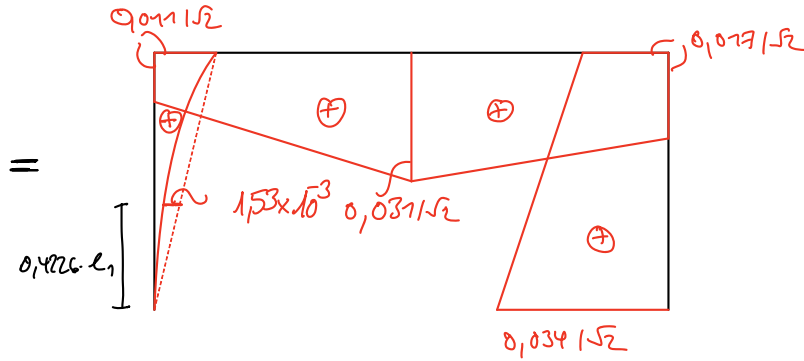
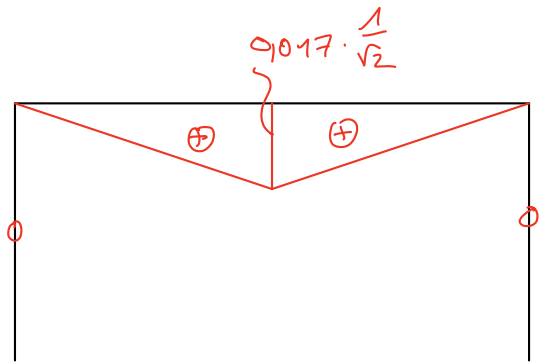
$$= 3m. 0,0056 + 0,017m = \underline{0,034m}$$




Einflusslinien:



+



* 

$$w_{\max} = \kappa \cdot \frac{3^2}{9\sqrt{3}} = -2,165 \times 10^{-3}$$

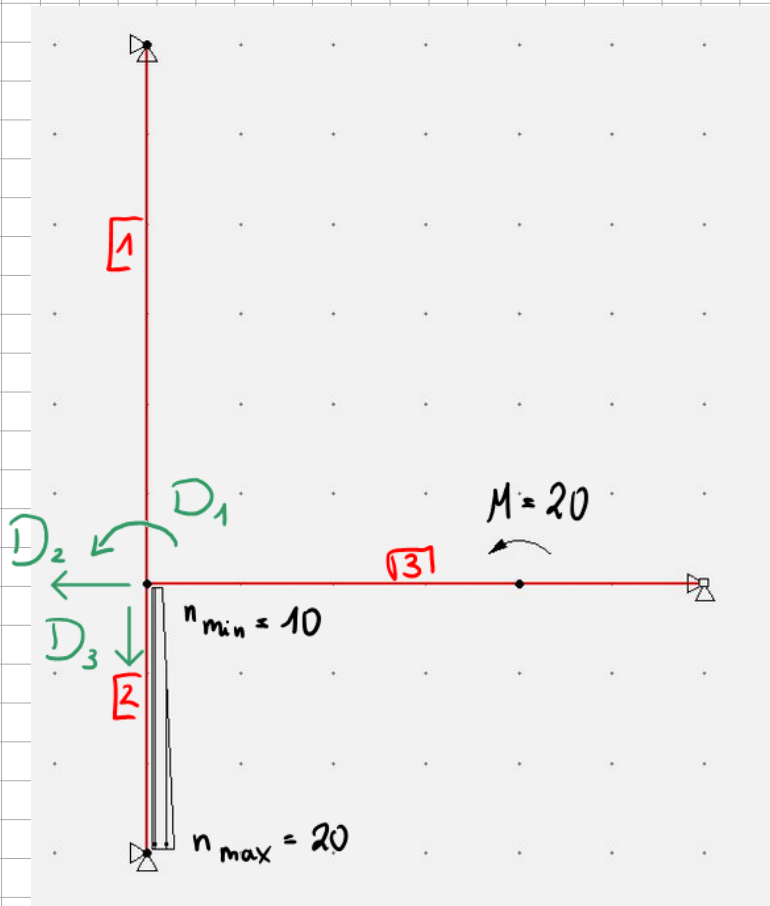
$$x_{\max} = 0,4226 \cdot l_1$$

$\kappa = -3/EI$

(M-Verlauf aus realem LF)

Statik Musterlösungen

Probeklausur 2 - Aufgabe 1



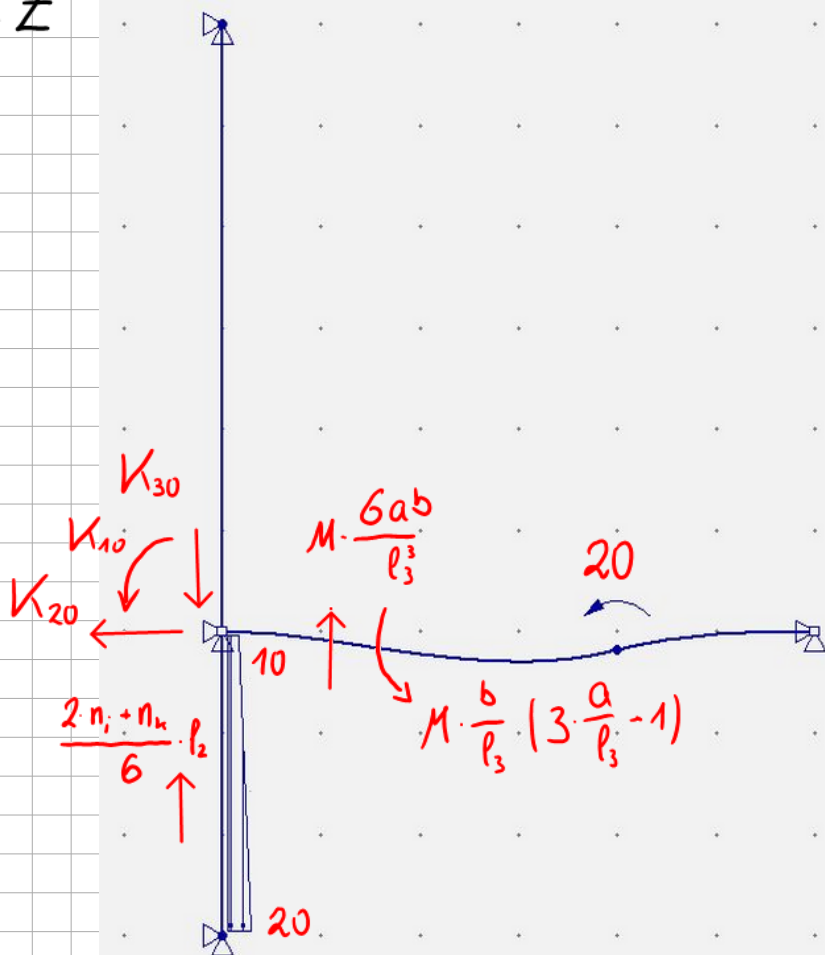
$$M = 20 \text{ kNm}$$

$$n = n_{\min} \div n_{\max} = 10 \div 20 \text{ kN/m}$$

$$EI = 2000 \text{ kNm}^2$$

$$EA = 1000 \text{ kN}$$

L2

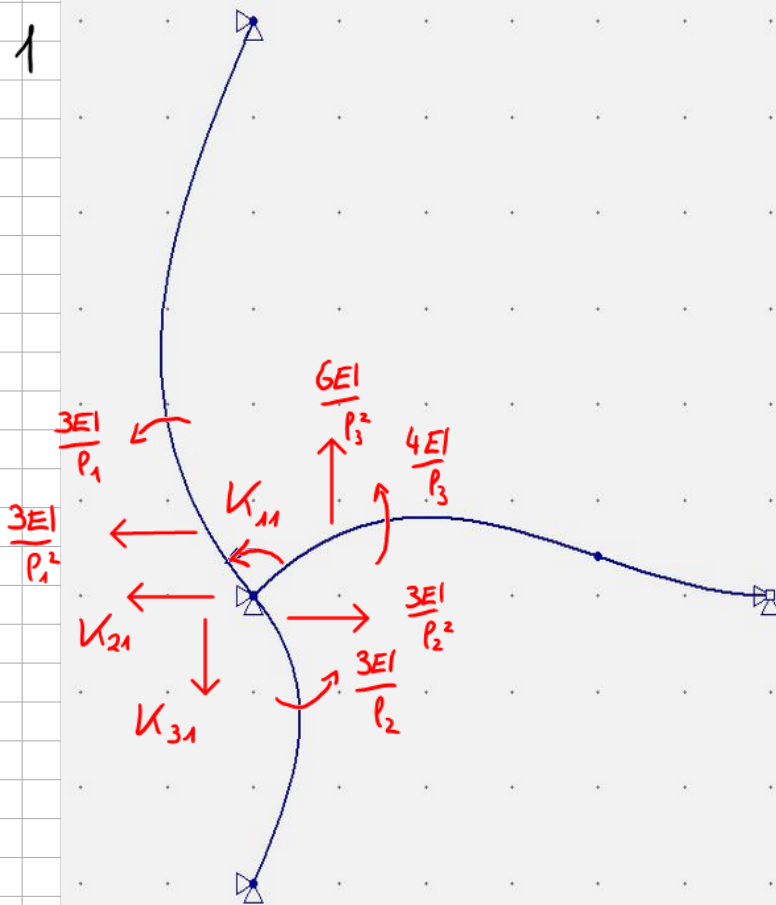


$$K_{10} = 20 \cdot \frac{2}{6} \cdot \left(3 \cdot \frac{4}{6} - 1 \right) = 6,6$$

$$K_{20} = 0$$

$$K_{30} = -\frac{2 \cdot 10 + 20}{6} \cdot 3 - 20 \cdot \frac{6 \cdot 24}{6^3} = -20 - 4,4 = -24,4$$

E2 1



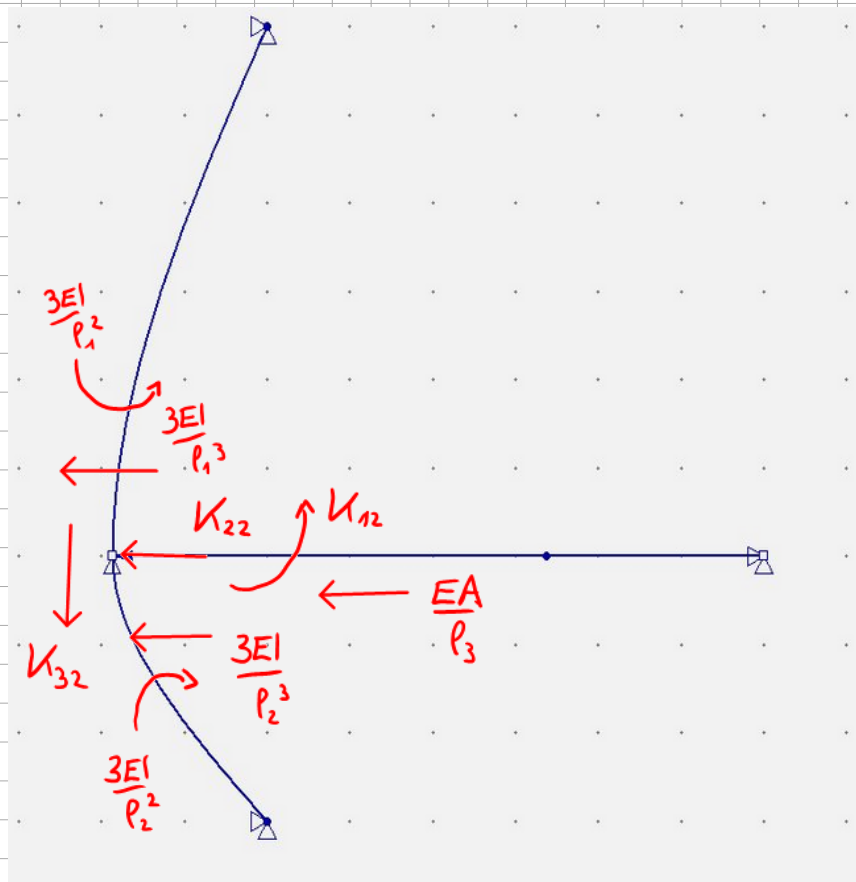
$$K_{11} = \frac{3EI}{l_1} + \frac{3EI}{l_2} + \frac{4EI}{l_3}$$

$$= 1000 + 2000 + 1333,3 = 4333,3$$

$$K_{21} = \frac{3EI}{l_1^2} - \frac{3EI}{l_2^2} = 166,6 - 666,6 = -500$$

$$K_{31} = -\frac{6EI}{l_3^2} = -333,3$$

E2 2



$$K_{12} = \frac{3EI}{l_1^2} - \frac{3EI}{l_2^2} = 166,6 - 666,6 = -500$$

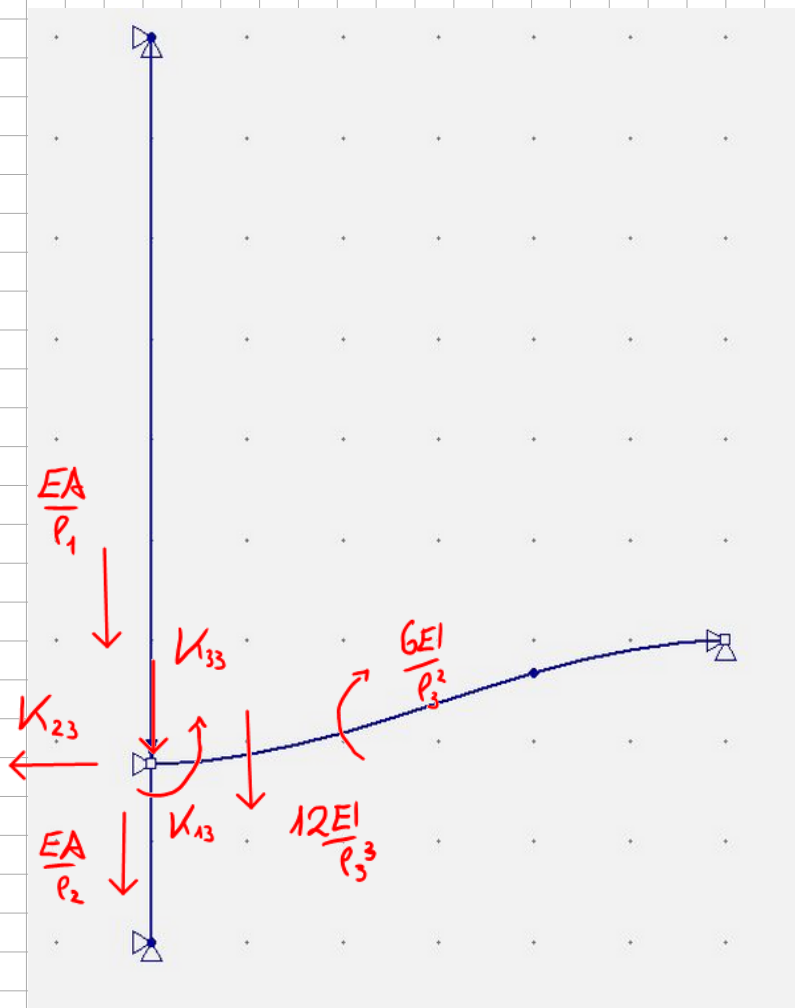
$$= K_{21}$$

$$K_{22} = \frac{3EI}{l_1^3} + \frac{3EI}{l_2^3} + \frac{EA}{l_3} =$$

$$= 27,7 + 222,2 + 166,6 = 416,6$$

$$K_{32} = 0$$

E2 3



$$K_{13} = -\frac{6EI}{l_3^2} = -333,3 = K_{31}$$

$$K_{23} = 0 = K_{32}$$

$$K_{33} = \frac{EA}{l_1} + \frac{EA}{l_2} + \frac{12EI}{l_3^3} =$$

$$= 166,6 + 333,3 + 111,1 = 611,1$$

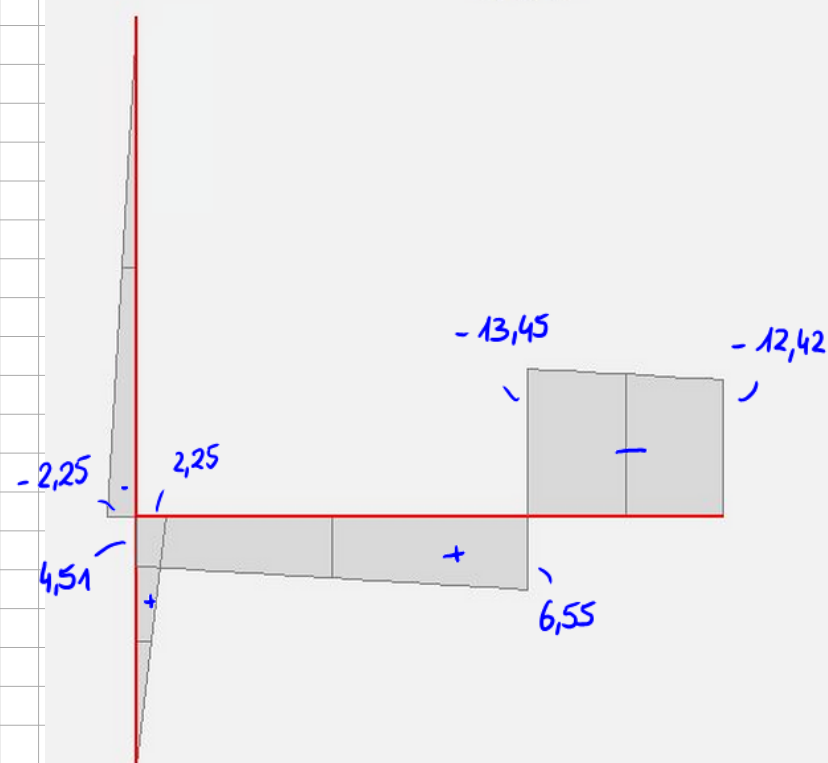
$$\underline{\underline{K}} = \begin{bmatrix} 4333,3 & -500 & -333,3 \\ -500 & 416,6 & 0 \\ -333,3 & 0 & 611,1 \end{bmatrix}$$

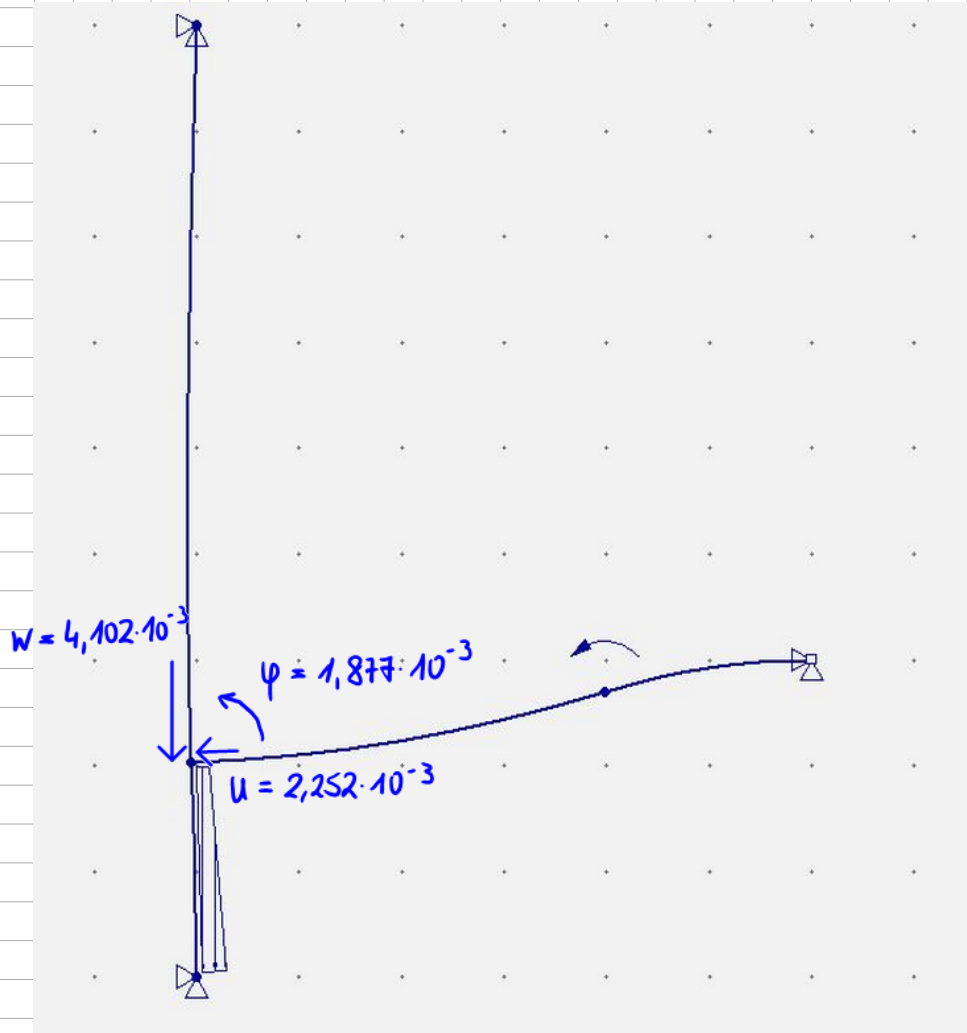
$$\underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} -66 \\ 0 \\ 24,4 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}}$$

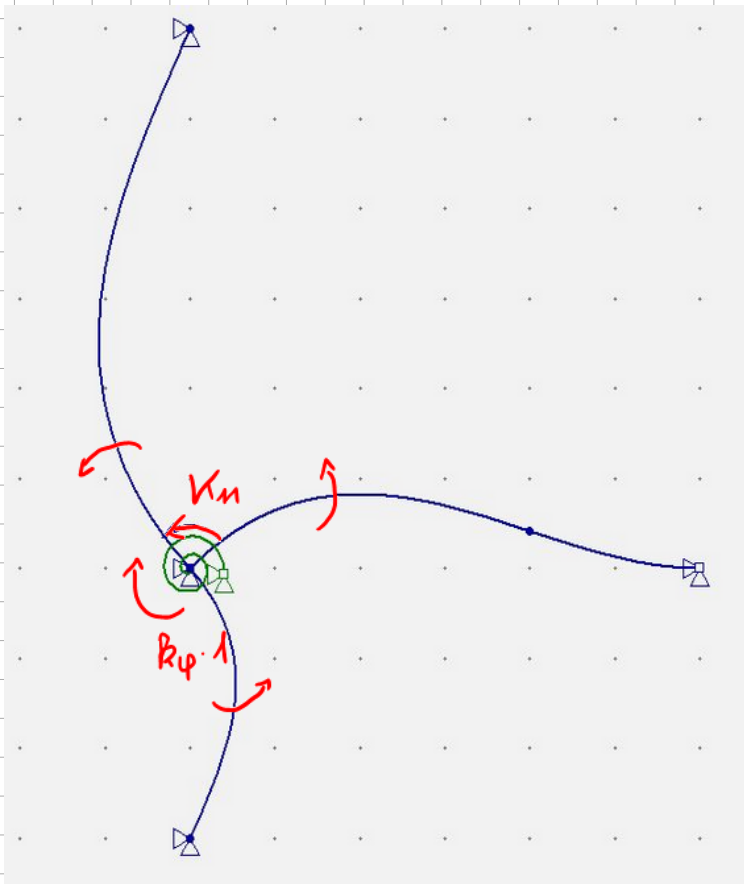
$$\Rightarrow \underline{\underline{u}} = \begin{bmatrix} 1,877 \cdot 10^{-3} \text{ rad} \\ 2,252 \cdot 10^{-3} \text{ m} \\ 4,102 \cdot 10^{-3} \text{ m} \end{bmatrix}$$

M





b) Die Feder hat nun eine Kraft, wenn sie verdrückt wird. Dies geschieht nun im EZ 1.



$$\psi_{2(b)} = \frac{1}{2} \cdot \psi_{2(a)} = \frac{1}{2} \cdot 1,877 \cdot 10^{-3} = 9,386 \cdot 10^{-4}$$

aus der 2. Zeile des Gleichungssystems folgt

$$-500 \cdot 9,386 \cdot 10^{-4} + 416,6 \cdot D_2 = 0$$

$$\Rightarrow D_2 = 1,126 \cdot 10^{-3}$$

und aus der 3. Zeile:

$$-333,3 \cdot 9,386 \cdot 10^{-4} + 611,1 \cdot D_3 = 24,4$$

$$\Rightarrow D_3 = 4,051 \cdot 10^{-2}$$

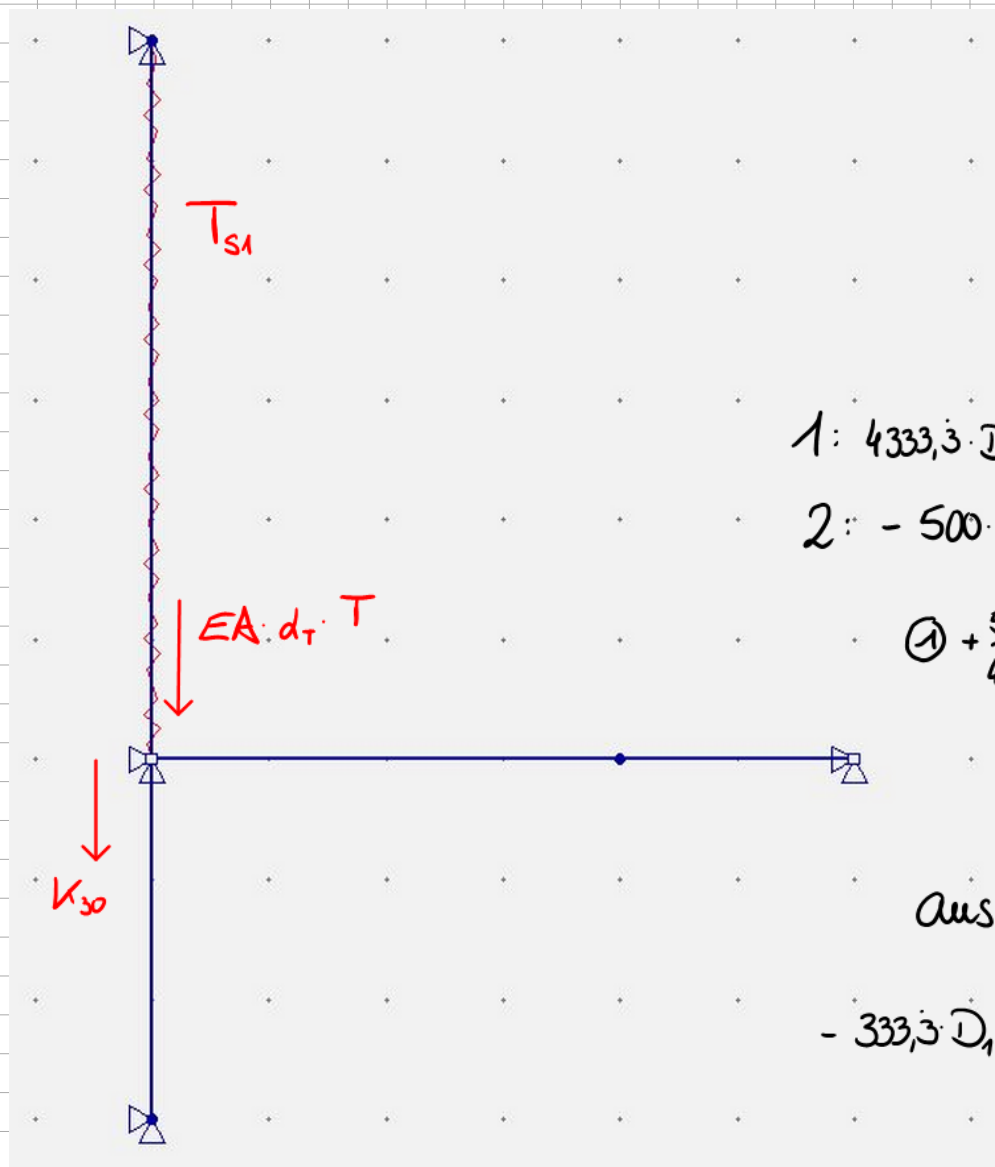
Somit ergibt sich aus der 1. Zeile

$$(4333,3 + k_p) \cdot 9,386 \cdot 10^{-4} - 500 \cdot 1,126 \cdot 10^{-3} - 333,3 \cdot 4,051 \cdot 10^{-2} = -6,6$$

$$\Rightarrow k_p = 3551,5 \text{ kNm/rad}$$

k_p muss also größer als 3551,5 kNm/rad sein, um den Grenzwert einzukalten!

c) Die Temperaturbelastung verändert nun den Lastzustand und dabei nun K_{30} .



$$K_{30} = -24,4 - EA \cdot d_T \cdot T$$

D_1 kann aus Zeile 1 & 2 bestimmt werden

$$1: 4333,3 \cdot D_1 - 500 \cdot D_2 - 333,3 \cdot D_3 \stackrel{0}{=} -6,6$$

$$2: -500 \cdot D_1 + 416,6 \cdot D_2 = 0$$

$$\textcircled{1} + \frac{500}{416,6} \textcircled{2}: (4333,3 - \frac{500^2}{416,6}) \cdot D_1 = -6,6$$

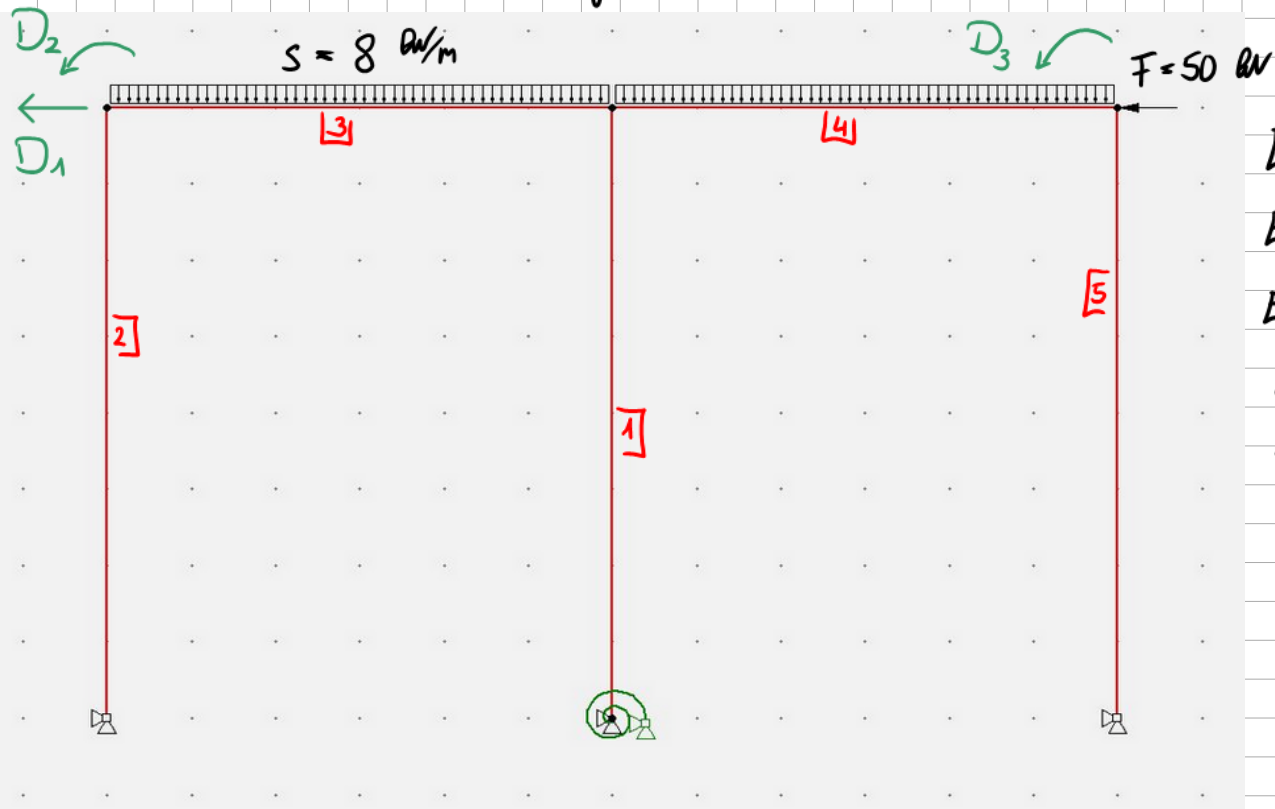
$$\Rightarrow D_1 = -1,79 \cdot 10^{-3}$$

aus Zeile 3 folgt nun

$$-333,3 \cdot D_1 + 611,1 \cdot D_3 \stackrel{0}{=} 24,4 + EA \cdot d_T \cdot T$$

$$\Rightarrow T = -23,85 \text{ K}$$

Probeklausur 2 - Aufgabe 2



$$EI_1 \rightarrow \infty$$

$$EI_{2-5} = 10 \text{ kN/m}^2$$

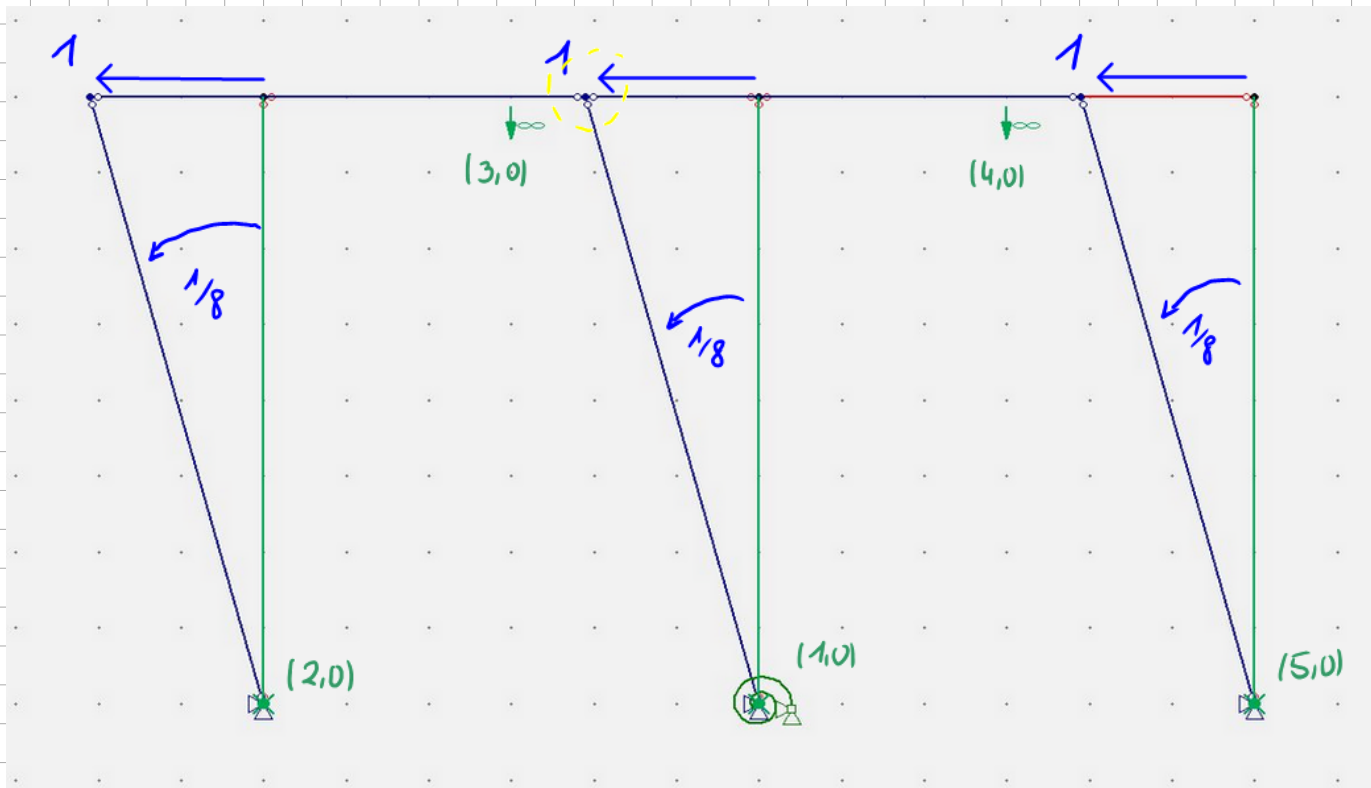
$$EA \rightarrow \infty$$

$$s = 8 \text{ kN/m}$$

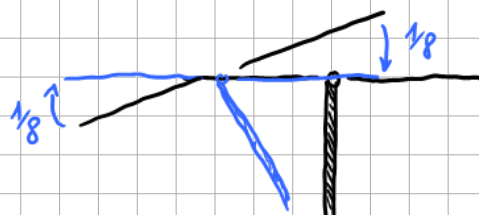
$$F = 50 \text{ kN}$$

$$c_\varphi = 5000 \text{ kN/m}$$

Gelenkfiguren

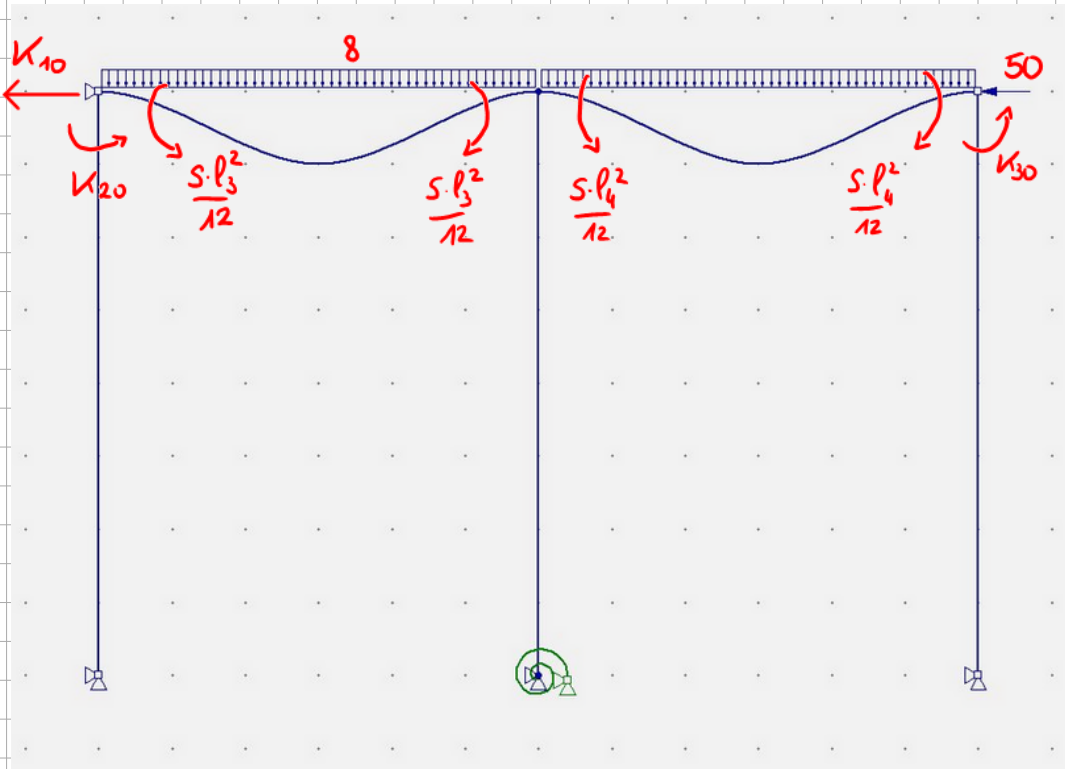


Detail



$$\varphi = \frac{1}{8}$$

L2



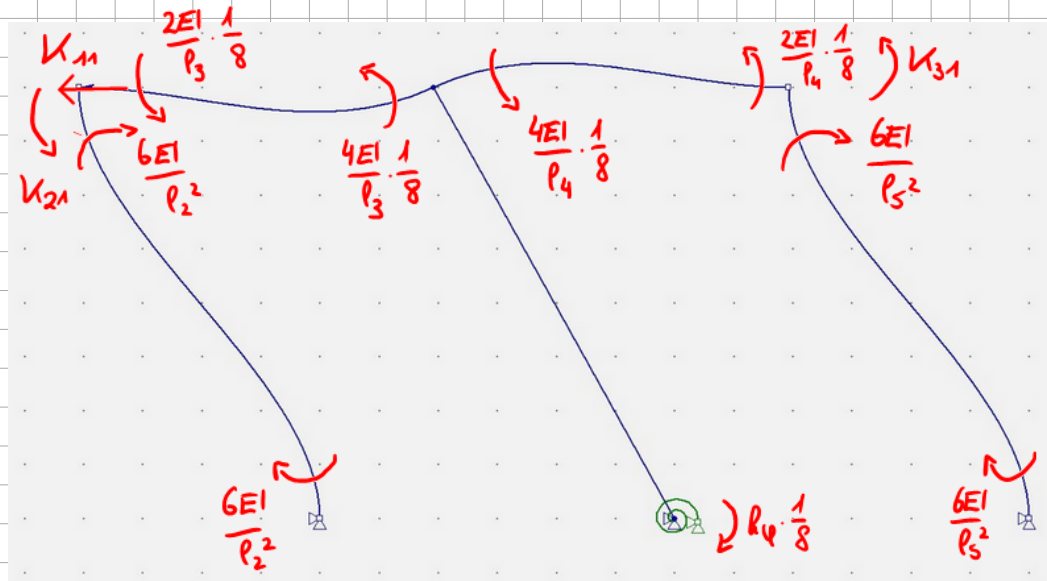
$$PV: K_{10} \cdot 1 + \left(\frac{5 \cdot l_3^2}{12} - \frac{5 \cdot l_4^2}{12} \right) \cdot \frac{1}{8} + 50 \cdot 1 = 0$$

$$K_{10} = -50$$

$$GG: K_{20} = \frac{5 \cdot l_3^2}{12} = 24$$

$$K_{30} = -\frac{5 \cdot l_4^2}{12} = -24$$

E2 1



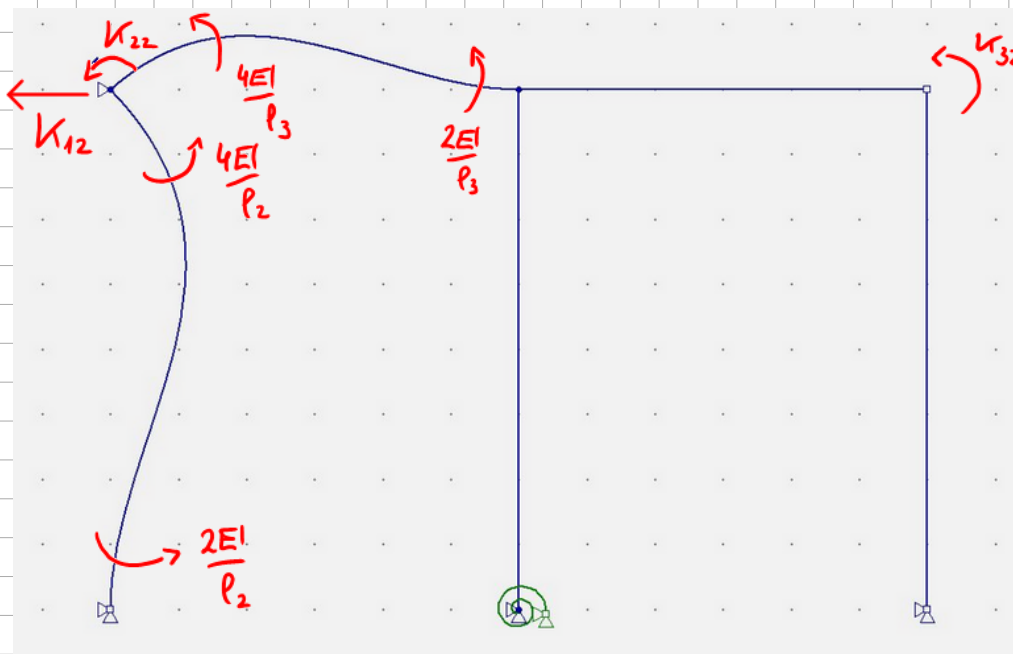
$$PV: K_{11} \cdot 1 - \frac{6EI}{l_2^2} \cdot \frac{1}{8} \cdot 2 - \frac{6EI}{l_5^2} \cdot \frac{1}{8} \cdot 2 - \frac{4EI}{l_4} \cdot \frac{1}{8} \cdot \frac{1}{8} \cdot 2 - 50 \cdot \frac{1}{8} \cdot \frac{1}{8} = 0$$

$$K_{11} = 234,375 + 234,375 + 208,3 + 78,125 = 755,208$$

$$GG: K_{21} = \frac{2EI}{6} \cdot \frac{1}{8} - \frac{6EI}{8^2} = 416,6 - 937,5 = -520,83$$

$$K_{31} = \frac{2EI}{6} \cdot \frac{1}{8} - \frac{6EI}{8^2} = 416,6 - 937,5 = -520,83$$

E2 2



$$P.V: K_{12} \cdot \bar{1} + \frac{4EI}{l_2} \cdot \frac{\bar{1}}{8} + \frac{2EI}{l_2} \cdot \frac{\bar{1}}{8} - \frac{2EI}{l_3} \cdot \frac{\bar{1}}{8} = 0$$

$$K_{12} = -625 - 312,5 + 416,6 = -520,83$$

$$G.G: K_{22} = \frac{4EI}{l_2} + \frac{4EI}{l_3} = 5000 + 666,6 = 11666,6$$

$$K_{32} = 0$$

E2 3: System ist symmetrisch, E2 3 ist E2 2 gespiegelt

$$K_{13} = -520,83 \quad (= K_{12})$$

$$K_{23} = 0 \quad (= K_{32})$$

$$K_{33} = 11666,6 \quad (= K_{22})$$

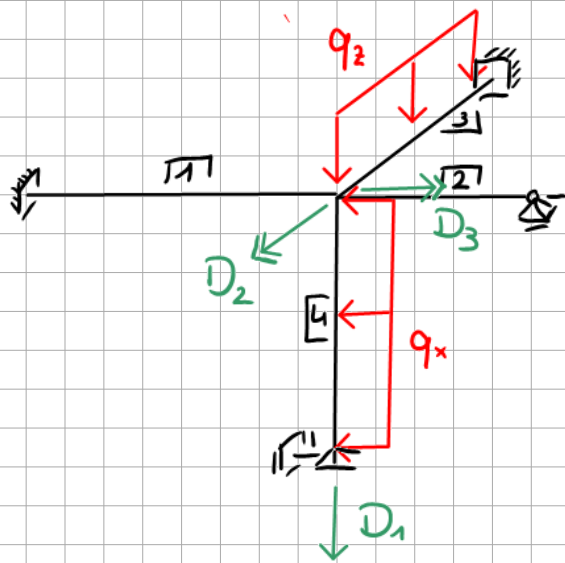
$$\underline{\underline{K}} = \begin{bmatrix} 755,208 & -520,83 & -520,83 \\ -520,83 & 11666,6 & 0 \\ -520,83 & 0 & 11666,6 \end{bmatrix}$$

$$\underline{\underline{F}} = -K_0 = \begin{bmatrix} 50 \\ -24 \\ 24 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}}$$

$$\Rightarrow \underline{\underline{u}} = \begin{bmatrix} 7,06 \cdot 10^{-2} \text{ m} \\ 1,09 \cdot 10^{-3} \text{ rad} \\ 5,21 \cdot 10^{-3} \text{ rad} \end{bmatrix}$$

Probeklausur 2 - Aufgabe 3



$$q_2 = 4 \text{ kN/m}$$

$$q_x = 5 \text{ kN/m}$$

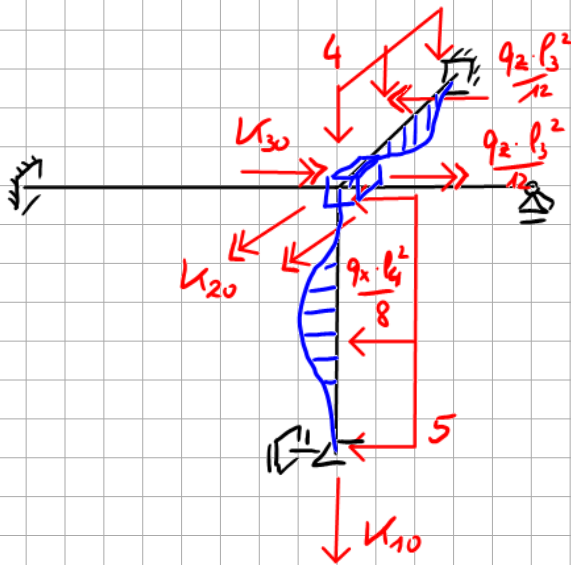
$$EI = 10000$$

$$G_{IT,1-3} = 5000$$

$$G_{IT,4} \rightarrow \infty$$

$$EA \rightarrow \infty$$

L2

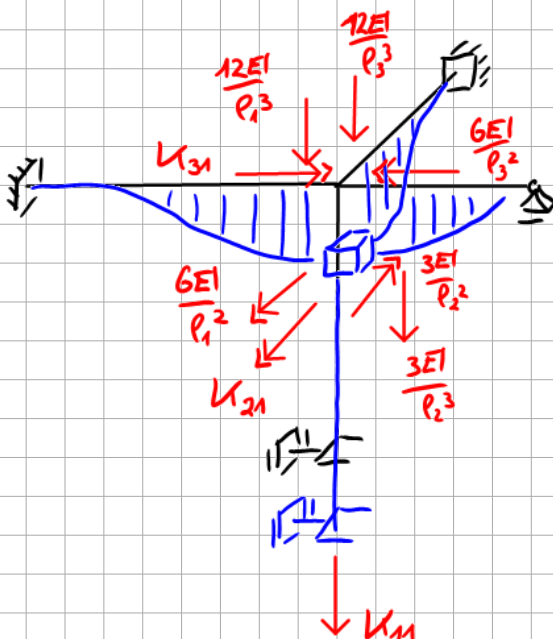


$$K_{10} = \frac{q_2 \cdot l_3^2}{2} = 8$$

$$K_{20} = \frac{q_x \cdot l_4^2}{8} = 15,625$$

$$K_{30} = \frac{q_2 \cdot l_3^2}{12} = 5,3$$

E2 1

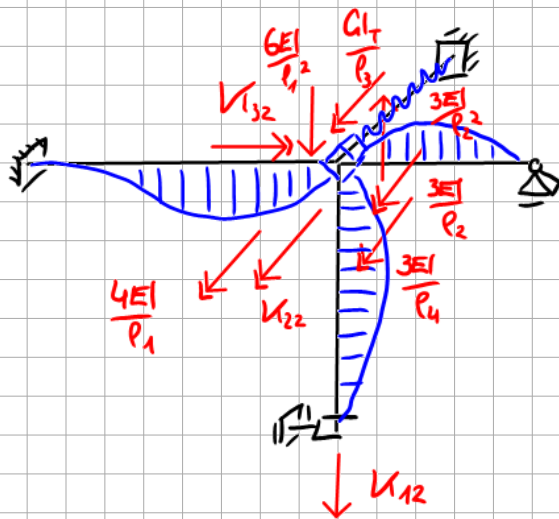


$$K_{11} = \frac{12EI}{l_1^3} + \frac{12EI}{l_3^3} + \frac{3EI}{l_2^3} = 960 + 1875 + 1111,1 = 3946,1$$

$$K_{21} = \frac{6EI}{l_1^2} - \frac{3EI}{l_2^2} = 2400 - 3333,3 = -933,3$$

$$K_{31} = -\frac{6EI}{l_3^2} = -3750$$

E2 2

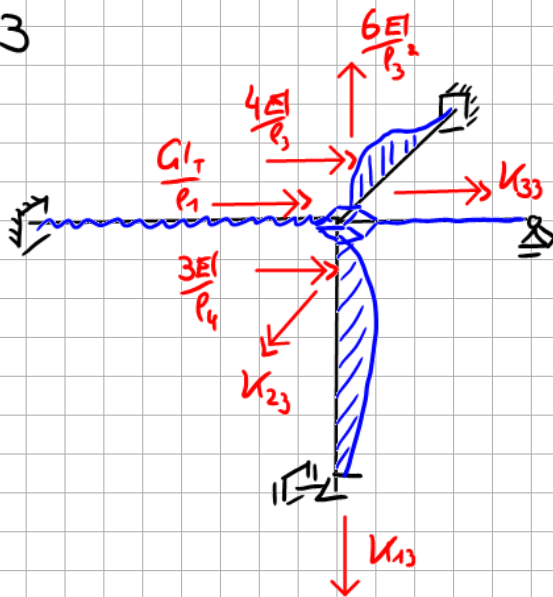


$$K_{12} = \frac{6EI}{l_1^2} - \frac{3EI}{l_2^2} = 2400 - 3333,3 = -933,3 = K_{21}$$

$$K_{22} = \frac{4EI}{l_1} + \frac{3EI}{l_2} + \frac{GI_T}{l_3} + \frac{3EI}{l_4} = 8000 + 10000 + 1250 + 6000 = 25250$$

$$K_{32} = 0$$

E2 3



$$K_{13} = -\frac{6EI}{l_3^2} = -3750$$

$$K_{23} = 0$$

$$K_{33} = \frac{GI_T}{l_1} + \frac{4EI}{l_3} + \frac{3EI}{l_4} = 1000 + 10000 + 6000 = 17000$$

$$\underline{K} = \begin{bmatrix} 3946,1 & -933,3 & -3750 \\ -933,3 & 25250 & 0 \\ -3750 & 0 & 17000 \end{bmatrix}$$

$$\underline{F} = -\underline{K}_0 = \begin{bmatrix} 8 \\ -15,625 \\ -5,3 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F}$$

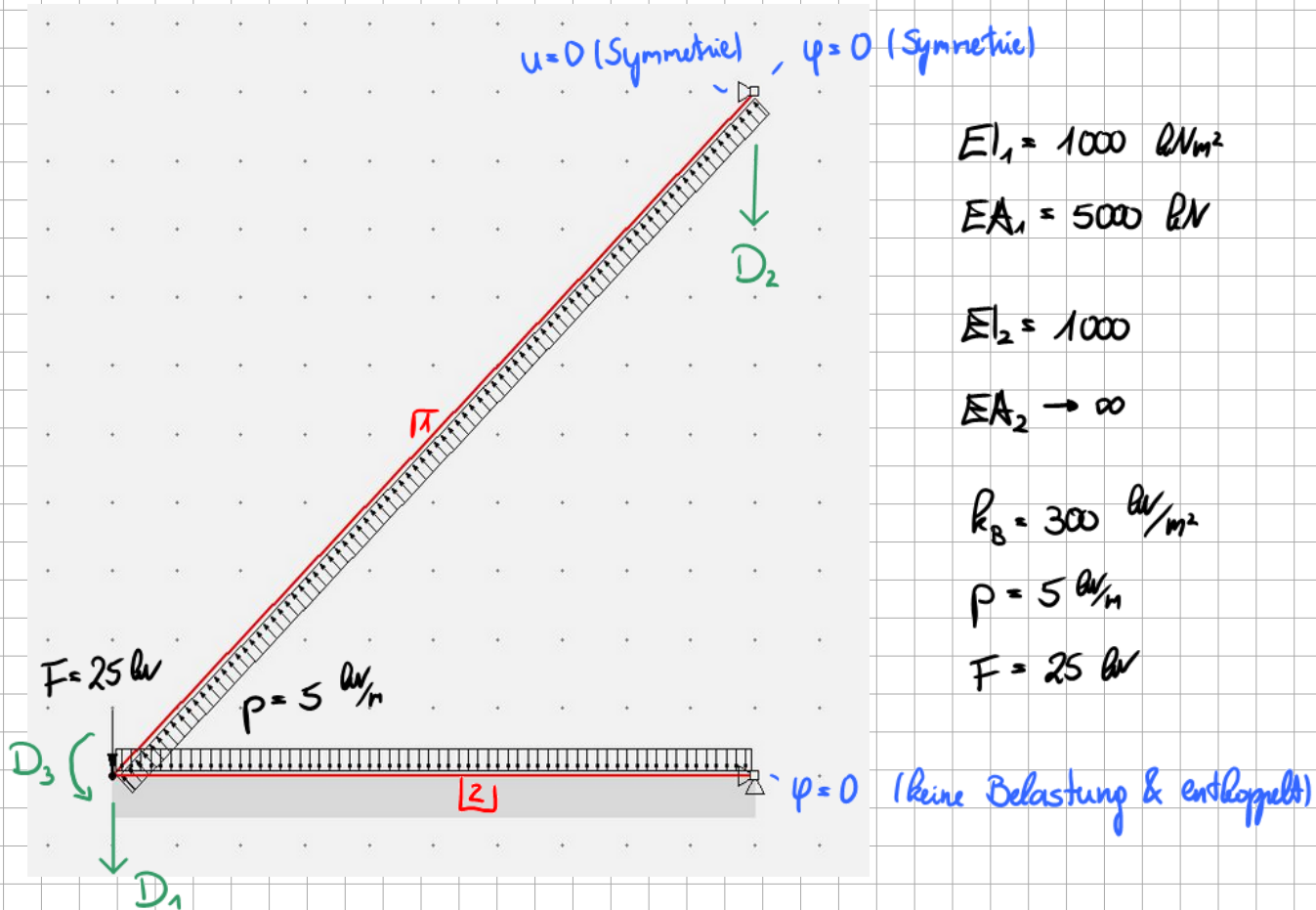
$$\Rightarrow \underline{u} = \begin{bmatrix} 2,03 \cdot 10^{-3} \text{ m} \\ -5,44 \cdot 10^{-4} \text{ rad} \\ 1,33 \cdot 10^{-4} \text{ rad} \end{bmatrix}$$

Probeklausur 2 - Aufgabe 4

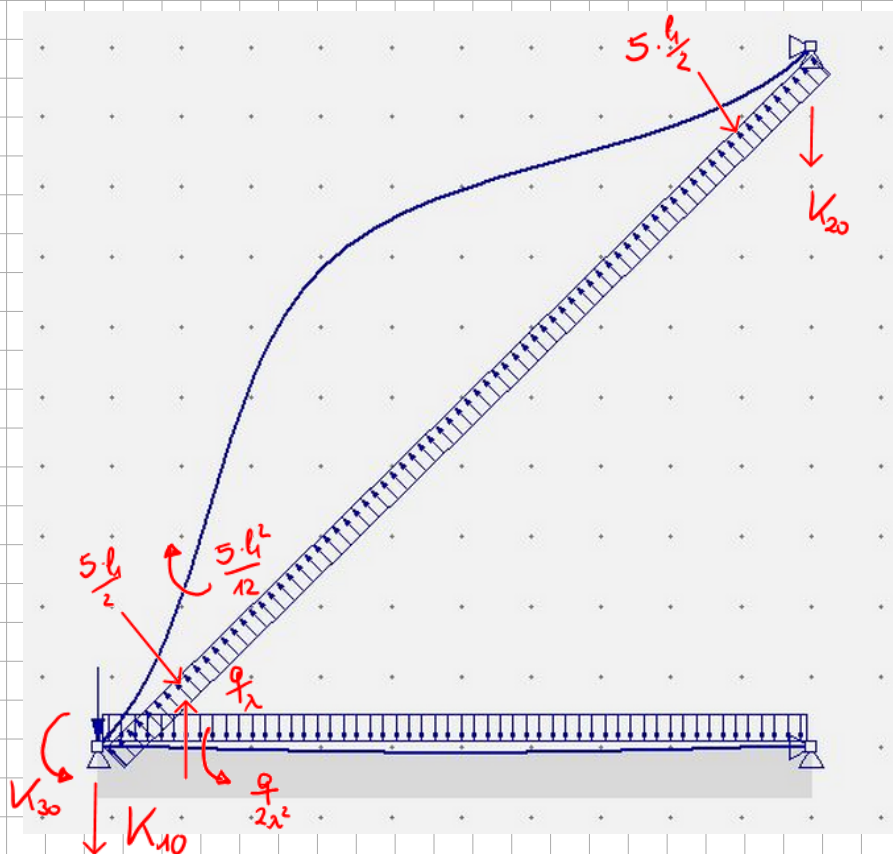
$$\lambda = \sqrt[4]{\frac{k}{EI_B}} = 0,523$$

$$\lambda \cdot l = 0,523 \cdot 10 = 5,23 > \pi \rightarrow \infty \text{ langer Balken}$$

Rechnung am halben System möglich



L2

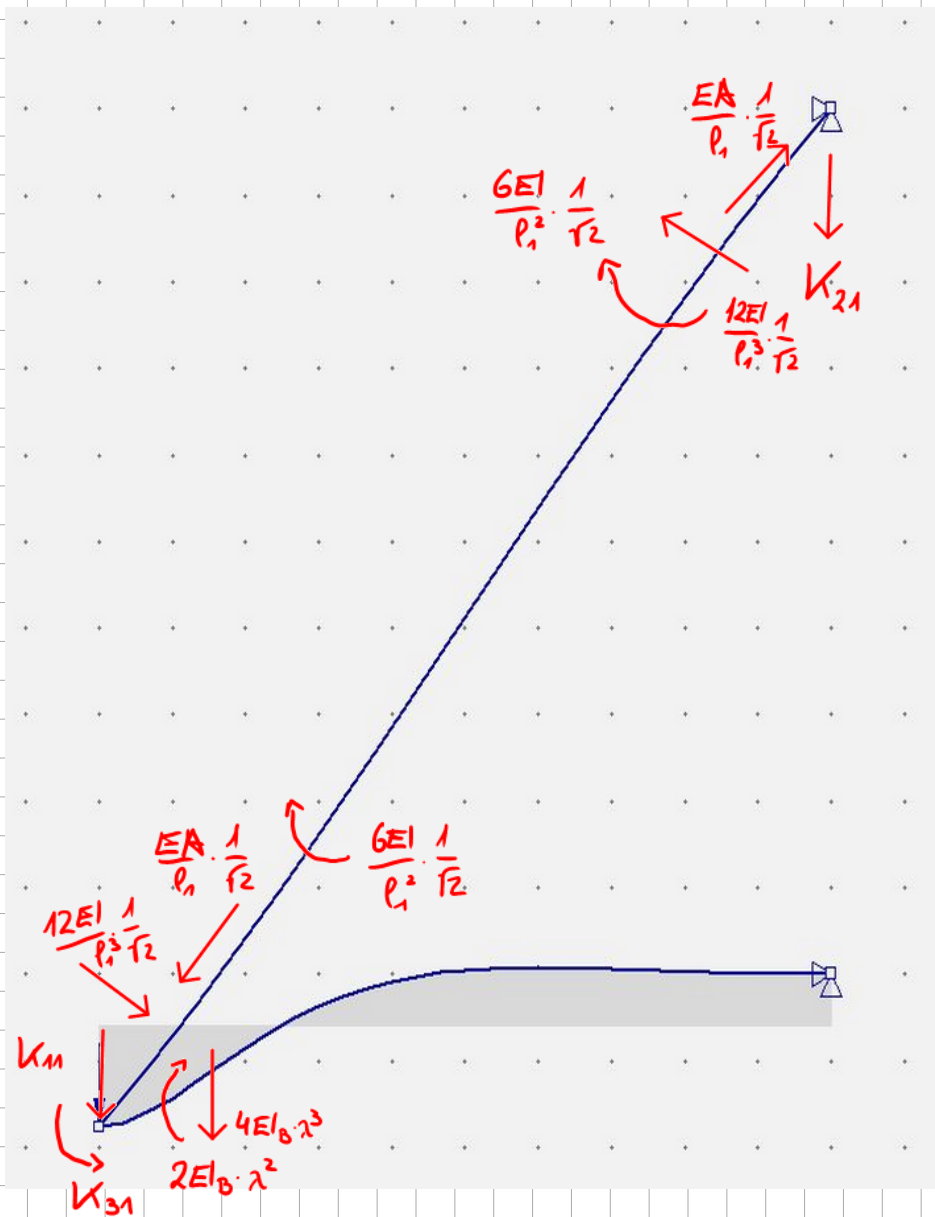


$$K_{10} = \frac{q \cdot l}{2} - F - \frac{q}{\lambda} = 25 - 25 - 9,6 = -9,6$$

$$K_{20} = \frac{q \cdot l}{2} = 25$$

$$K_{30} = -\frac{q \cdot l^2}{12} + \frac{q}{2\lambda^2} = -83,3 + 9,14 = -74,19$$

E2 1



$$K_{11} = \frac{EA}{l_1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{12EI}{l_1^3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + 4EI_0 \cdot \lambda^3 =$$

$$= 176,777 + 2,121 + 573,266 =$$

$$= 752,164$$

$$K_{21} = -\frac{EA}{l_1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{12EI}{l_1^3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} =$$

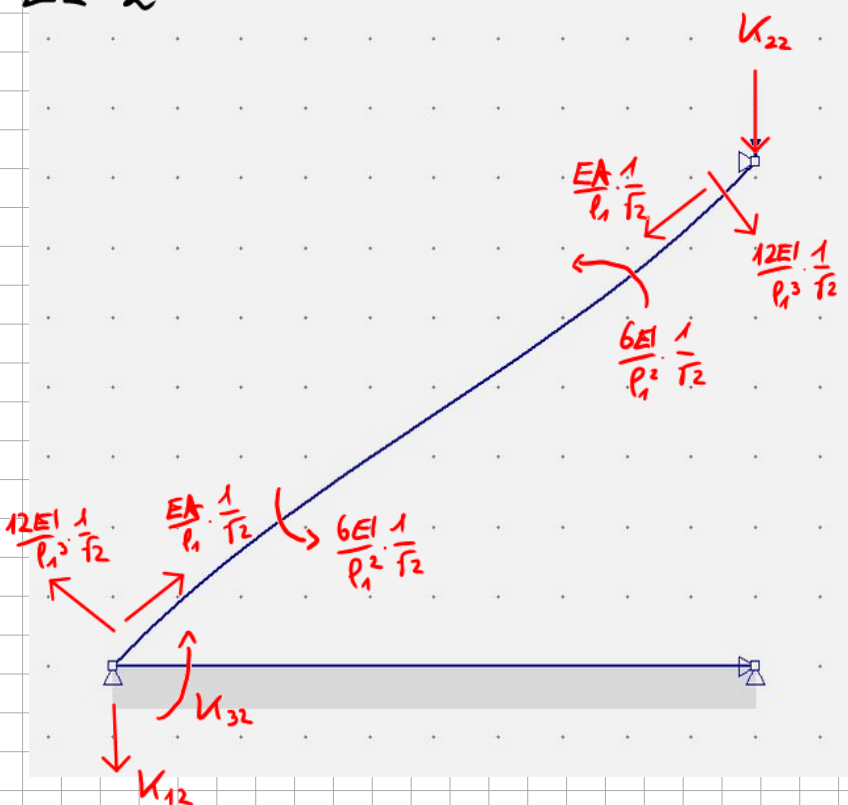
$$= -176,777 - 2,121 = -178,898$$

$$K_{31} = -\frac{6EI}{l_1^2} \cdot \frac{1}{\sqrt{2}} - 2EI_0 \cdot \lambda^2 =$$

$$= -21,21 - 547,723 =$$

$$= -568,933$$

E2 2



$$K_{12} = -\frac{EA}{l_1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{12EI}{l_1^3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} =$$

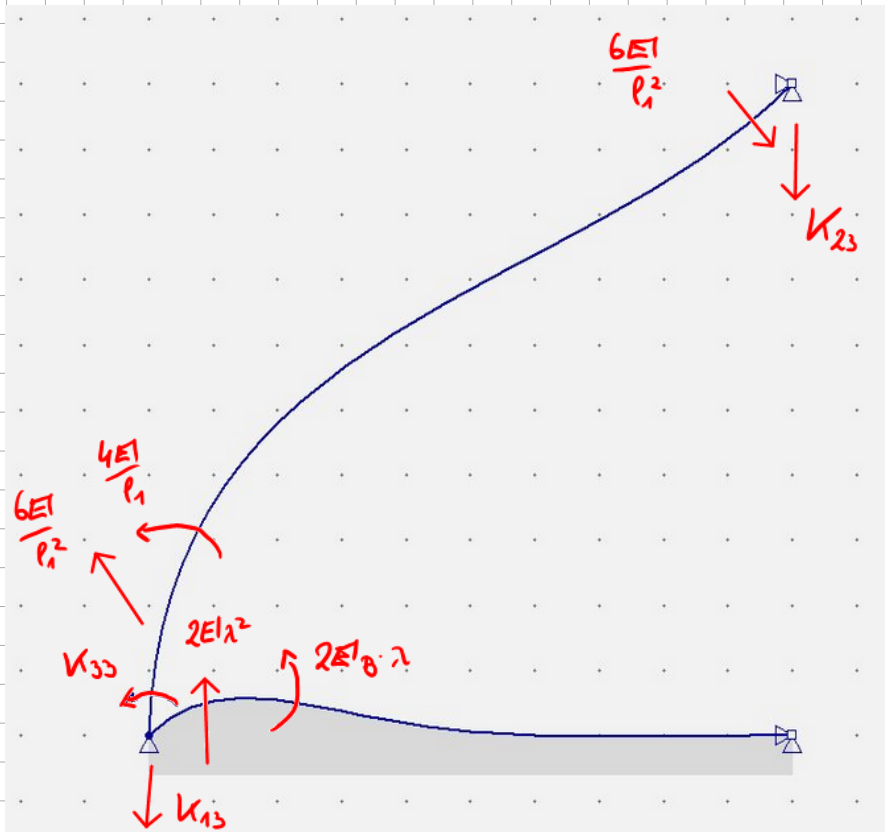
$$= -176,777 - 2,121 = -178,898 = K_{21}$$

$$K_{22} = \frac{EA}{l_1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{12EI}{l_1^3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} =$$

$$= 176,777 + 2,121 = 178,898$$

$$K_{32} = \frac{6EI}{l_1^2} \cdot \frac{1}{\sqrt{2}} = 21,21$$

Ex 3



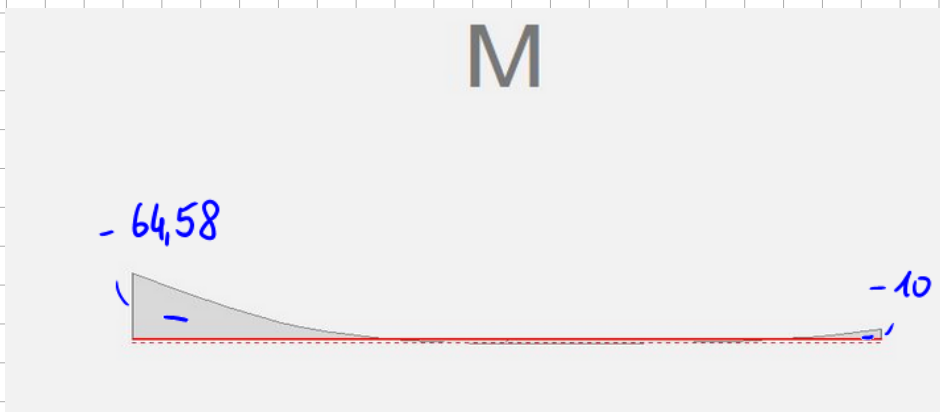
$$K_{13} = -\frac{6EI}{l_1^2} \cdot \frac{1}{\sqrt{2}} - 2EI_B \cdot \lambda^2 = -21,21 - 547,723 = -568,933 = K_{31}$$

$$K_{23} = \frac{6EI}{l_1^2} \cdot \frac{1}{\sqrt{2}} = 21,21 = K_{32}$$

$$K_{33} = \frac{4EI}{l_1} + 2EI_B \cdot \lambda = 282,843 + 1046,635 = 1329,478$$

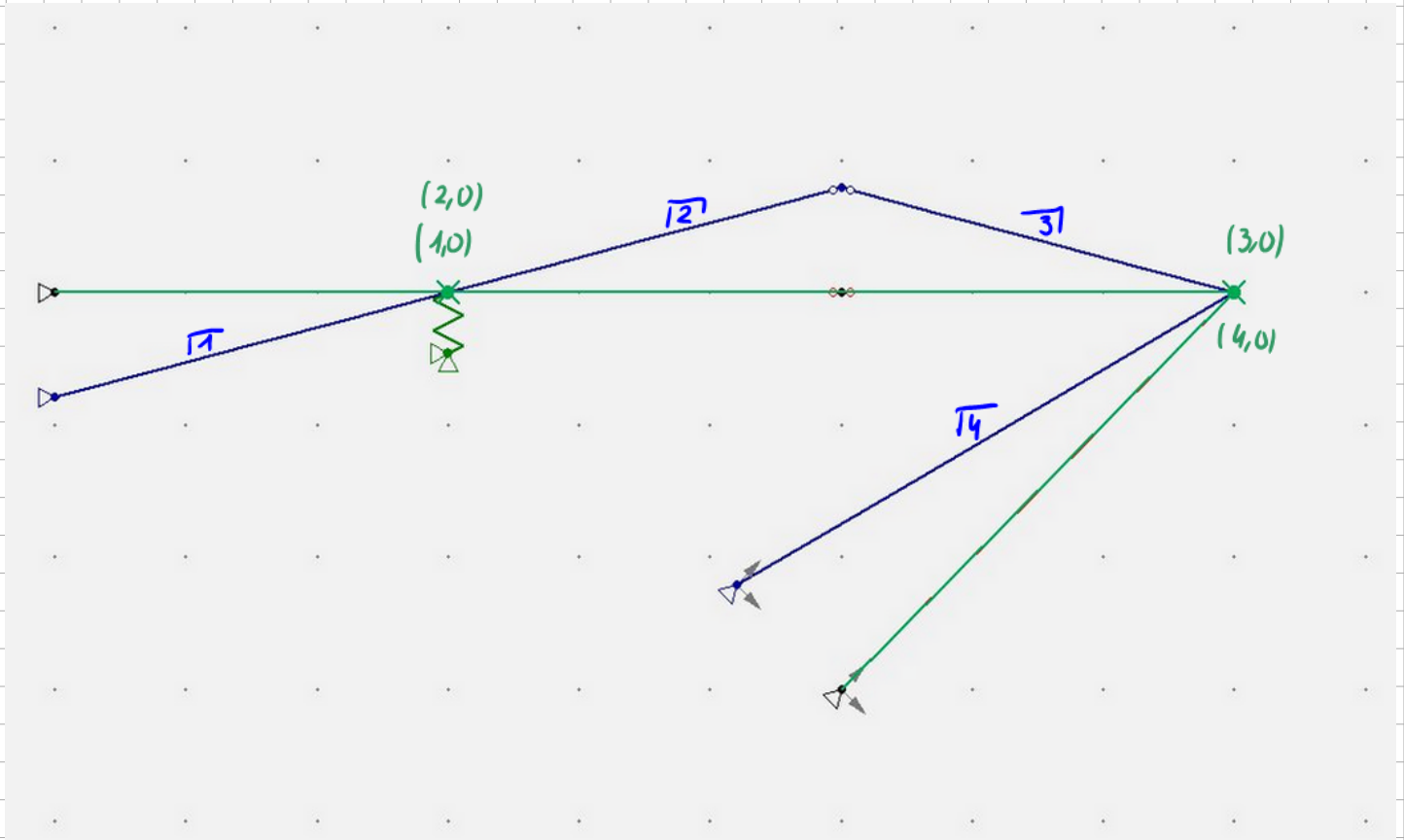
$$\underline{\underline{K}} = \begin{bmatrix} 752,164 & 178,898 & -568,933 \\ 178,898 & 178,898 & 21,21 \\ -568,933 & 21,21 & 1329,478 \end{bmatrix} \quad \underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 9,6 \\ -25 \\ 74,2 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \Rightarrow \underline{\underline{u}} = \begin{bmatrix} 4,717 \cdot 10^{-2} \text{ m} \\ -1,017 \cdot 10^{-1} \text{ m} \\ 7,749 \cdot 10^{-2} \text{ rad} \end{bmatrix}$$

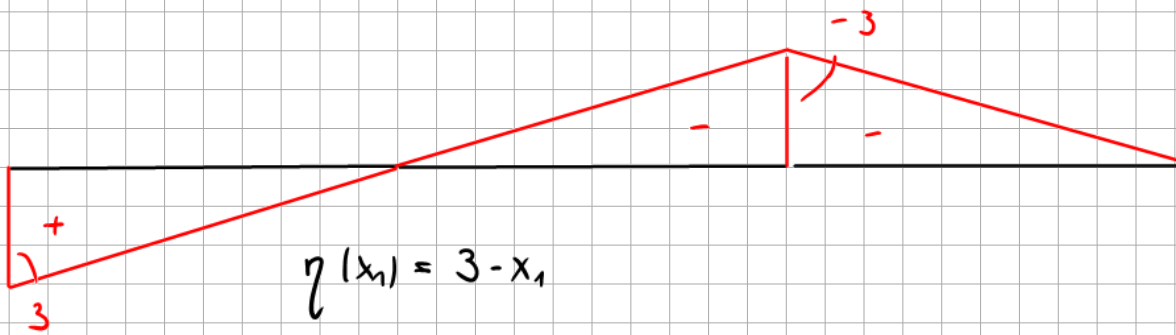


Probeklausur 2 - Aufgabe 5

(a)



Einflusslinie

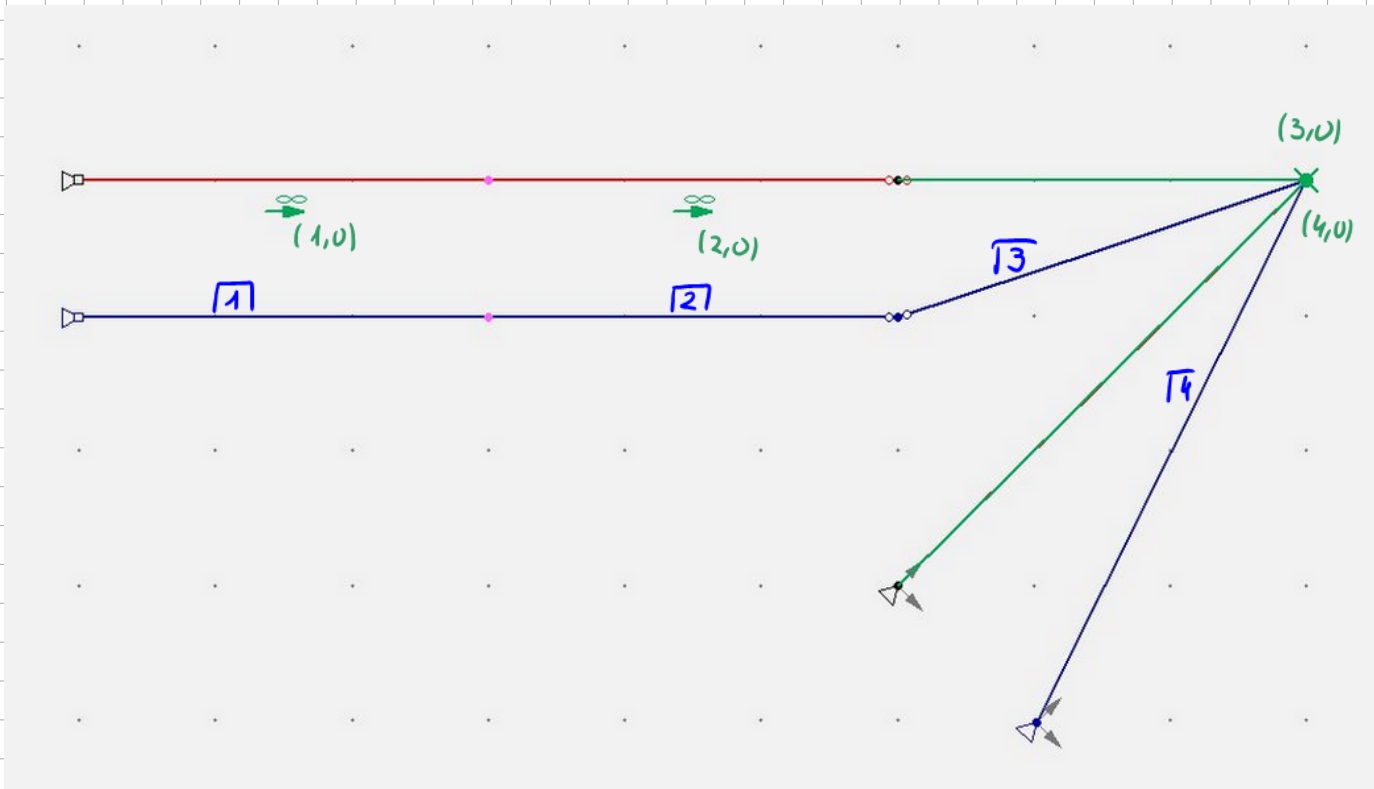


$$\eta(x_1) = 3 - x_1$$

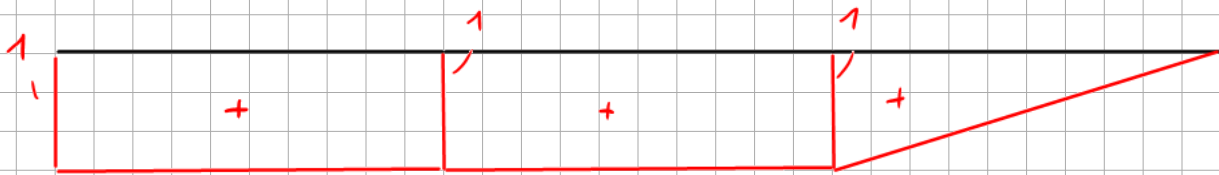
$$\eta(x_2) = x_2$$

$$\eta(x_3) = -3 + x_3$$

(b)



Einflusslinie

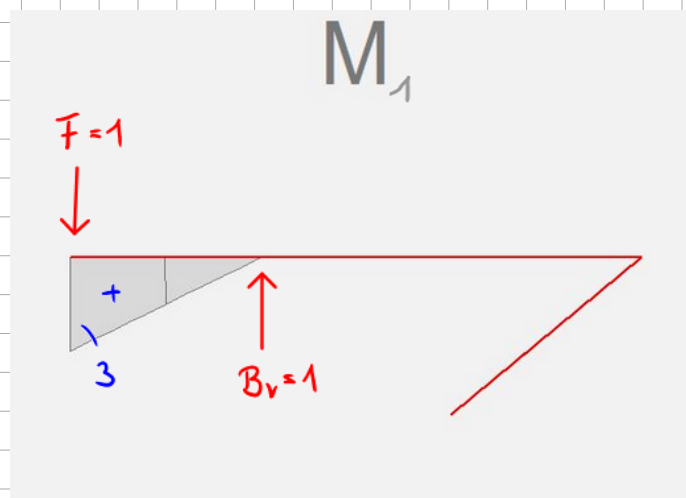
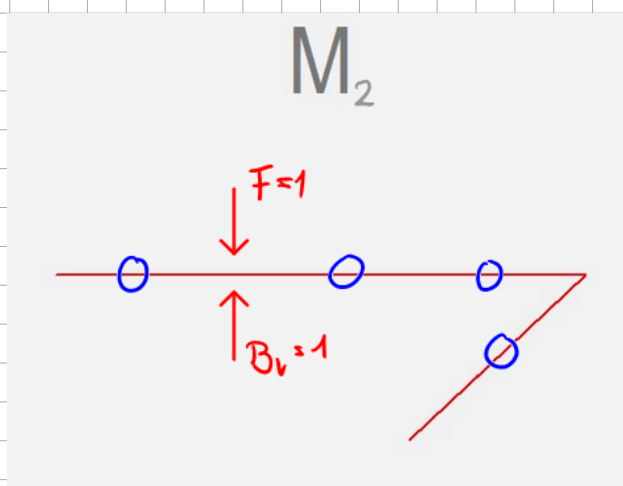
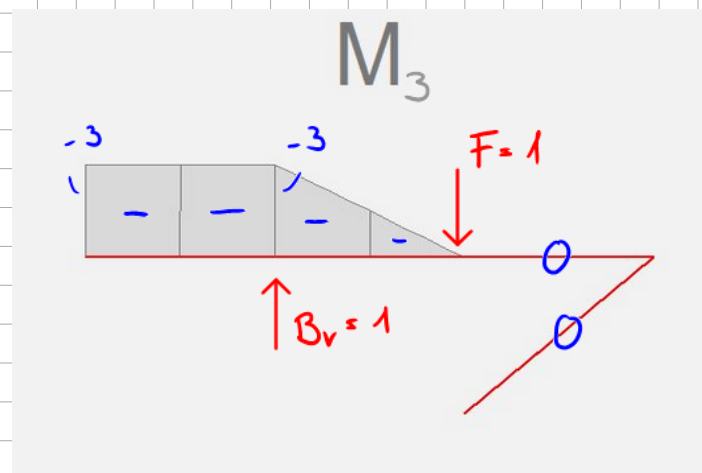
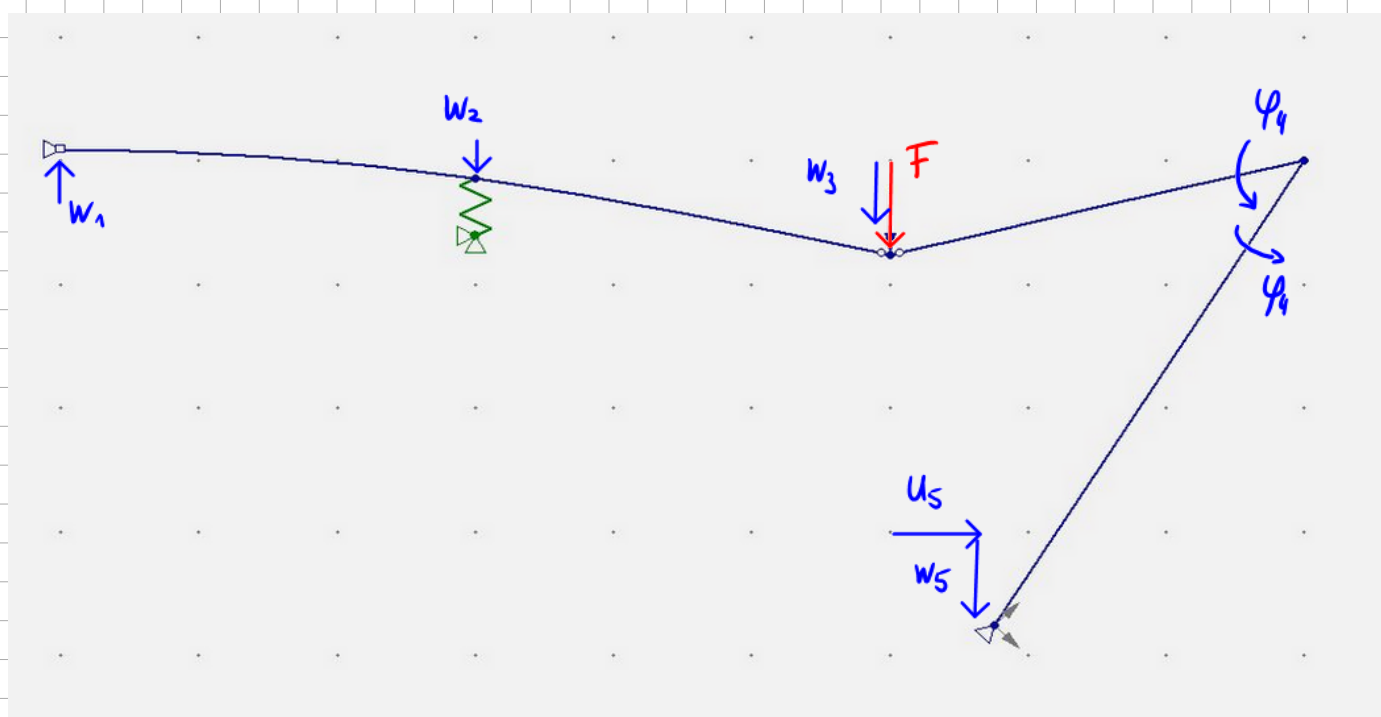


$$\eta(x_1) = 1$$

$$\eta(x_2) = 1$$

$$\eta(x_3) = 1 - \frac{x_3}{3}$$

(c)

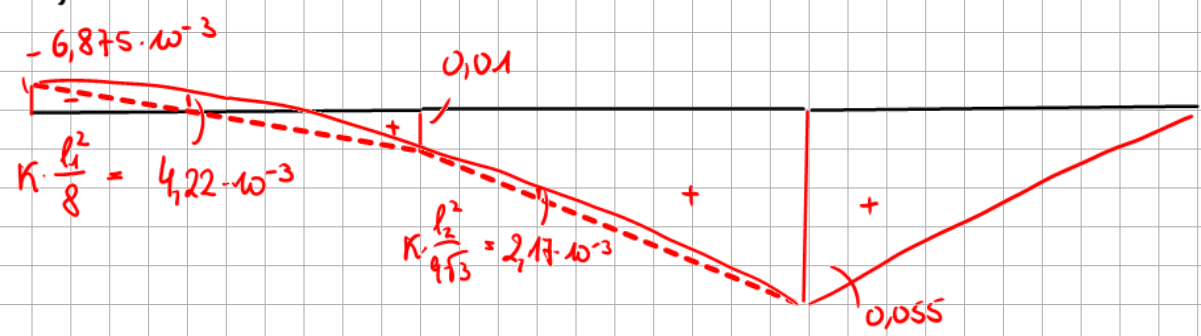


$$w_3 = (-3)^2 \cdot \frac{3}{EI} + \frac{1}{3} \cdot (-3)^2 \cdot \frac{3}{EI} + \frac{1^2}{EI} = 0,055 \text{ m}$$

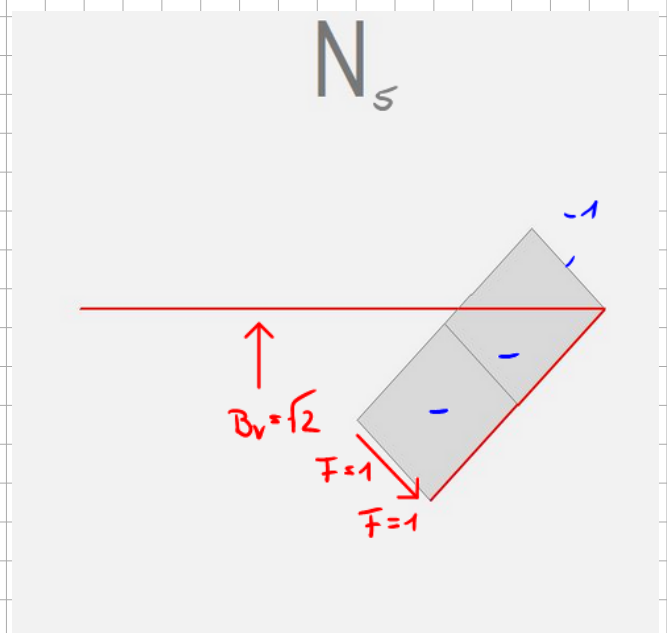
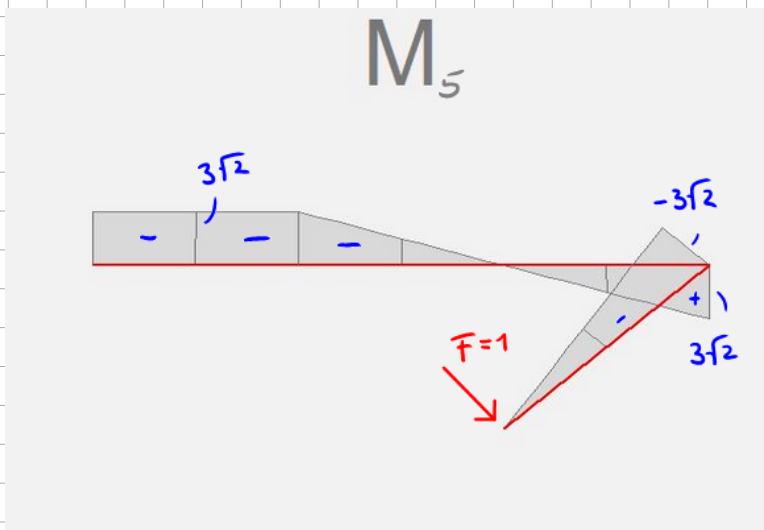
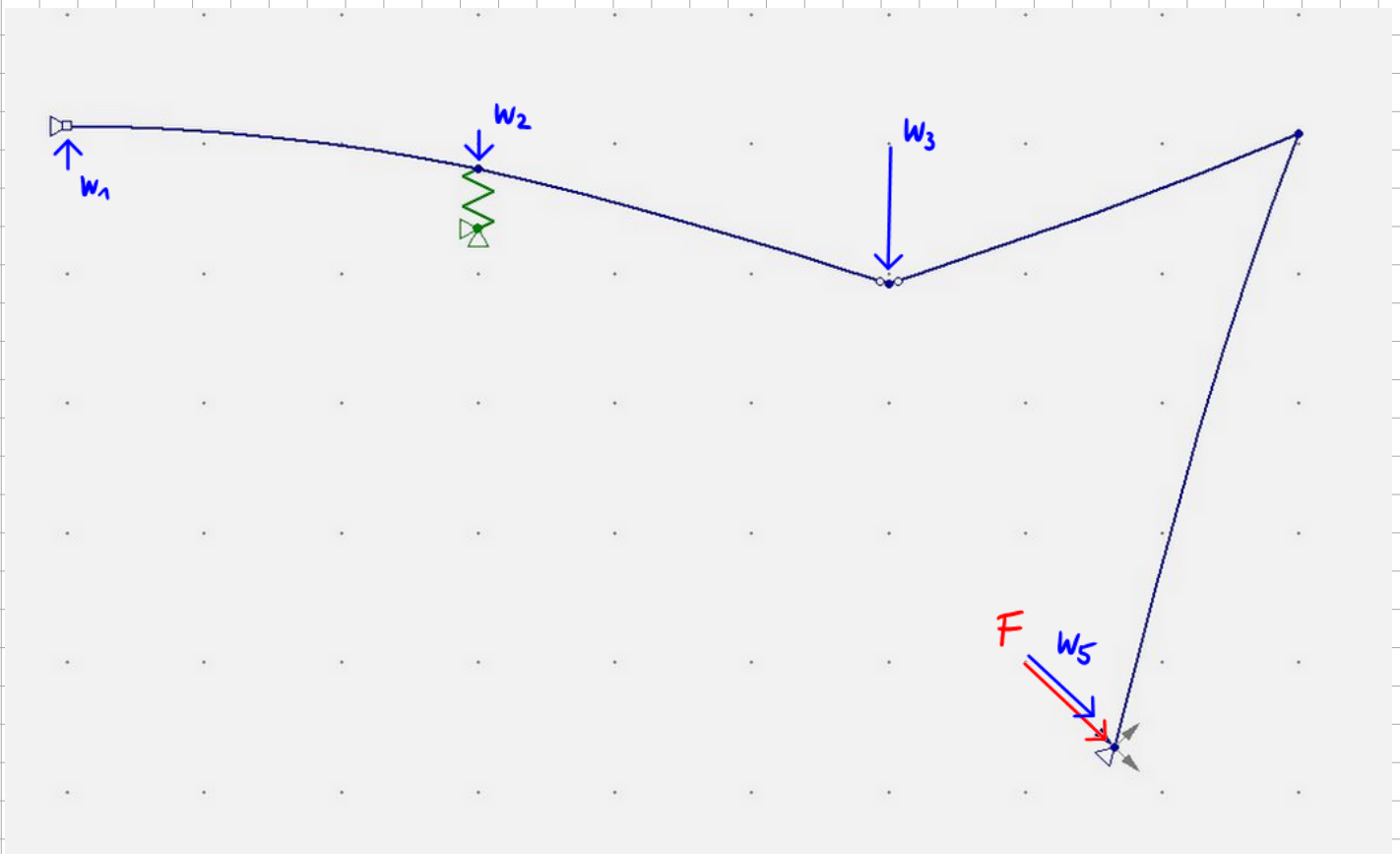
$$w_2 = \frac{1}{EI} = 0,01 \text{ m}$$

$$w_1 = \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{3}{EI} + \frac{1 \cdot 1}{EI} = -6,875 \cdot 10^{-3} \text{ m}$$

Einflusslinie



(d)

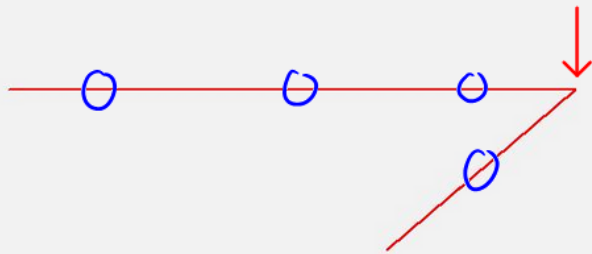


$$w_5 = (-3\sqrt{2})^2 \cdot \frac{3}{EI} + \frac{1}{3} (-3\sqrt{2})^2 \cdot \frac{3}{EI} \cdot 2 + \frac{1}{3} \cdot (-3\sqrt{2})^2 \cdot \frac{3\sqrt{2}}{EI} + (-1)^2 \cdot \frac{3\sqrt{2}}{EA} + \frac{\sqrt{2} \cdot \sqrt{2}}{k} = 0,169 \text{ m}$$

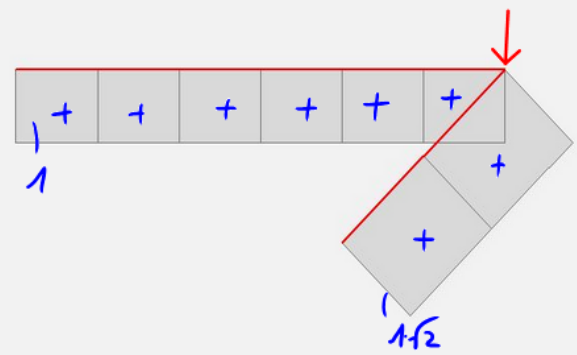
$$w_2 = \frac{\sqrt{2}}{k} = 0,0141 \text{ m}$$

$$w_3 = (-3\sqrt{2})/3 \cdot \frac{3}{EI} + \frac{1}{3} \cdot (-3\sqrt{2}) \cdot (-3) \cdot \frac{3}{EI} + \frac{1 \cdot \sqrt{2}}{k} = 0,078 \text{ m}$$

M_4

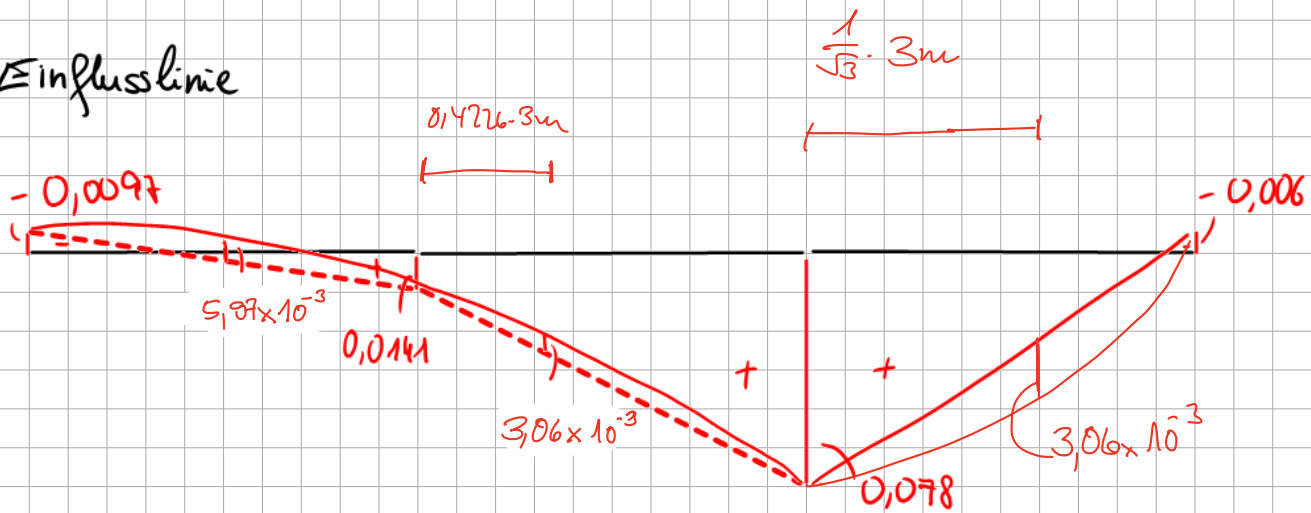


N_4

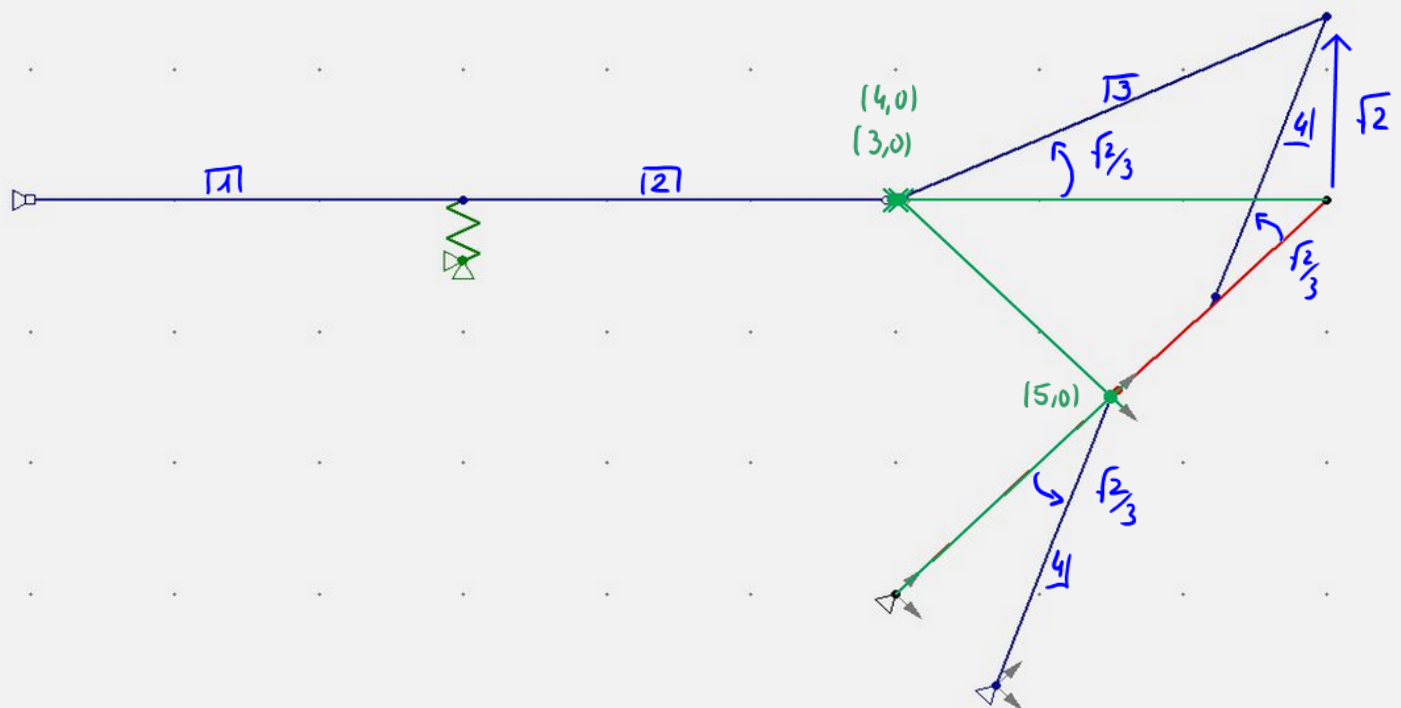


$$w_4 = (-1) \cdot 1\sqrt{2} \cdot \frac{3f_2}{EA} = -0,006 \text{ m}$$

Einflusslinie



(e)

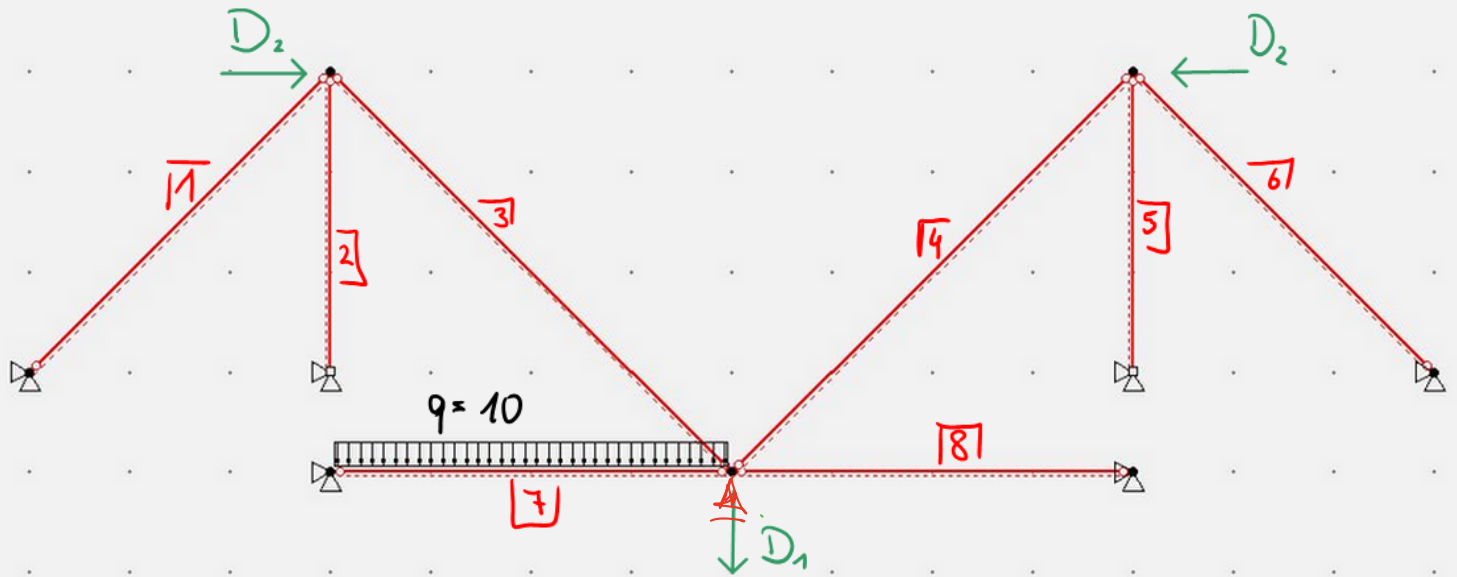


Einflusslinie



Statik Muskelösungen

Probeklausur 3 - Aufgabe 1



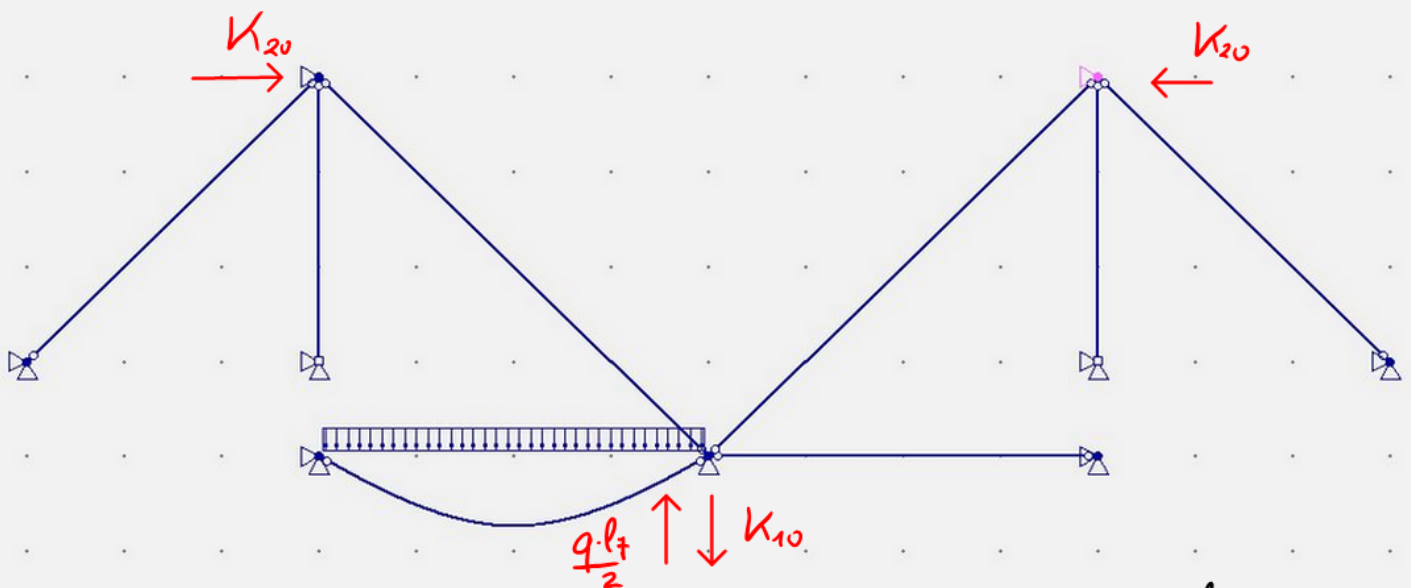
$$EI = 1000 \text{ Nm}^2$$

$$EA_{1,3,4,6} = 1000 \text{ N}$$

$$EA_{2,5,7,8} \rightarrow \infty$$

(a)

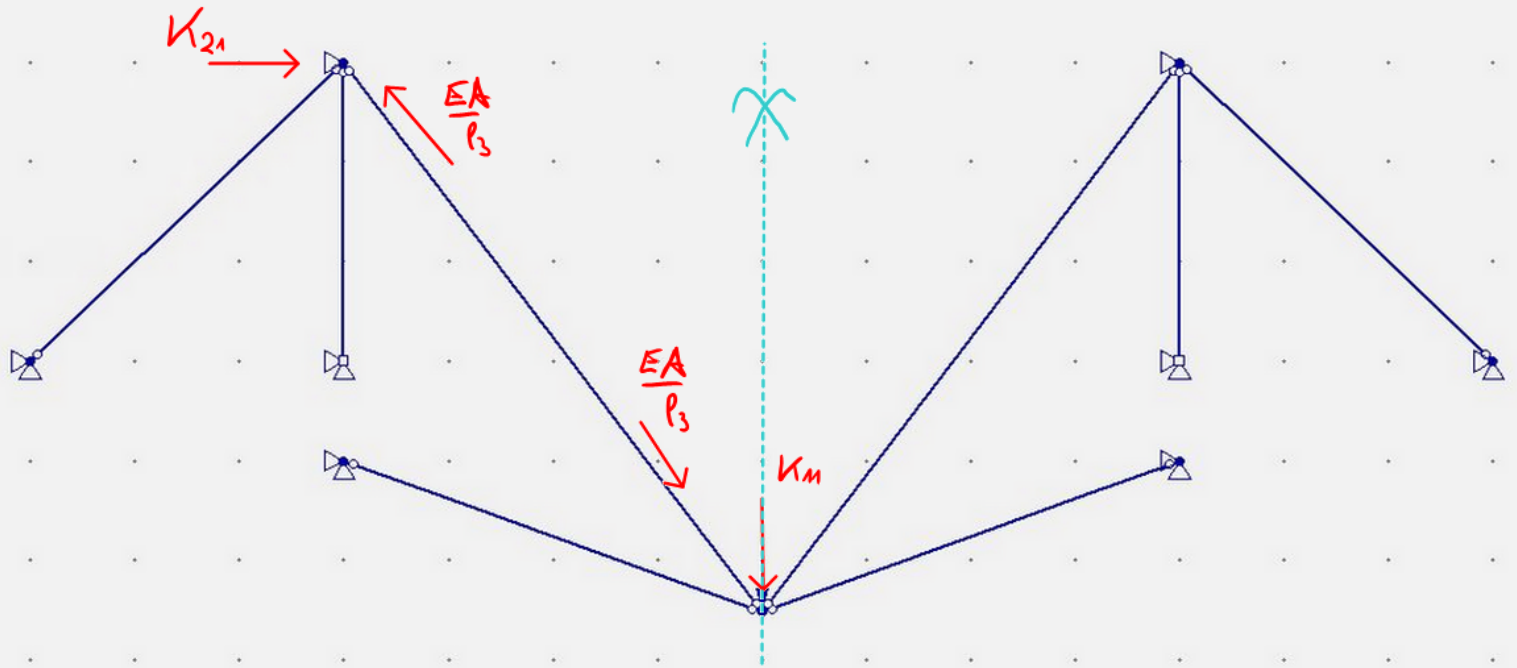
L7



$$K_{10} = \frac{q \cdot l_f}{2} - 20$$

$$K_{20} = 0$$

E2 1

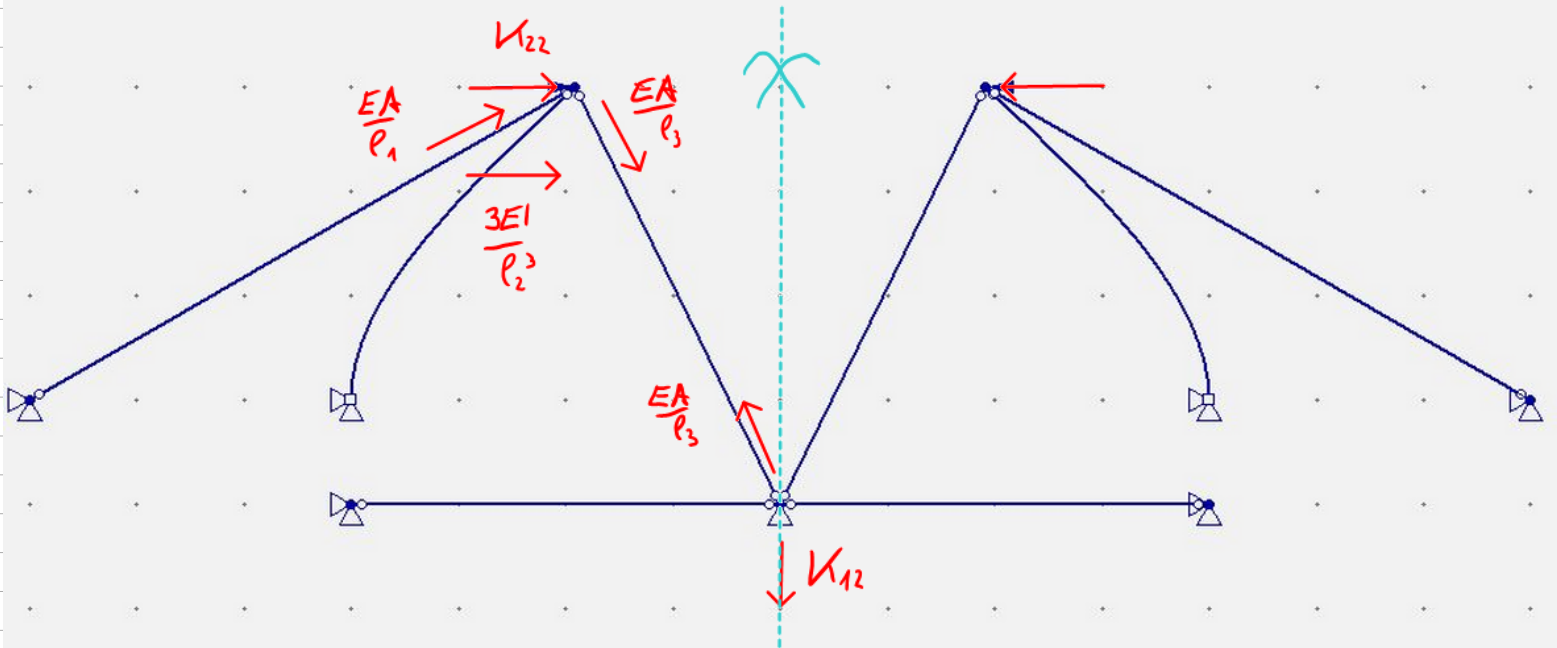


$$K_{11} = \frac{EA}{l_3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 2 = 88,388 \cdot 2 = 176,777$$

Symmetrie

$$K_{21} = - \frac{EA}{l_3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 2 = -88,388 \cdot 2 = -176,777$$

E2 2



$$K_{12} = - \frac{EA}{l_3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 2 = -88,388 \cdot 2 = -176,777 = K_{21}$$

$$K_{22} = \left(\frac{EA}{l_1} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{3EI}{l_2^3} + \frac{EA}{l_3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right) \cdot 2 = (117,851 + 111,1 + 88,388) \cdot 2 = 634,7$$

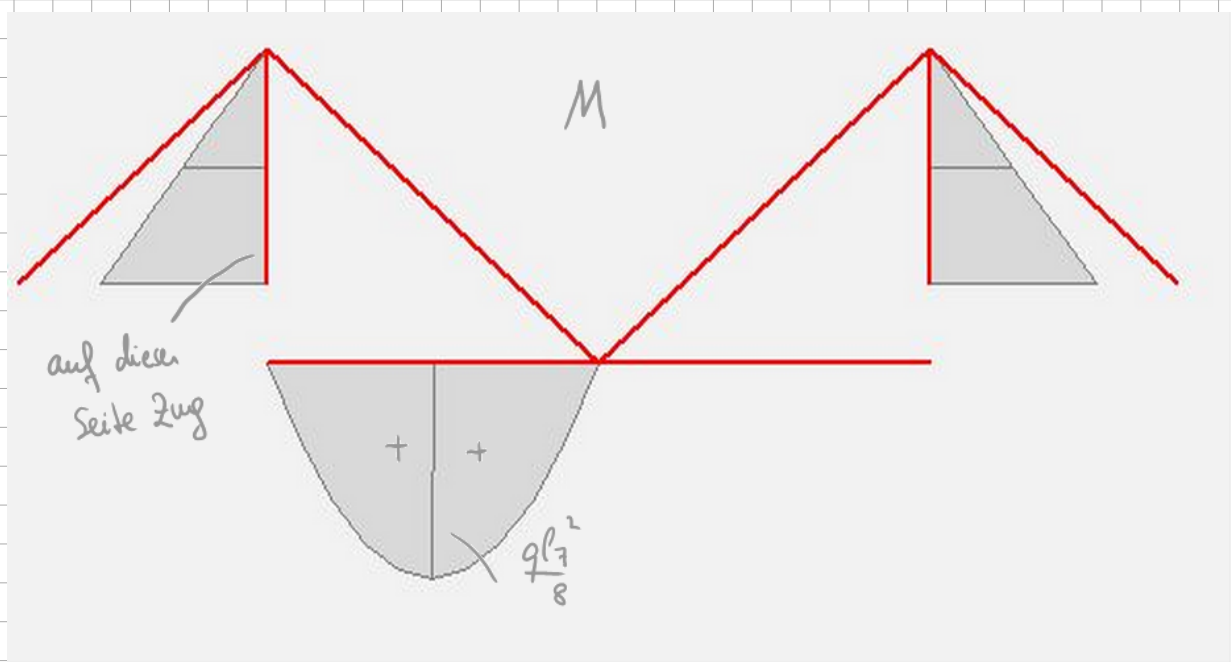
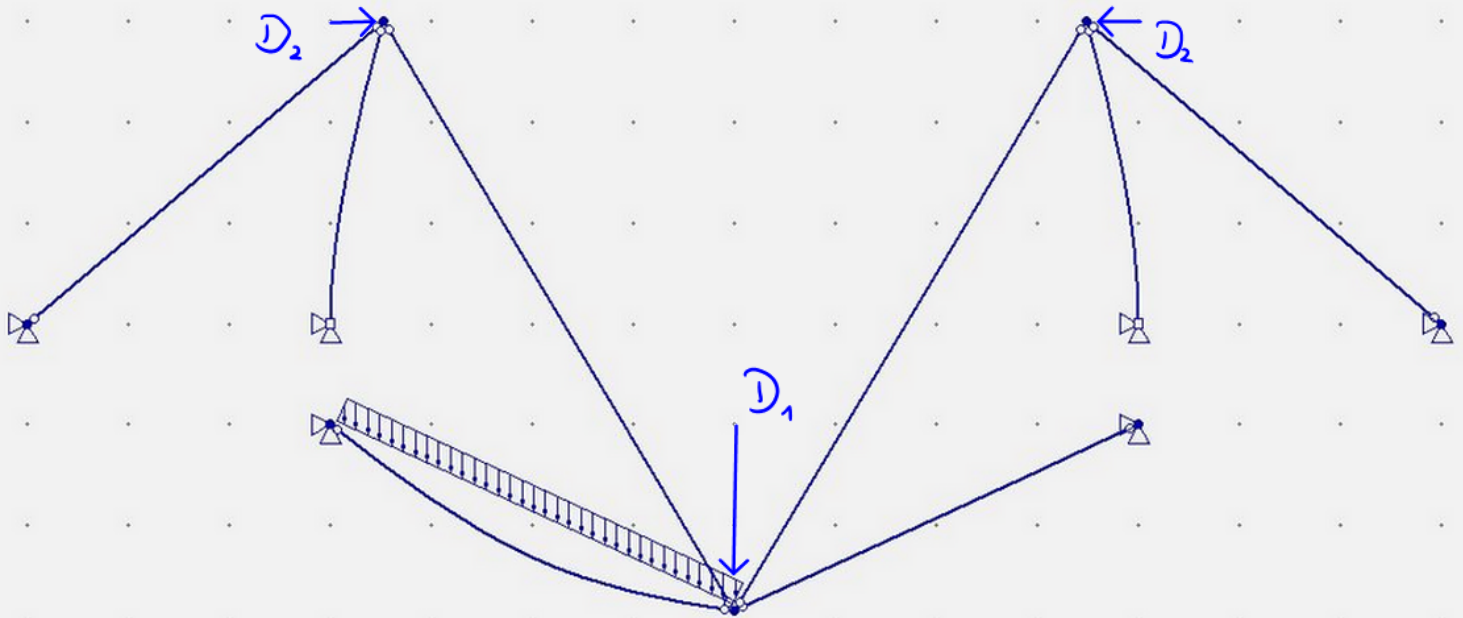
$$\underline{K} = \begin{bmatrix} 176,777 & -176,777 \\ -176,777 & 634,7 \end{bmatrix}$$

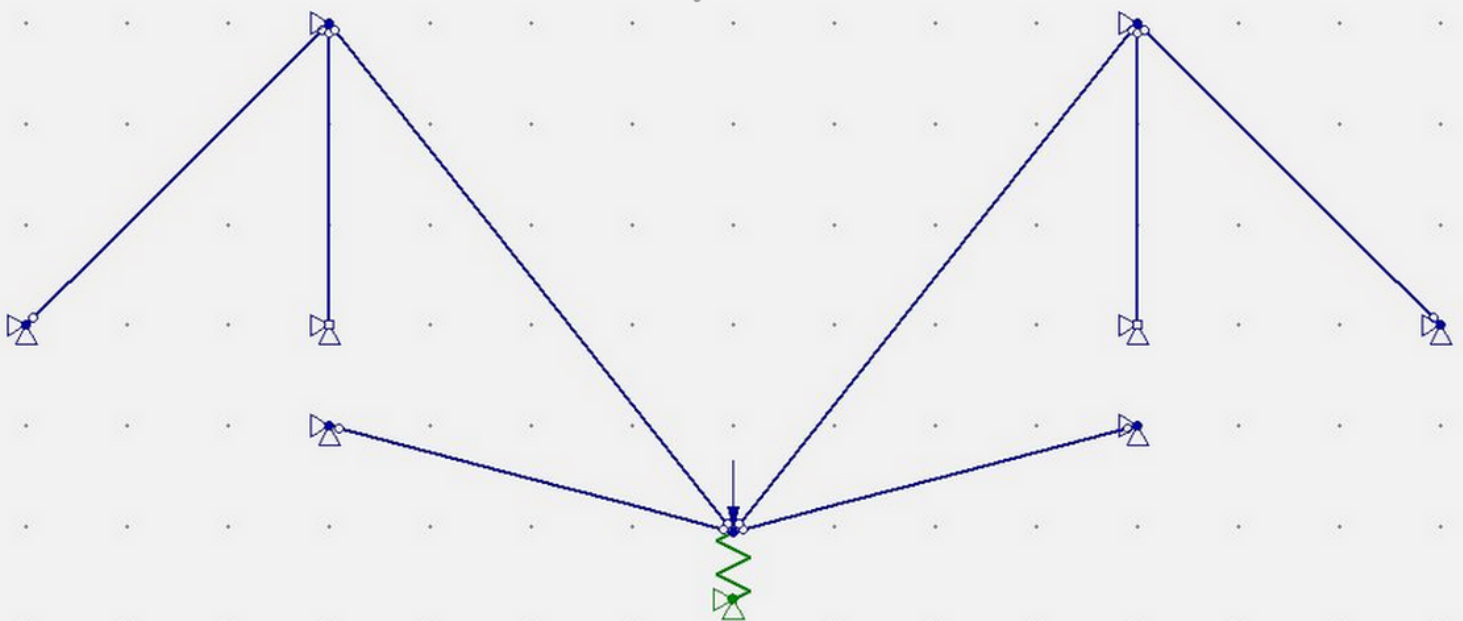
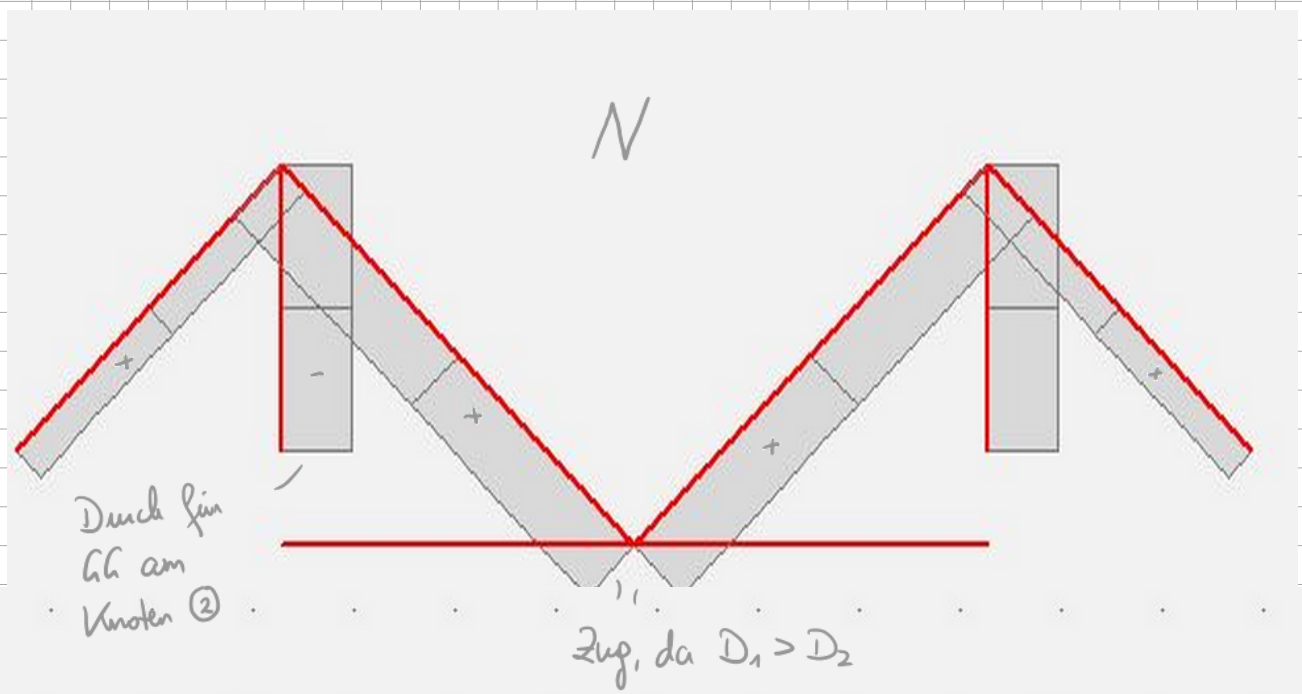
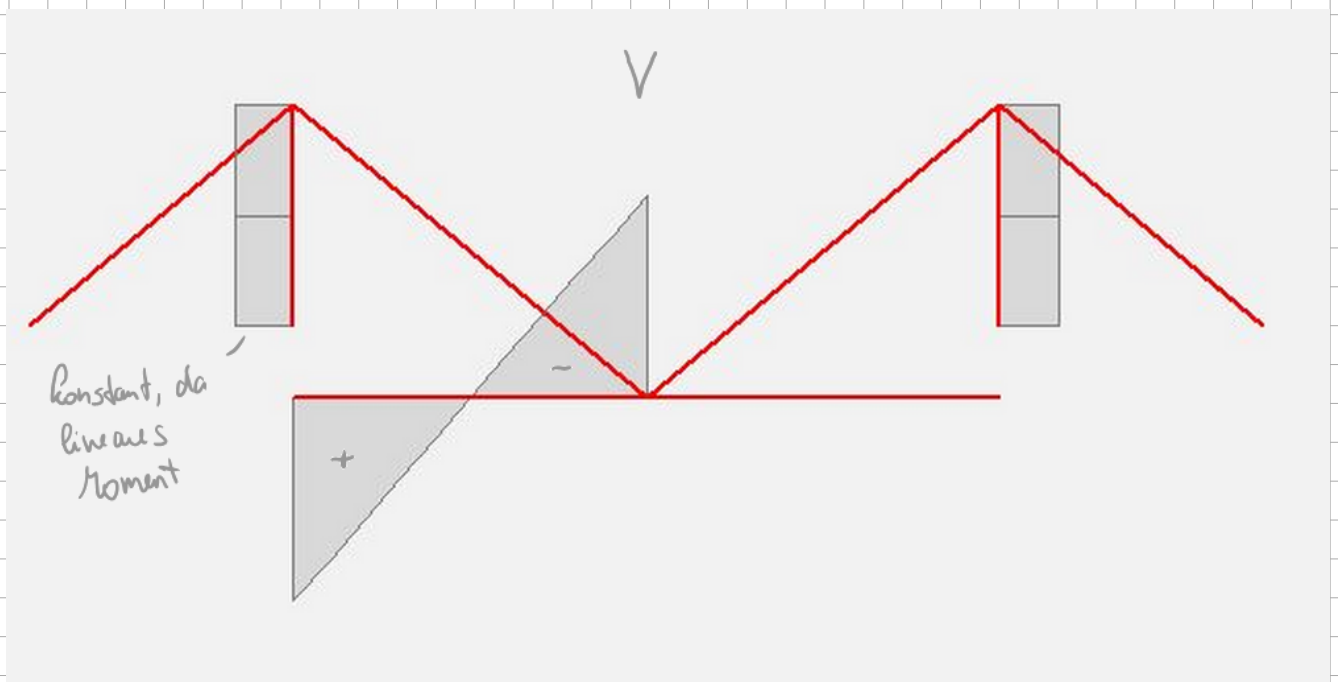
$$\underline{F} = -\underline{K}_0 \cdot \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F}$$

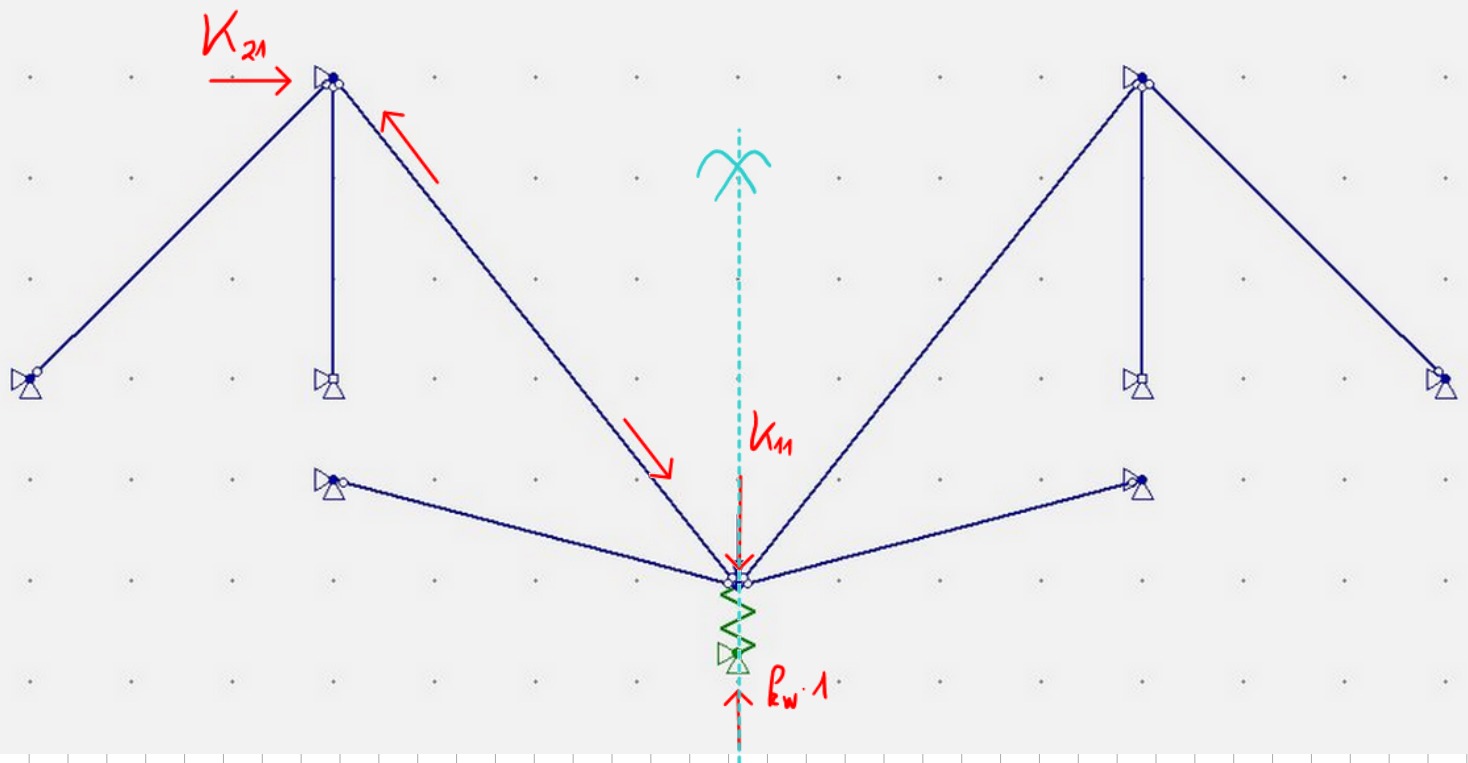
$$\Rightarrow \underline{u} = \begin{bmatrix} 1,56 \cdot 10^{-1} \text{ m} \\ 4,37 \cdot 10^{-2} \text{ m} \end{bmatrix}$$

(b)





(c) Feder geht nur in E2 1 ein



$$K_m = 176,777 + k_w$$

Gleichungssystem:

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \begin{bmatrix} 176,777 + k_w & -176,777 \\ -176,777 & 634,7 \end{bmatrix} \begin{bmatrix} 0,075 \\ D_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

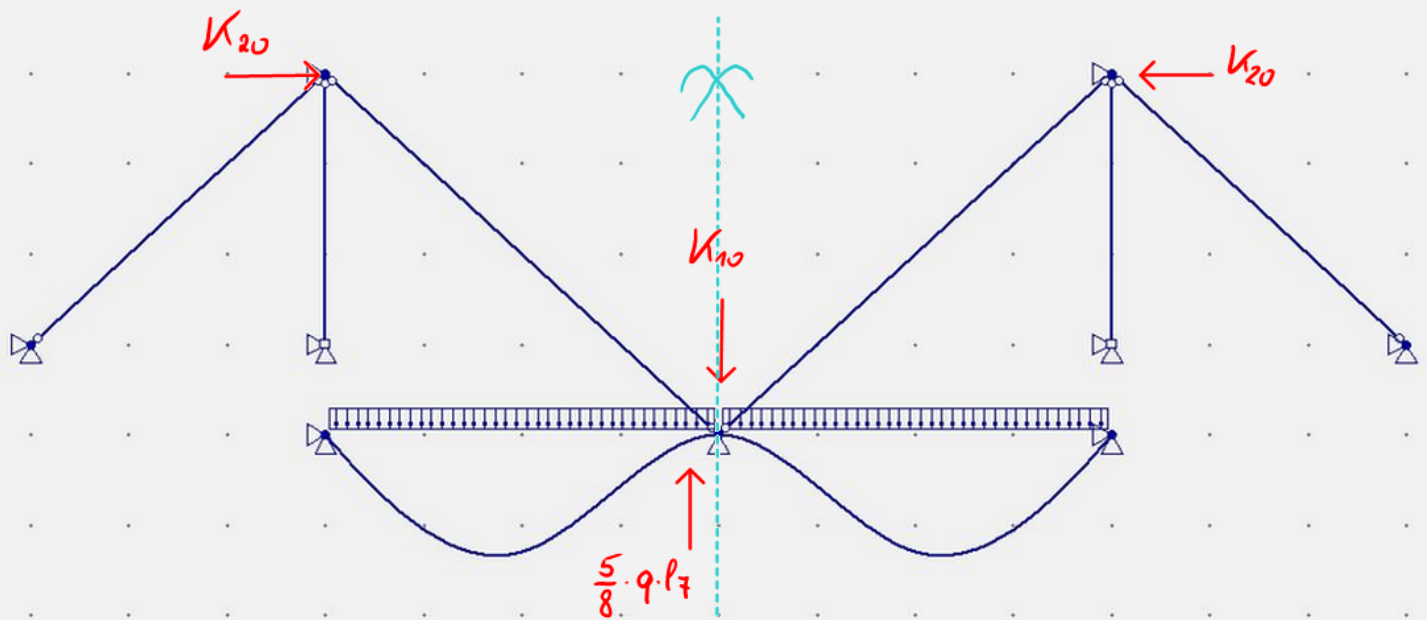
aus II folgt: $-176,777 \cdot 0,075 + 634,7 \cdot D_2 = 0$

$$\Rightarrow D_2 = 2,08 \cdot 10^{-2} \text{ m}$$

aus I folgt damit: $(176,777 + k_w) \cdot 0,075 - 176,777 \cdot 2,08 \cdot 10^{-2} = 20$

$$\Rightarrow k_w = \underline{\underline{139,13 \frac{\text{N}}{\text{m}}}}$$

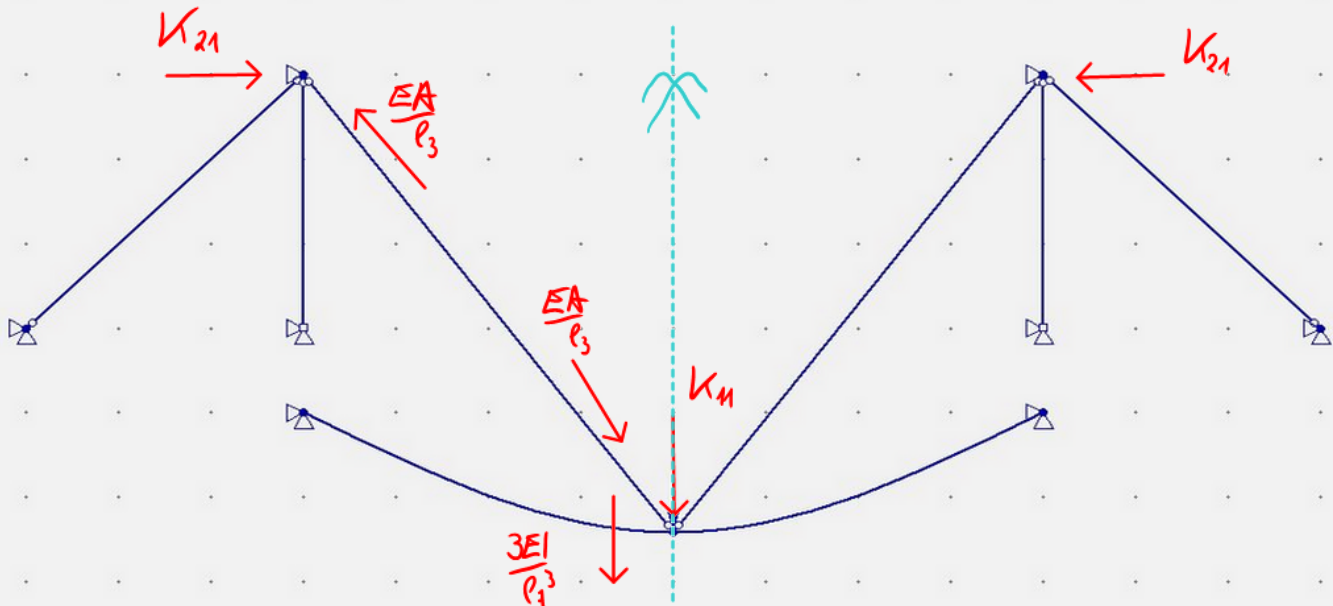
(d) neues System: LZ



$$K_{10} = -\frac{5}{8} \cdot q \cdot l_7 \cdot 2 = -25 \cdot 2 = -50$$

$$K_{20} = 0$$

E2 1



$$K_{11} = \left(\frac{EA}{l_3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{3EI}{l_1^3} \right) \cdot 2 = (88,388 + 46,875) \cdot 2 = 270,526$$

$$K_{21} = -\frac{EA}{l_3} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 2 = -88,388 \cdot 2 = -176,777$$

E2 2 bleibt unverändert

$$\Rightarrow K_{12} = -176,777$$

$$K_{22} = 634,7$$

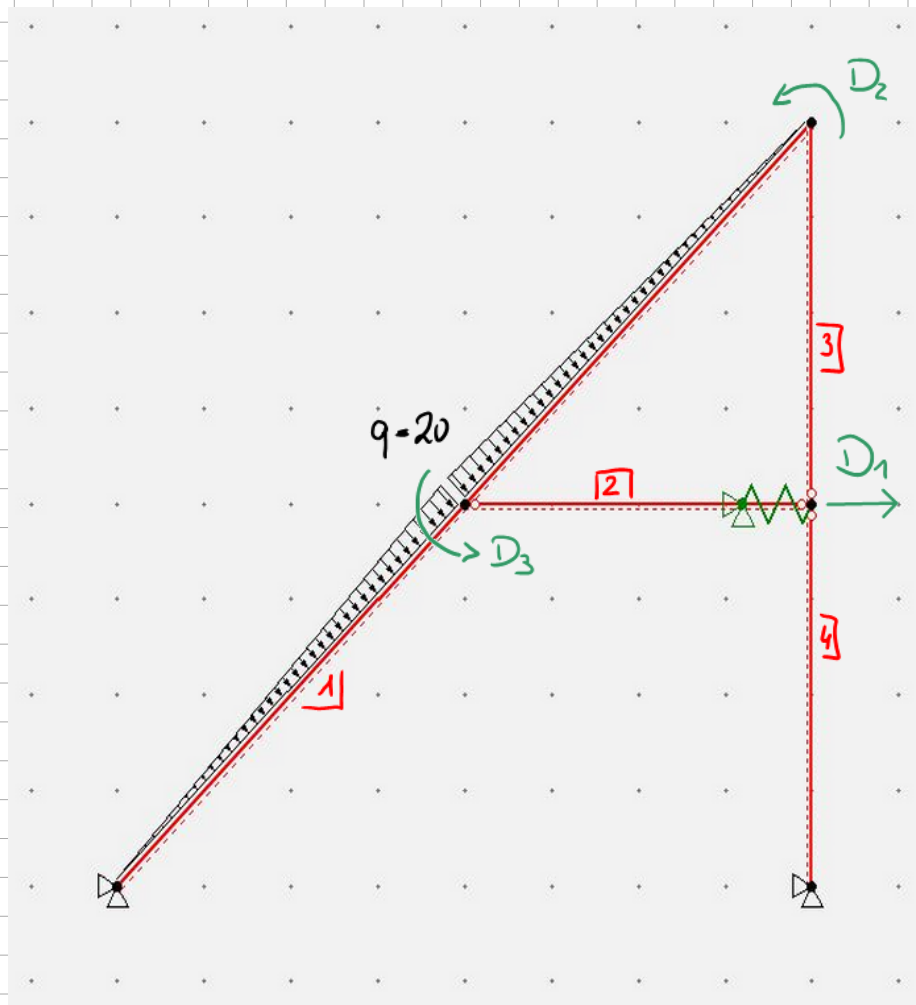
Kein weiteres E2 erforderlich, da die Verschiebung am Knoten ⑥ aufgrund der Symmetrie 0 ist!

$$\underline{K} = \begin{bmatrix} 270,527 & -176,777 \\ -176,777 & 634,7 \end{bmatrix} \quad \underline{F} = -\underline{K}_0 = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F}$$

$$\Rightarrow \underline{u} = \begin{bmatrix} 2,26 \cdot 10^{-1} \text{ m} \\ 6,29 \cdot 10^{-2} \text{ m} \end{bmatrix}$$

Probeklausur 3 - Aufgabe 2

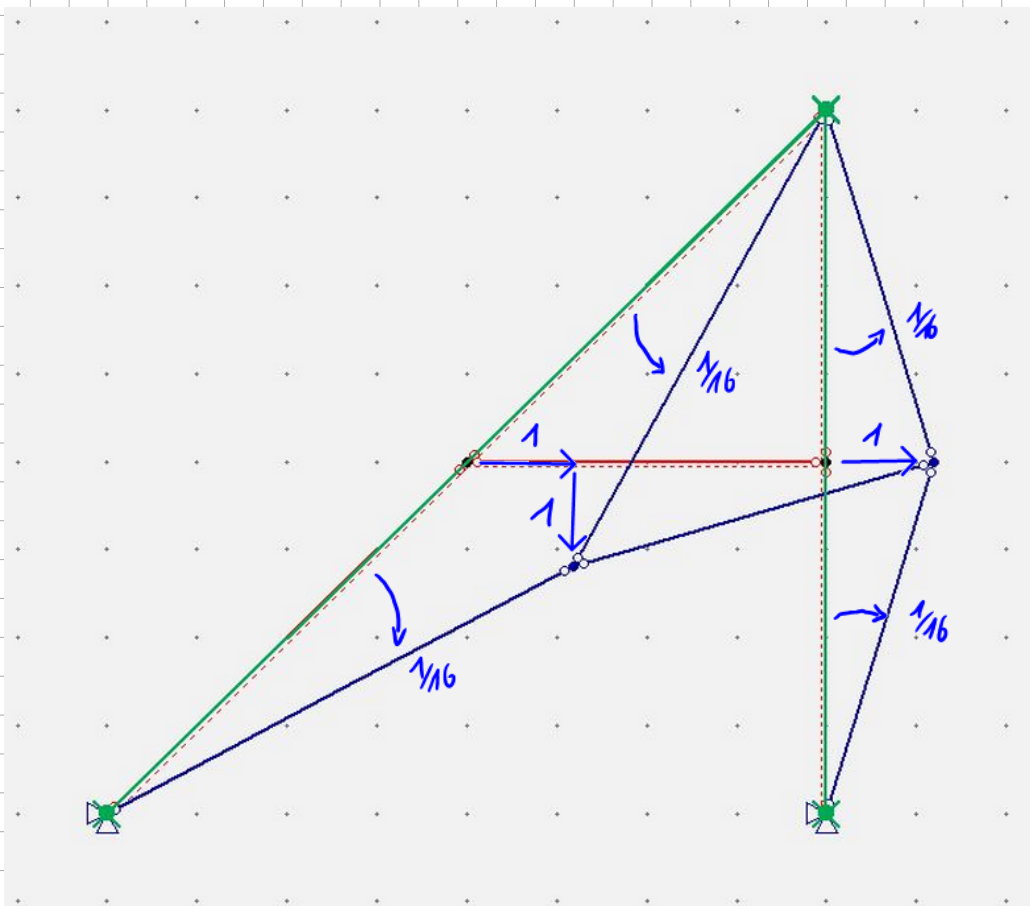


$$EA \rightarrow \infty$$

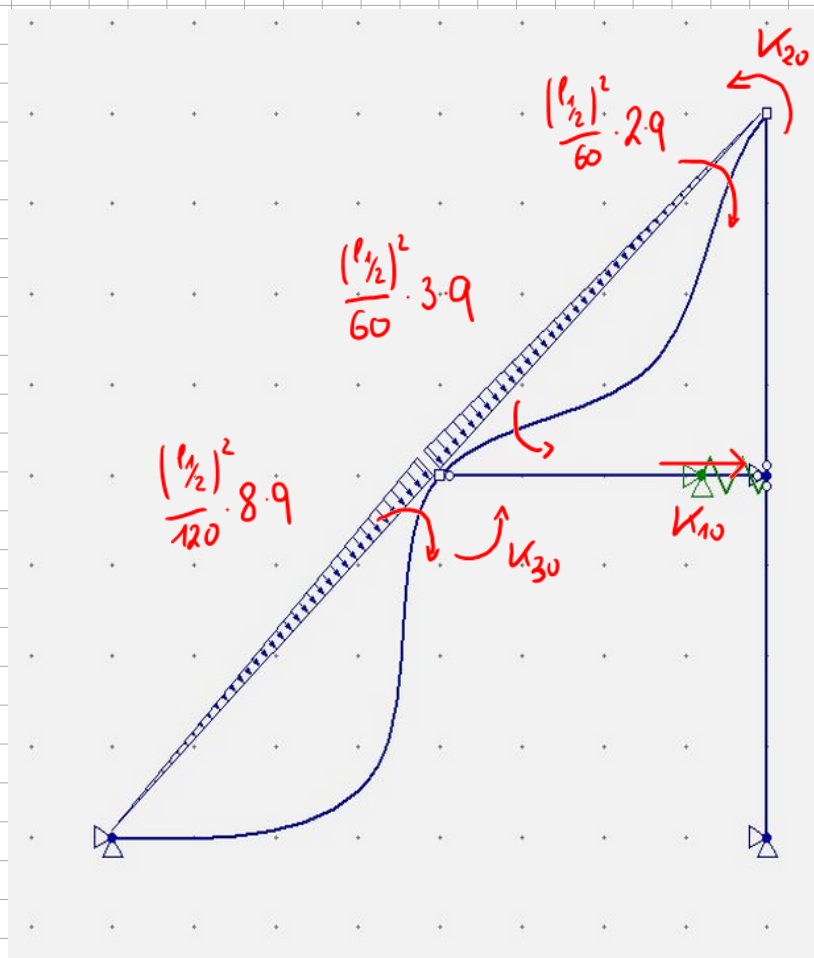
$$EI = 10000 \text{ Nm}^2$$

$$k_w = 7500 \text{ N/m}$$

Gelenkfigur



L2



$$P.V.: K_{10} \cdot \bar{1} + \frac{(\frac{l}{2})^2}{120} \cdot 8q \cdot \frac{\bar{1}}{16} + \frac{(\frac{l}{2})^2}{60} \cdot 3q \cdot \frac{\bar{1}}{16} -$$

$$- \frac{(\frac{l}{2})^2}{60} \cdot 2q \cdot \frac{\bar{1}}{16} + \frac{1}{2} \cdot q \cdot (\frac{l}{2}) \cdot \frac{2}{3} \cdot \sqrt{2} \cdot 2$$

dreieckige Last
Resultierende greift
im Drittelpunkt an
obere und
untere Hlfte
der Last gleich

Rueverschiebung des Knoten ②

$$K_{10} = -42,6 - 32 + 21,3 - 213,3 \cdot 2 =$$

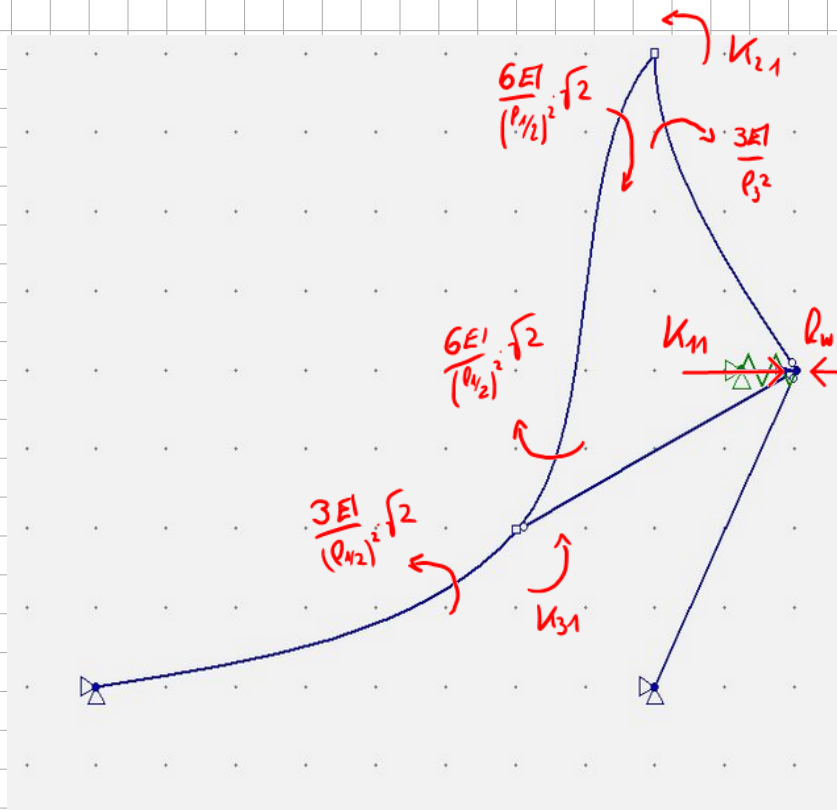
$$= -480$$

$$G.G.: K_{20} = - \frac{(\frac{l}{2})^2}{60} \cdot 2q = -341,3$$

$$K_{30} = \frac{(\frac{l}{2})^2}{60} \cdot 3q - \frac{(\frac{l}{2})^2}{120} \cdot 8q =$$

$$= 512 - 682,6 = -170,6$$

B2 1



$$P.V.: K_{11} \cdot \bar{1} - q \cdot \bar{1} - \frac{3EI}{(\frac{l}{2})^2} \cdot \sqrt{2} \cdot \frac{\bar{1}}{16} - \frac{6EI}{(\frac{l}{2})^2} \cdot \sqrt{2} \cdot \frac{\bar{1}}{16} \cdot 2 -$$

$$- \frac{3EI}{l^2} \cdot \frac{\bar{1}}{16} = 0$$

$$K_{11} = 7500 + 5,179 + 10,358 \cdot 2 + 7,324$$

$$= 7533,219$$

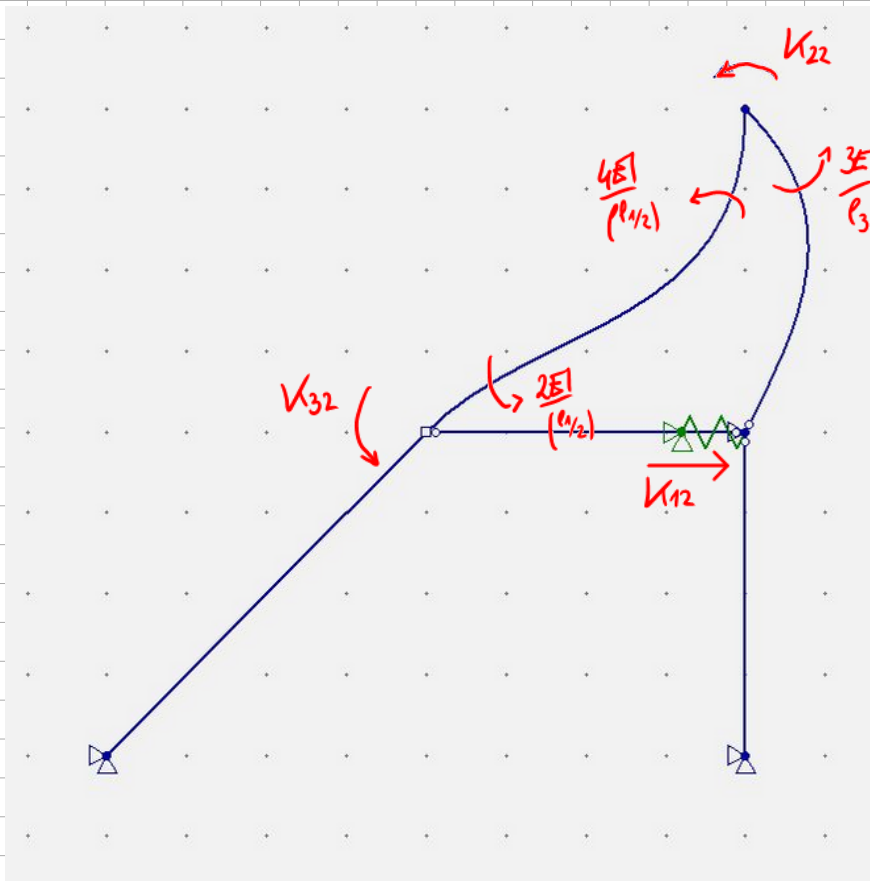
$$G.G.: K_{21} = - \frac{3EI}{l^2} - \frac{6EI}{(\frac{l}{2})^2} \cdot \sqrt{2} = -117,188 - 165,728 =$$

$$= -282,916$$

$$K_{31} = \frac{3EI}{(\frac{l}{2})^2} \cdot \sqrt{2} - \frac{6EI}{(\frac{l}{2})^2} \cdot \sqrt{2} = 82,864 - 165,728 =$$

$$= -82,864$$

E2 2



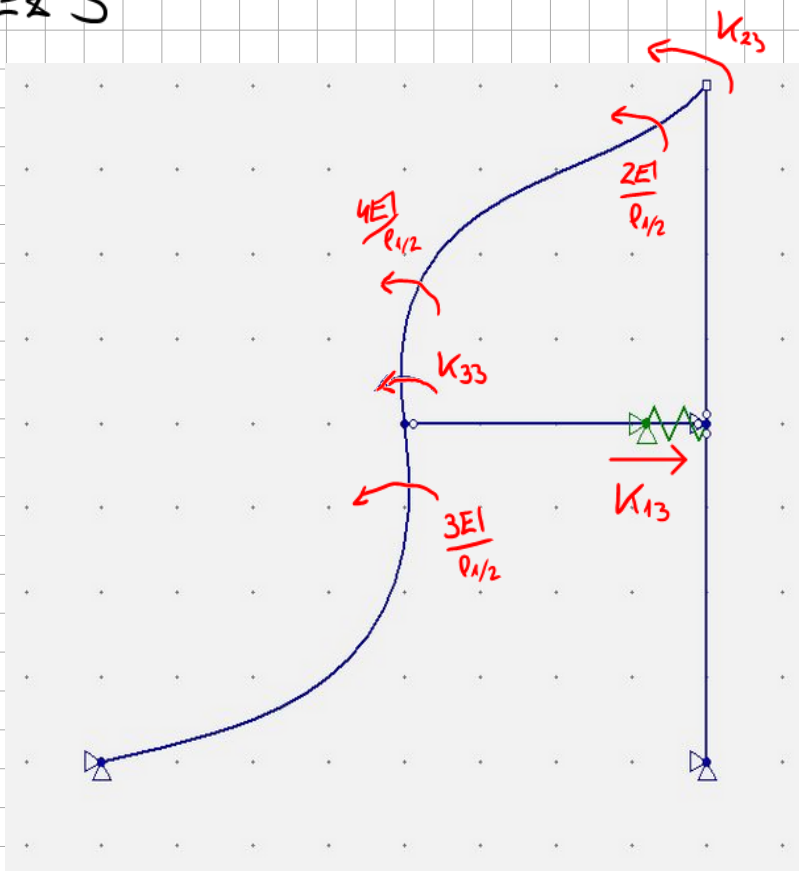
$$P.V.: K_{12} \cdot \bar{1} + \left(\frac{4EI}{l_{1/2}} + \frac{2EI}{l_{1/2}} \right) \cdot \frac{\bar{1}}{16} + \frac{3EI}{l_3} \cdot \frac{\bar{1}}{16} = 0$$

$$K_{12} = -165,728 - 117,188 = -282,916$$

$$G.G.: K_{22} = \frac{4EI}{l_{1/2}} + \frac{3EI}{l_3} = 1767,767 + 1875 = 3642,767$$

$$K_{33} = \frac{2EI}{l_{1/2}} = 883,883$$

E2 3



$$P.V.: K_{13} \cdot \bar{1} + \frac{2EI}{l_{1/2}} \cdot \frac{\bar{1}}{16} + \frac{4EI}{l_{1/2}} \cdot \frac{\bar{1}}{16} - \frac{3EI}{l_{1/2}} \cdot \frac{\bar{1}}{16} = 0$$

$$K_{13} = -55,243 - 110,485 + 82,864 = -82,864$$

$$G.G.: K_{23} = \frac{2EI}{l_{1/2}} = 883,883$$

$$K_{33} = \frac{4EI}{l_{1/2}} + \frac{3EI}{l_{1/2}} = 1767,767 + 1325,825 = 3093,592$$

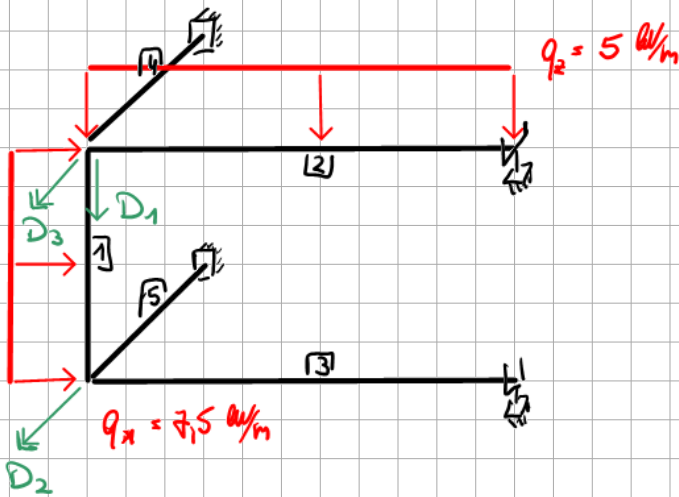
$$\underline{\underline{K}} = \begin{bmatrix} 7533,219 & -282,916 & -82,865 \\ -282,916 & 3642,767 & 883,883 \\ -82,865 & 883,883 & 3093,59 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_b = \begin{bmatrix} 480 \\ 341,3 \\ 170,6 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}}$$

$$\Rightarrow \underline{\underline{u}} = \begin{bmatrix} 6,75 \cdot 10^{-2} \text{ m} \\ 9,15 \cdot 10^{-2} \text{ rad} \\ 3,08 \cdot 10^{-2} \text{ rad} \end{bmatrix}$$

Probeklausur 3 - Aufgabe 3



$$EI = 10\,000 \text{ kNm}^2$$

$$GI_{T,2,3} \rightarrow \infty$$

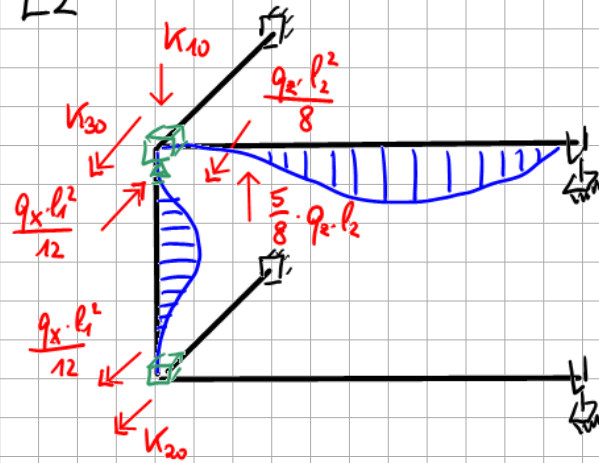
$$GI_{T, \text{Rost}} = 5\,000 \text{ kNm}^2$$

$$EA \rightarrow \infty$$

$$q_2 = 5 \text{ kN/m}$$

$$q_x = 7,5 \text{ kN/m}$$

L2

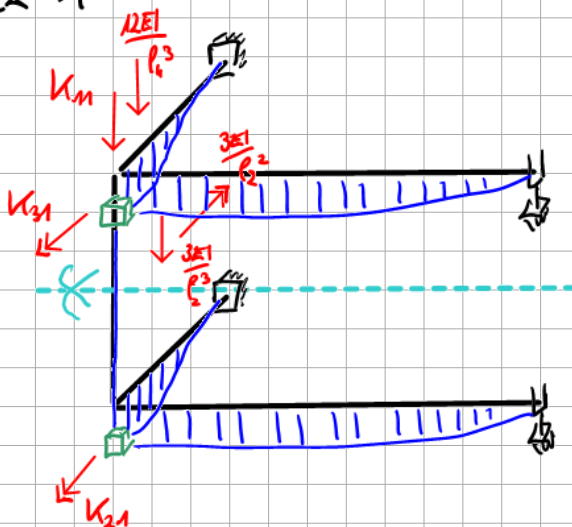


$$K_{10} = -\frac{5}{8} \cdot q_2 \cdot l_2^2 = -9,375$$

$$K_{20} = \frac{q_x \cdot l_1^2}{12} = 1,406$$

$$K_{30} = \frac{q_2 \cdot l_2^2}{8} - \frac{q_x \cdot l_1^2}{12} = 5,625 - 1,406 = 4,219$$

Ex 1



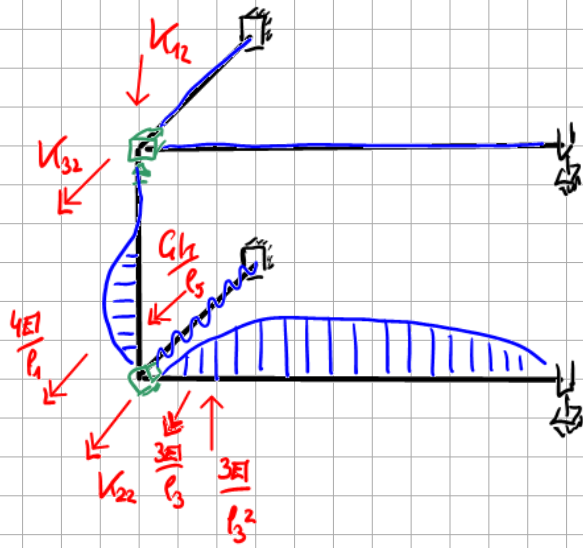
Symmetrie

$$K_{10} = 2 \cdot \left[\frac{12EI}{l_1^3} + \frac{3EI}{l_2^3} \right] = 2 \cdot [35\,555,5 + 1\,111,1] = 73\,333,3$$

$$K_{20} = -\frac{3EI}{l_2^2} = -3\,333,3$$

$$K_{30} = -\frac{3EI}{l_2^2} = -3\,333,3$$

E2 2

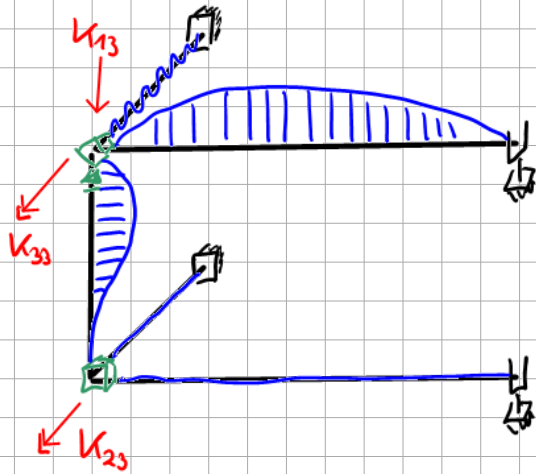


$$K_{12} = -\frac{3EI}{l_3^2} = -3\,333,3$$

$$K_{32} = \frac{4EI}{l_1} + \frac{6EI}{l_3} + \frac{3EI}{l_3^2} = 26\,666,6 + 3\,333,3 + 10\,000 = 40\,000$$

$$K_{33} = \frac{2EI}{l_1} = 13\,333,3$$

E2 3



Symmetrisches System \rightarrow E2 2 & E2 3 sind symmetrisch

$$\Rightarrow K_{13} = K_{12} = -3\,333,3$$

$$K_{23} = K_{32} = 13\,333,3$$

$$K_{33} = K_{22} = 40\,000$$

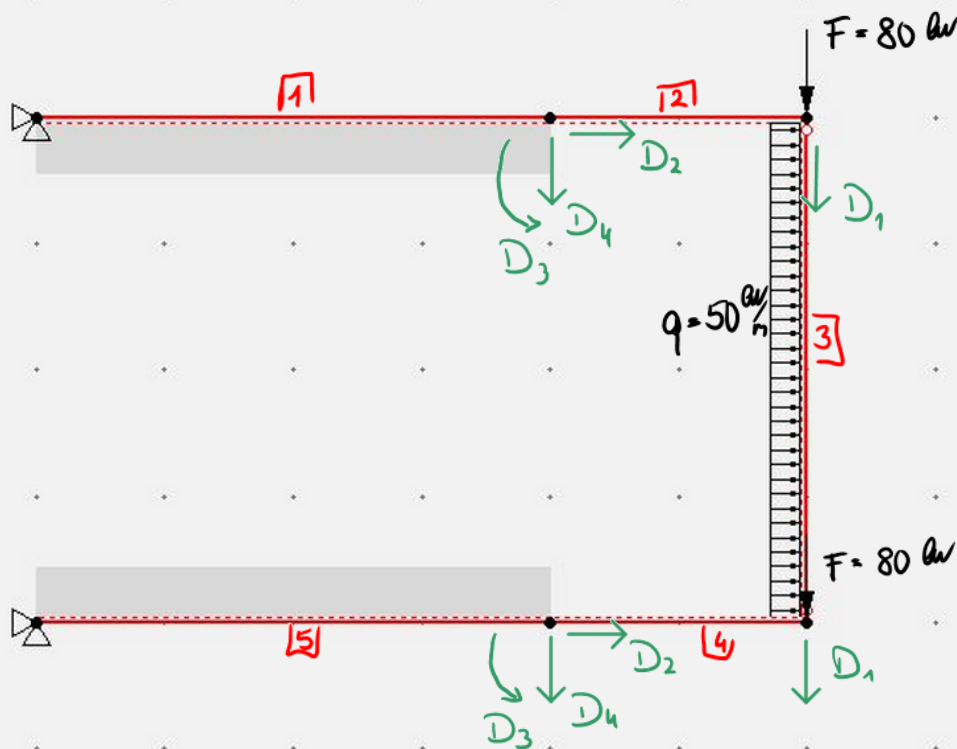
$$\underline{K} = \begin{bmatrix} 73\,333,3 & -3\,333,3 & -3\,333,3 \\ -3\,333,3 & 40\,000 & 13\,333,3 \\ -3\,333,3 & 13\,333,3 & 40\,000 \end{bmatrix} \quad \underline{F} = -\underline{K}_0 = \begin{bmatrix} 9,375 \\ -1,406 \\ -4,219 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \Rightarrow \underline{u} = \begin{bmatrix} 1,237 \cdot 10^{-4} \text{ m} \\ 7,737 \cdot 10^{-6} \text{ rad} \\ -9,774 \cdot 10^{-5} \text{ rad} \end{bmatrix}$$

Probeklausur 3 - Aufgabe 4

$$\lambda = \sqrt[4]{\frac{12000}{4 \cdot 5000}} = 0,880$$

$$\lambda \cdot l = 0,88 \cdot 4 = 3,52 > \pi \rightarrow \infty \text{ langer Balken}$$



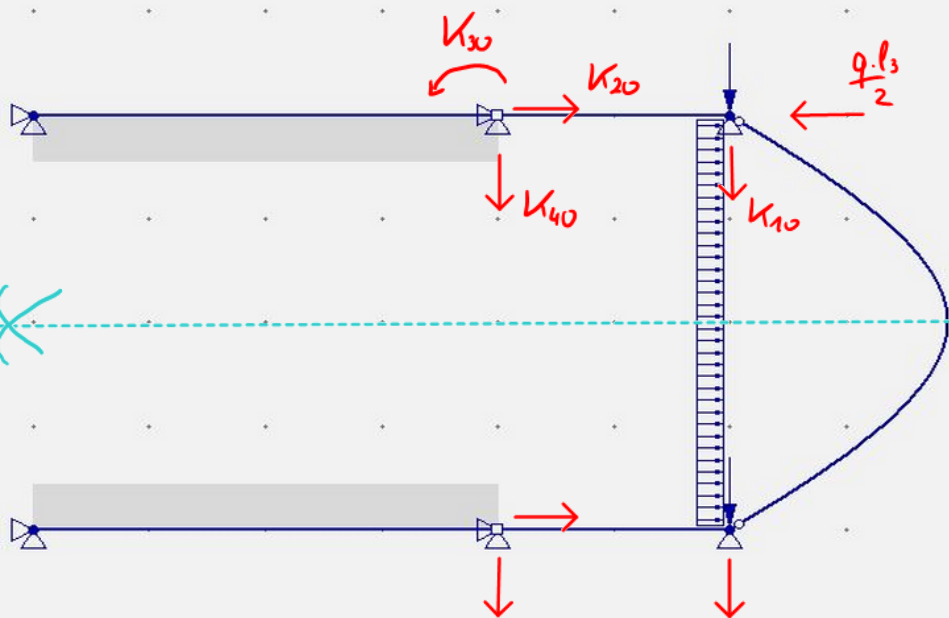
$$EI = 5000 \text{ kNm}^2$$

$$EA_{1,5} = 50000 \text{ kN}$$

$$EA_{2,3,4} \rightarrow \infty$$

$$k = 12000 \text{ kN/m}^2$$

L2



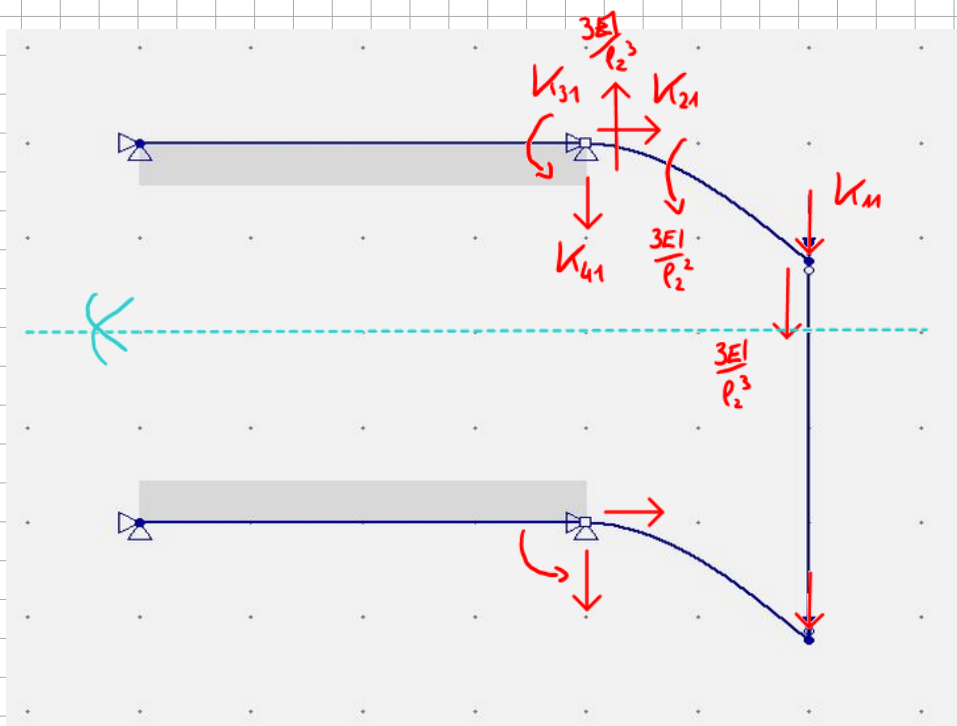
$$K_{10} = -80$$

$$K_{20} = -\frac{q \cdot l^3}{2} = -100$$

$$K_{30} = 0$$

$$K_{40} = 0$$

E2 1



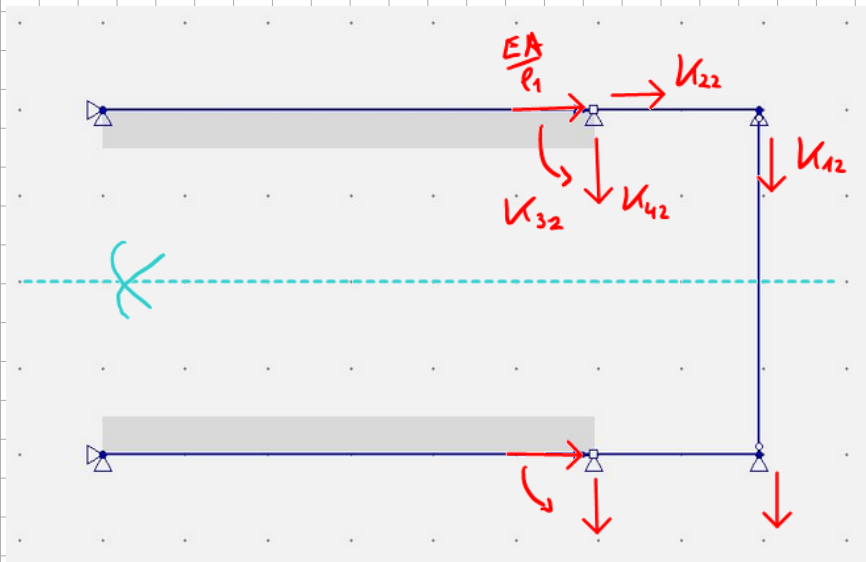
$$K_{11} = \frac{3EI}{l_2^3} = 1875$$

$$K_{21} = 0$$

$$K_{31} = \frac{3EI}{l_2^2} = 3750$$

$$K_{41} = -\frac{3EI}{l_2^3} = -1875$$

E2 2



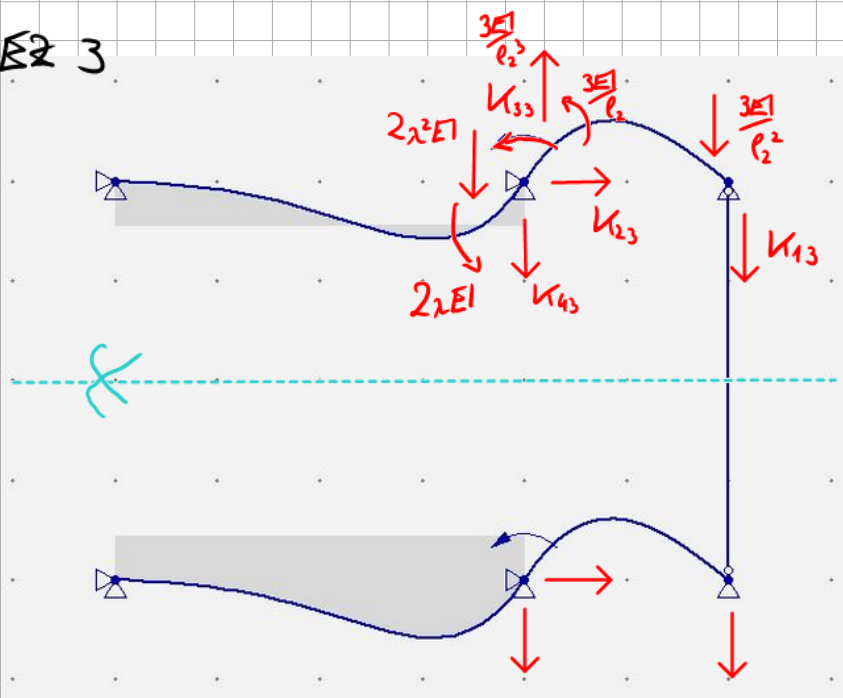
$$K_{12} = 0$$

$$K_{22} = \frac{EA}{l_1} = 12500$$

$$K_{32} = 0$$

$$K_{42} = 0$$

E2 3



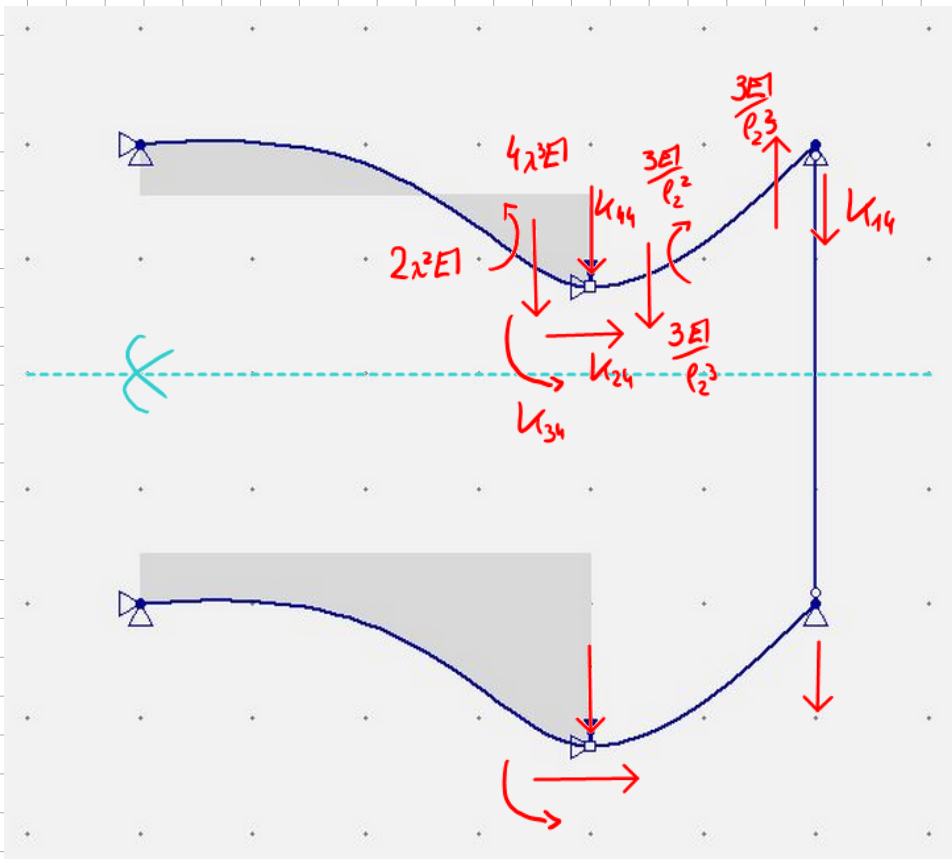
$$K_{13} = \frac{3EI}{l_2^2} = 3750$$

$$K_{23} = 0$$

$$K_{33} = 2\lambda^2 EI + \frac{3EI}{l_2} = 8801,17 + 7500 = 16301,17$$

$$K_{43} = 2\lambda^2 EI - \frac{3EI}{l_2} = 7745,967 - 3750 = -3995,957$$

E2 4



$$K_{14} = -\frac{3EI}{l_2^3} \cdot -1875$$

$$K_{24} = 0$$

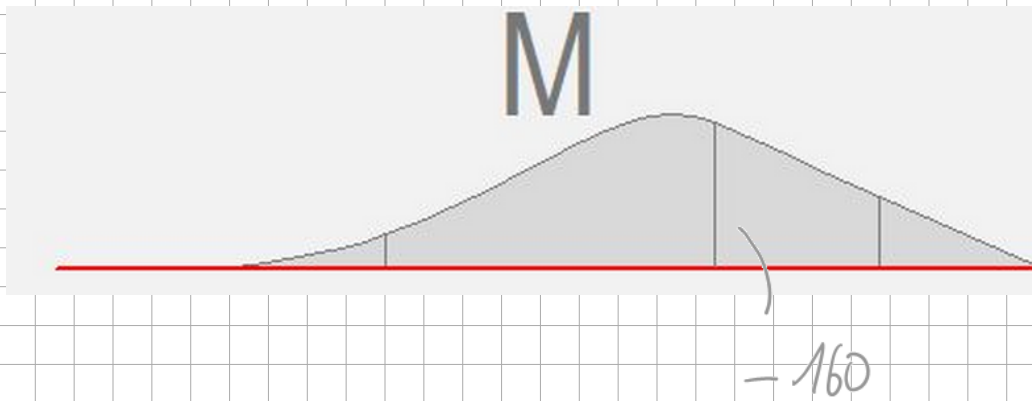
$$K_{34} = 4\lambda^2 EI + \frac{3EI}{l_2^3} = 13634,63 + 1875 = 15509,632$$

$$K_{44} = 2\lambda^2 EI - \frac{3EI}{l_2^3} = 7745,967 - 3750 = 3995,967$$

$$\underline{K} = \begin{bmatrix} 1875 & 0 & 3750 & -1875 \\ 0 & 12600 & 0 & 0 \\ 3750 & 0 & 16301,117 & 3995,967 \\ -1875 & 0 & 3995,967 & 15509,632 \end{bmatrix} \quad \underline{F} = -\underline{K}_0 = \begin{bmatrix} 80 \\ 100 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F}$$

$$\Rightarrow \underline{u} = \begin{bmatrix} 1,68 \cdot 10^{-1} \text{ m} \\ 8 \cdot 10^{-3} \text{ m} \\ -4,67 \cdot 10^{-2} \text{ rad} \\ 3,24 \cdot 10^{-2} \text{ m} \end{bmatrix}$$

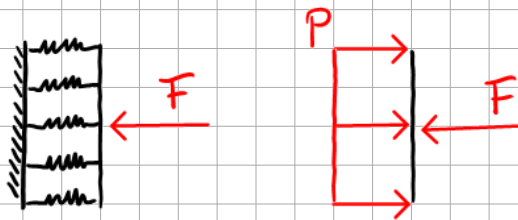


(b)

$$\lambda = \sqrt[4]{\frac{12\,000}{4 \cdot 100\,000}} = 0,416$$

$$\lambda \cdot l = 0,416 \cdot 0,5 = 0,2 < \frac{\pi}{2}$$

\Rightarrow sehr kurzer Balken, keine Staukörperrotation

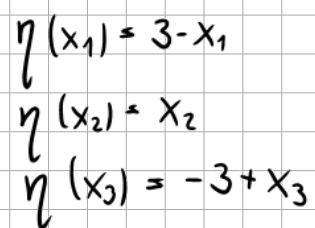
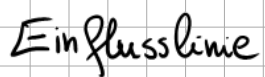


$$P = \frac{F}{l} = \frac{1\,000}{0,5\,\text{m}} = 2\,000\,\text{N/m}$$

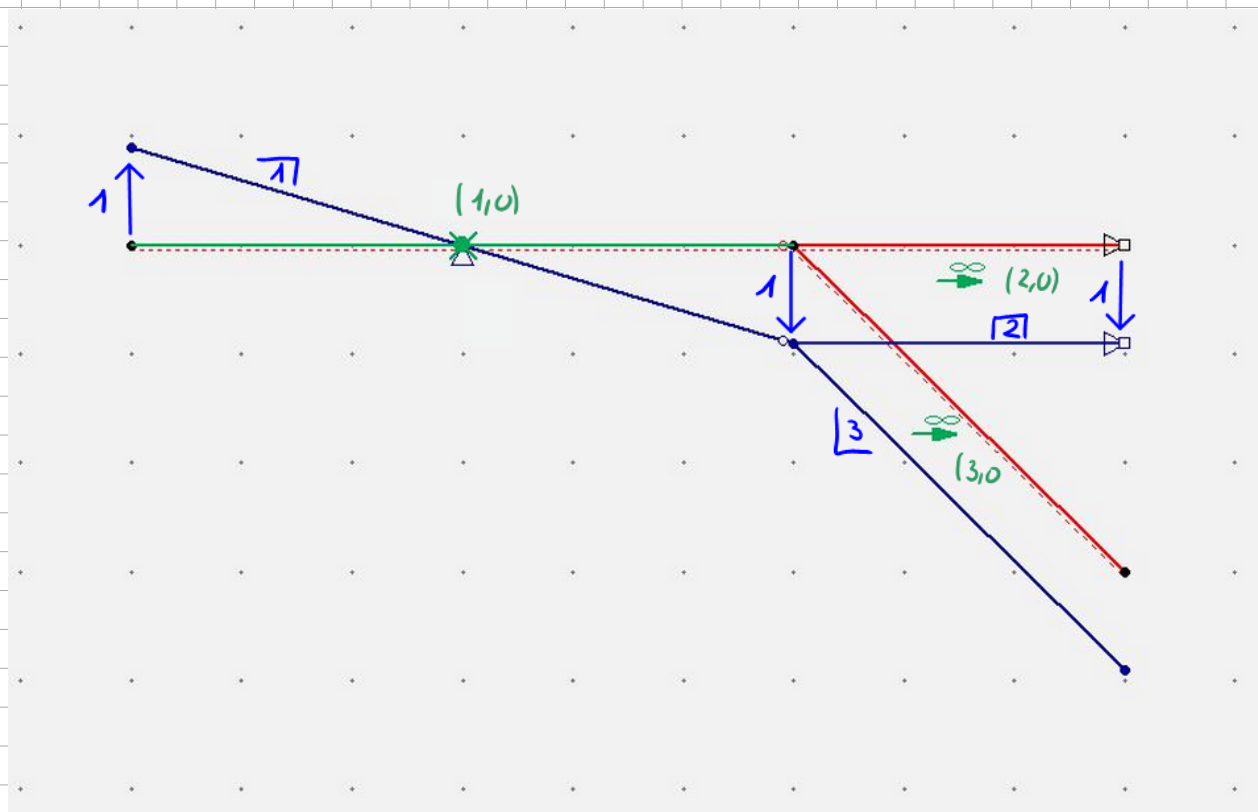
$$W = \frac{P}{k} = \frac{2\,000\,\text{N/m}}{12\,000\,\text{N/m}^2} = 1,6 \cdot 10^{-4}\,\text{m}$$

$$k_w = \frac{F}{W} = \frac{1\,000}{1,6 \cdot 10^{-4}\,\text{m}} = 6\,250\,000\,\text{N/m}$$

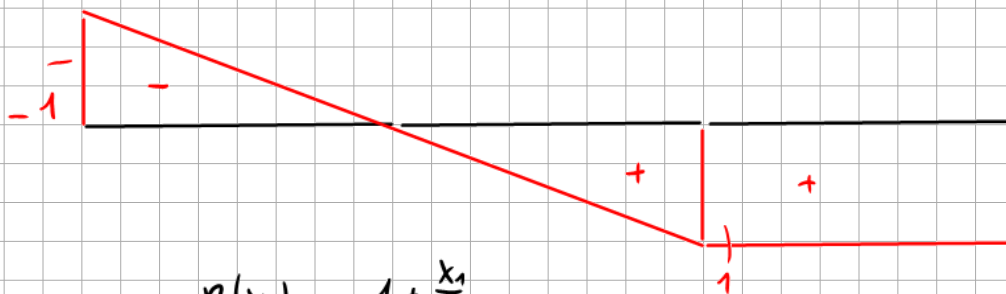
(a)



(b)



Einflusslinie

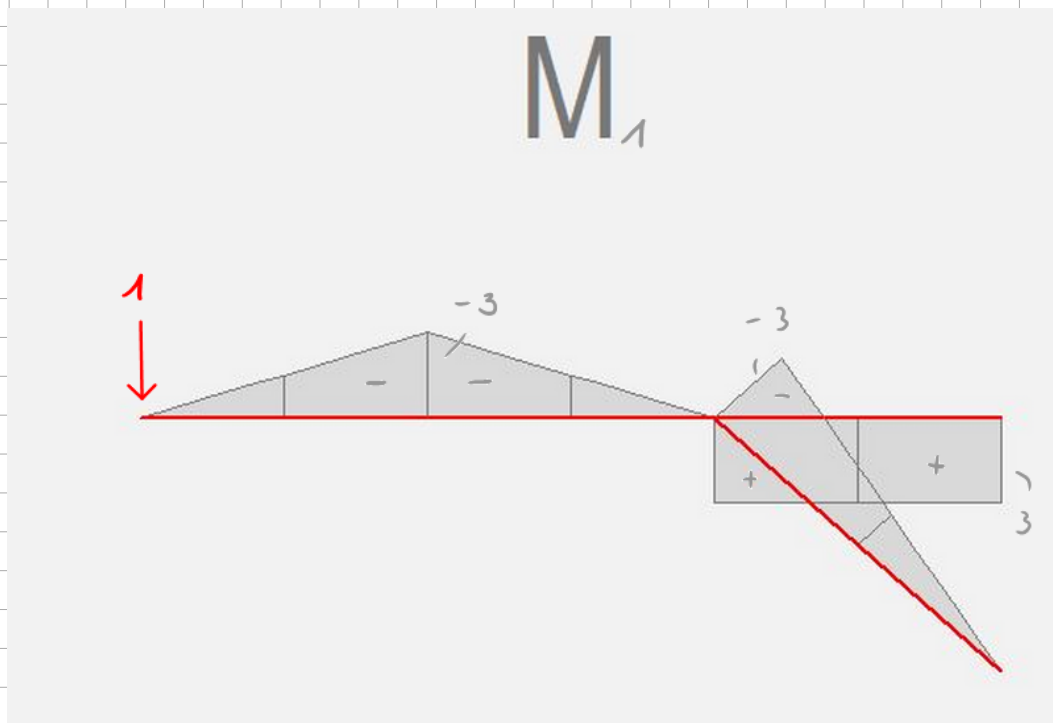


$$\eta(x_1) = -1 + \frac{x_1}{3}$$

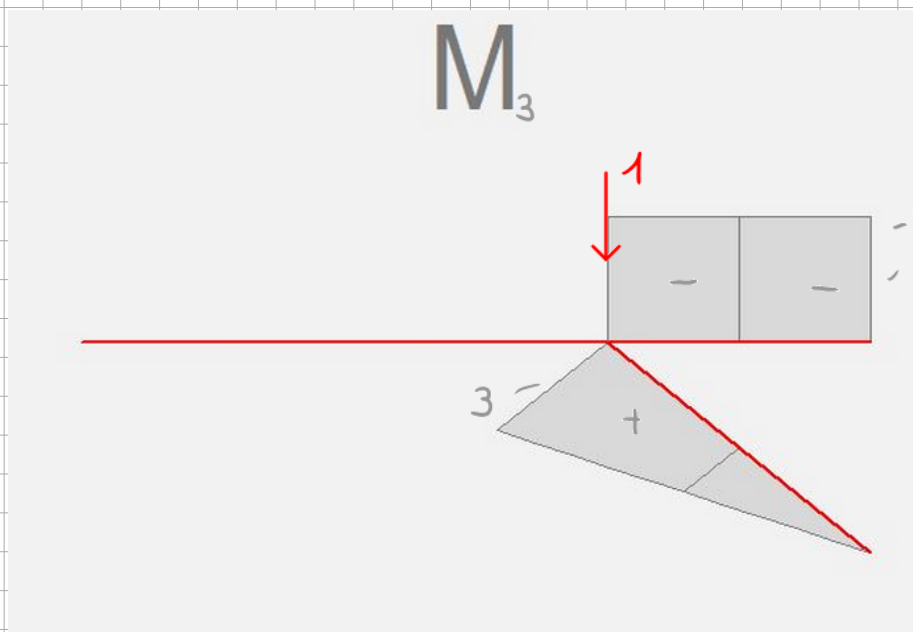
$$\eta(x_2) = \frac{x_2}{3}$$

$$\eta(x_3) = 1$$

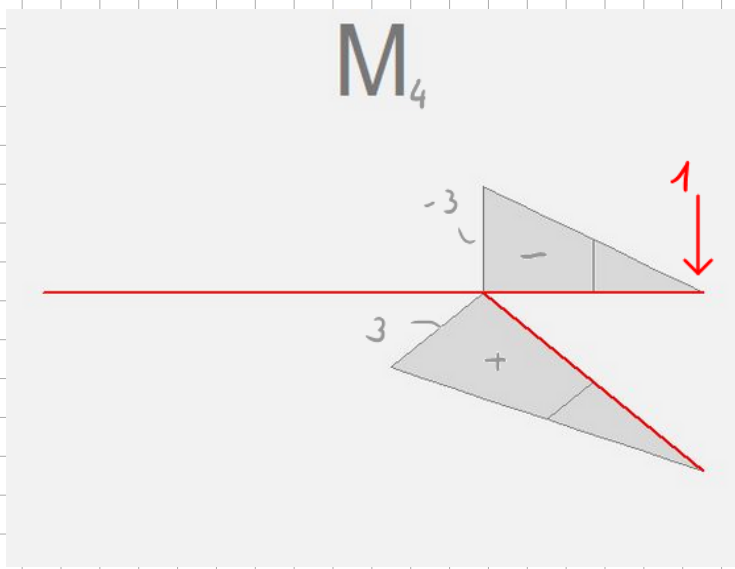
(c)



$$W_1 = \frac{1}{3} \cdot (-3)^2 \cdot \frac{3}{EI} \cdot 2 + 3^2 \cdot \frac{3}{EI} + \frac{1}{3} \cdot (-3)^2 \cdot \frac{3\sqrt{2}}{EI} = 0,072 \text{ m}$$

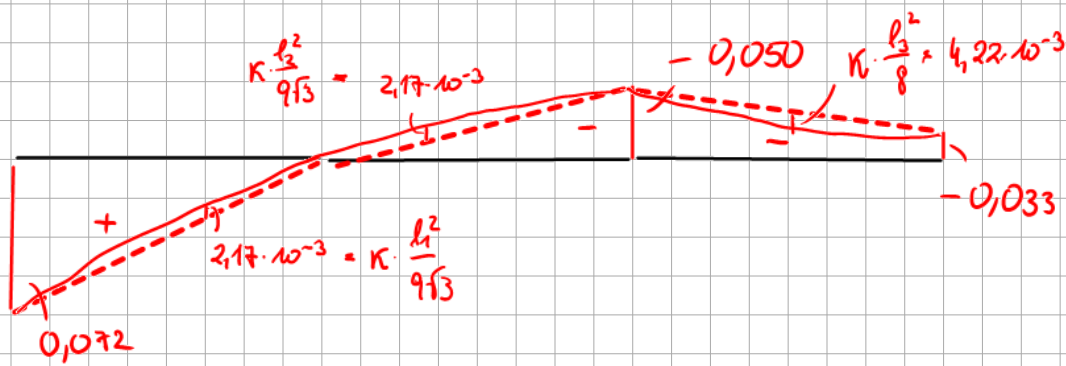


$$W_3 = (-3) \cdot 3 \cdot \frac{3}{EI} + \frac{1}{3} \cdot 3 \cdot (-3) \cdot \frac{3\sqrt{2}}{EI} = -0,050 \text{ m}$$



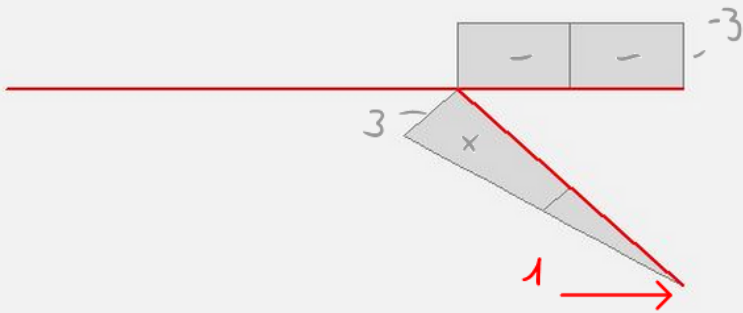
$$W_4 = \frac{1}{2} \cdot 3 \cdot (-3) \cdot \frac{3}{EI} + \frac{1}{3} \cdot 3 \cdot (-3) \cdot \frac{3\sqrt{2}}{EI} = 0,033n$$

Einflusslinie

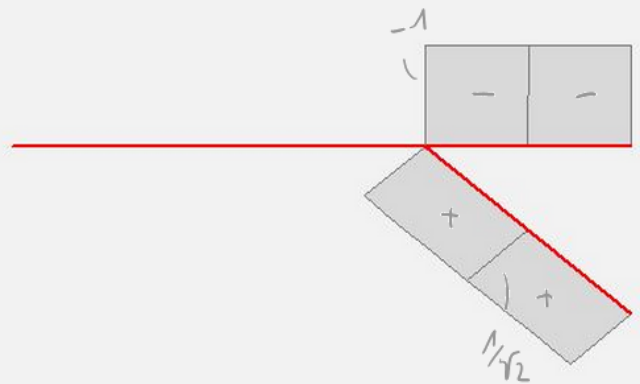


(d)

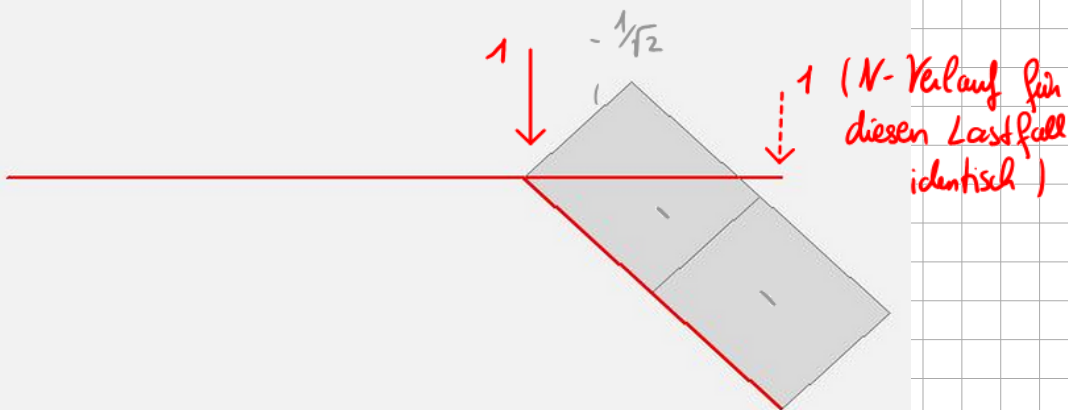
M



N_{u5}



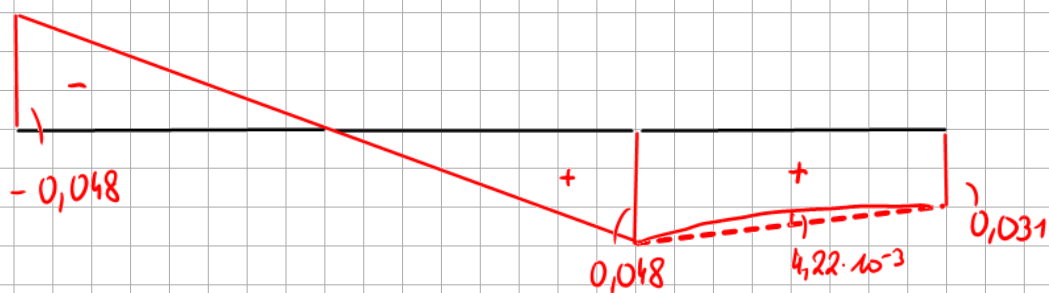
N₃ = N₄



$$W_3 = (-3) \cdot (-3) \cdot \frac{3}{EI} + \frac{1}{3} \cdot 3 \cdot 3 \cdot \frac{3\sqrt{2}}{EI} + \left(-\frac{1}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} \cdot \frac{3\sqrt{2}}{EI} = 0,048 \text{ m}$$

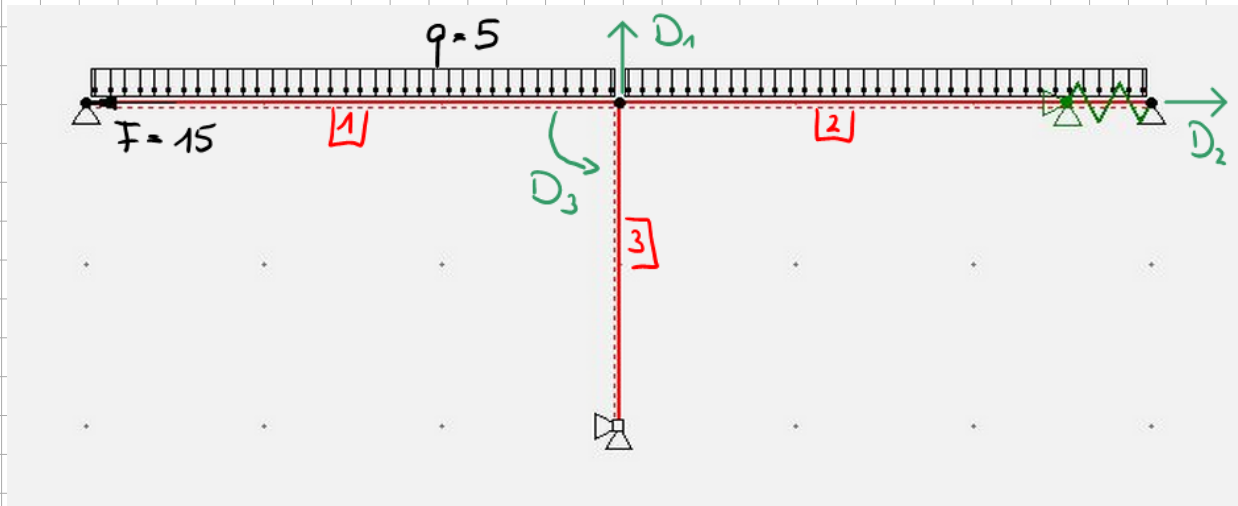
$$W_4 = \frac{1}{2} \cdot (-3) \cdot (-3) \cdot \frac{3}{EI} + \frac{1}{3} \cdot 3 \cdot 3 \cdot \frac{3\sqrt{2}}{EI} + \left(-\frac{1}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}} \cdot \frac{3\sqrt{2}}{EI} = 0,031 \text{ m}$$

Einflusslinie



Statik Musteralösungen

Probeklausur 4 - Aufgabe 1



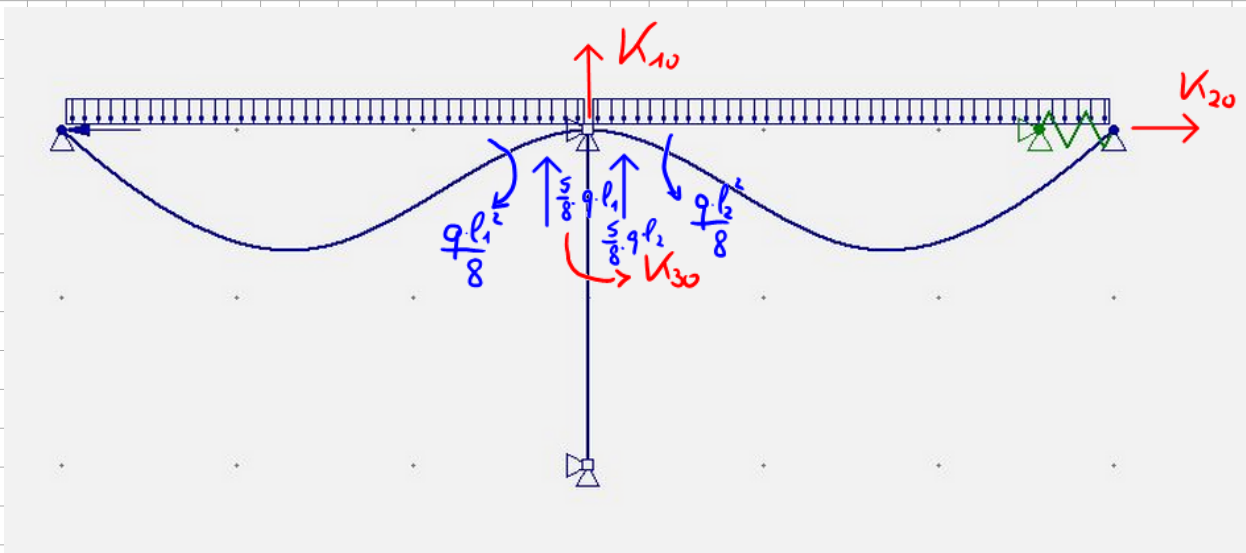
$$EI = 10\,000$$

$$EA_{1,2} \rightarrow \infty$$

$$EA_3 = 15\,000$$

$$k = 100$$

L7

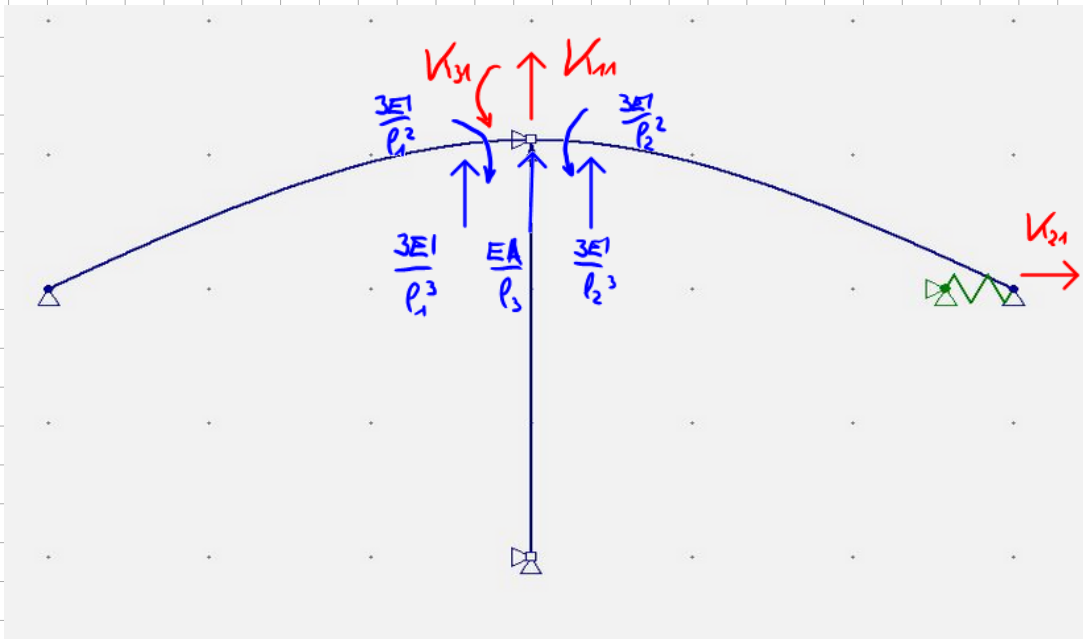


$$K_{10} = 2 \cdot \frac{5}{8} \cdot q \cdot l_1 = 18,75$$

$$K_{20} = 15$$

$$K_{30} = \frac{q l_1^2}{8} - \frac{q l_2^2}{8} = 0$$

E2 1



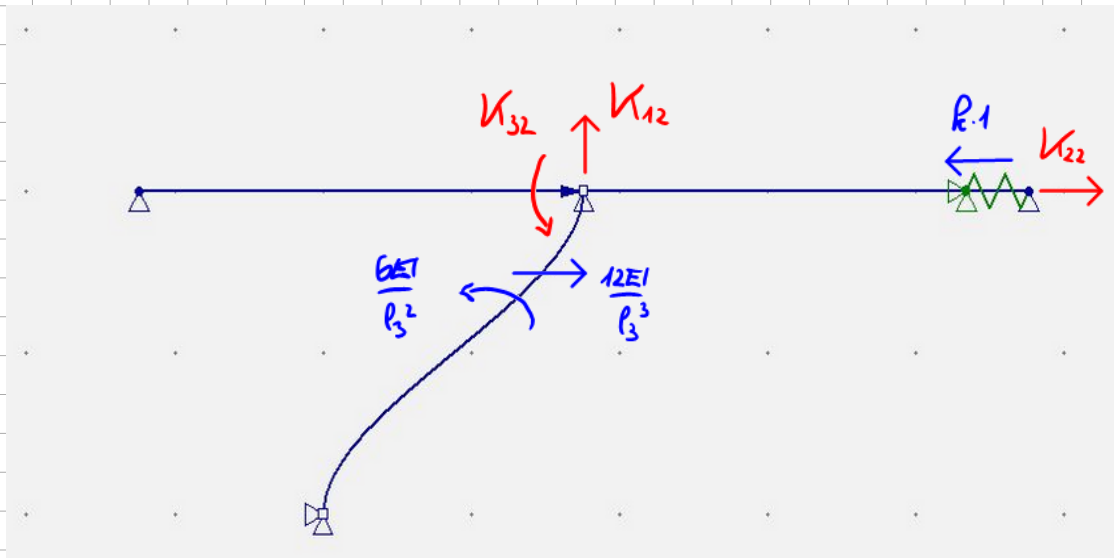
$$K_{11} = 2 \frac{3EI}{l_1^3} + \frac{EA}{l_3} =$$

$$= 2 \cdot 111,1 + 7500 = 9722,2$$

$$K_{21} = 0$$

$$K_{31} = \frac{3EI}{l_1^2} - \frac{3EI}{l_2^2} = 0$$

E2 2



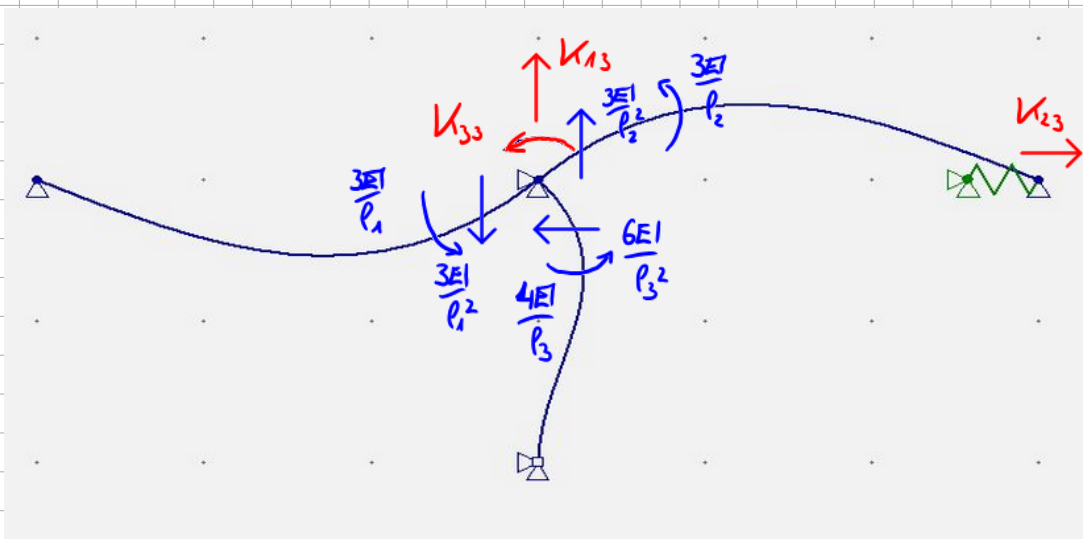
$$K_{12} = 0 = K_{21}$$

$$K_{22} = \frac{12EI}{l_3^3} + l_1 \cdot 15000 + 1000 =$$

$$= 15100$$

$$K_{32} = \frac{6EI}{l_3^2} \cdot 15000$$

E2 3



$$K_{13} = \frac{3EI}{l_1^2} - \frac{3EI}{l_1^2} = 0$$

$$K_{23} = \frac{6EI}{l_3^2} = 15000$$

$$K_{33} = \frac{3EI}{l_1} + \frac{3EI}{l_2} + \frac{4EI}{l_3} =$$

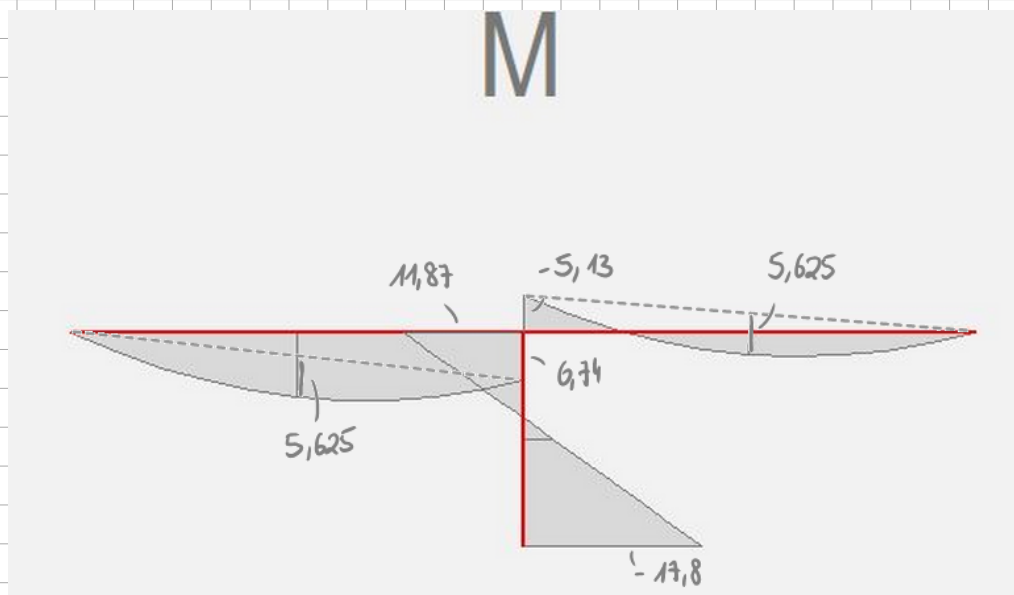
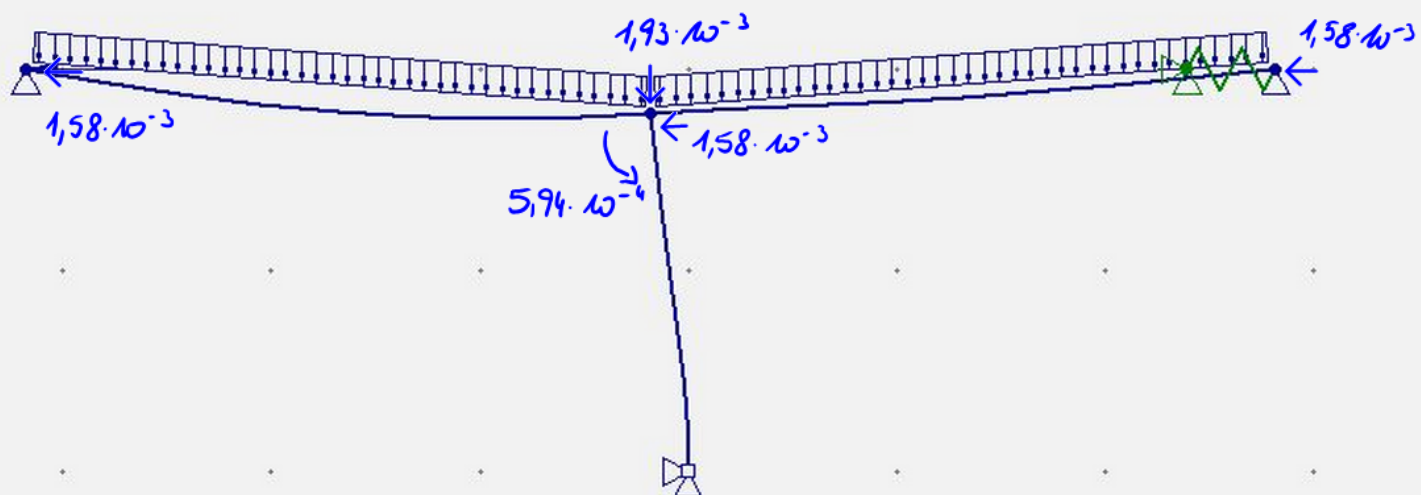
$$= 10000 + 10000 + 20000 =$$

$$= 40000$$

$$\underline{\underline{K}} = \begin{bmatrix} 9722,2 & 0 & 0 \\ 0 & 15\,000 & 15\,000 \\ 0 & 15\,000 & 40\,000 \end{bmatrix} \quad \underline{\underline{K}}_0 = -\underline{\underline{F}} = \begin{bmatrix} -18,75 \\ -15 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \quad \Rightarrow \quad \underline{\underline{u}} = \begin{bmatrix} -1,93 \cdot 10^{-3} \text{ m} \\ -1,58 \cdot 10^{-3} \text{ m} \\ 5,94 \cdot 10^{-4} \text{ rad} \end{bmatrix}$$

(b)



(c) • ja, anhand der Verformungsfigur lässt sich erkennen, dass in den Stäben 1 und 2 zusätzliche Momente wirken müssen. Zudem wirkt Stab 3 wie eine Feder, also bekommt er eine Normalkraft.

• Federsteifigkeit von Stab 3: $\frac{EA}{l} = \frac{15000}{2} = 7500 \frac{\text{N}}{\text{m}}$

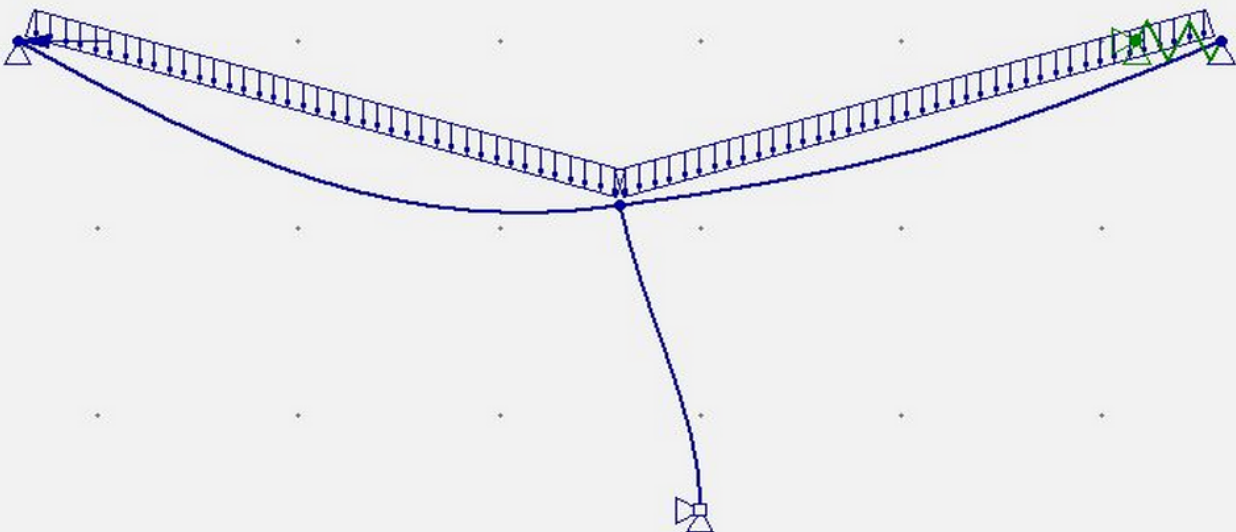
$$F = k \cdot u = 7500 \frac{\text{N}}{\text{m}} \cdot 0,002 \text{ m} = 15 \text{ N}$$

$$\Rightarrow K_{10} = 18,75 + 15 = 33,75$$

$$\Rightarrow \underline{K} \text{ bleibt gleich} \quad \underline{F} = -\underline{K}_0 = \begin{bmatrix} -33,75 \\ -15 \\ 0 \end{bmatrix}$$

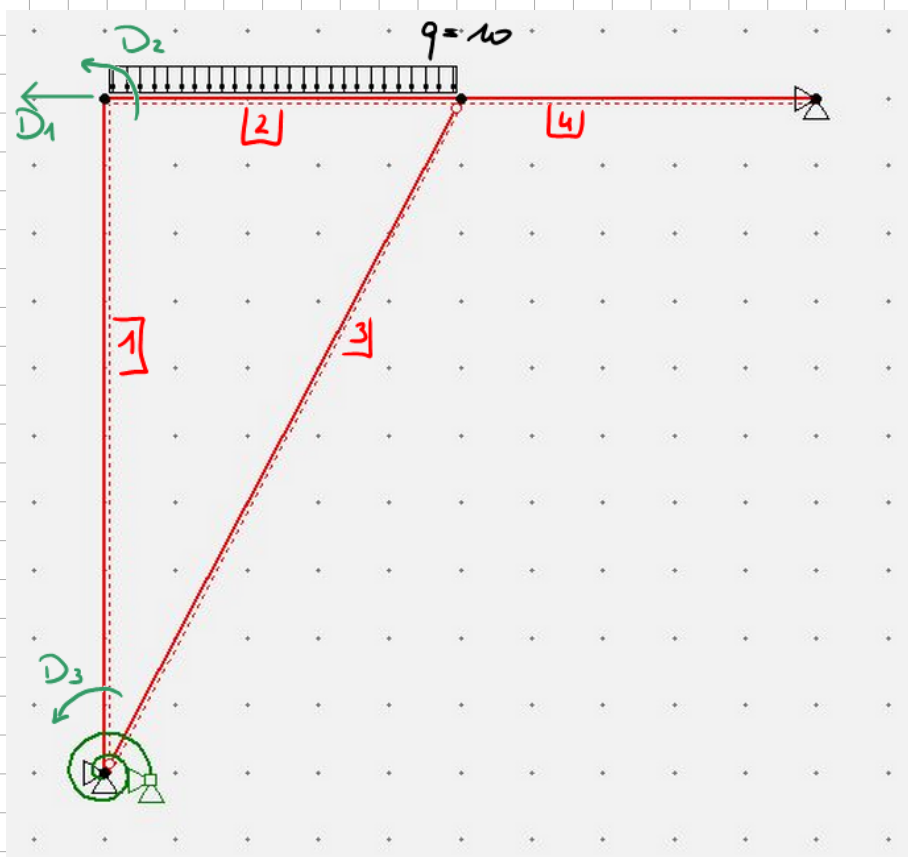
$$\underline{K} \cdot \underline{u} = \underline{F} \quad \Rightarrow \quad \underline{u} = \begin{bmatrix} -3,47 \cdot 10^{-3} & \text{m} \\ -1,58 \cdot 10^{-3} & \text{m} \\ 5,94 \cdot 10^{-3} & \text{rad} \end{bmatrix}$$

Verformung:



• die Stützensenkung wirkt wie eine äußere Last und hat somit nun auf den Lastvektor einen Einfluss

Probeklausur 4 - Aufgabe 2



$$EI_{1,2,3} = 1000$$

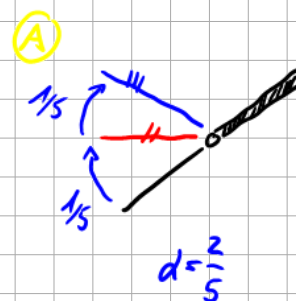
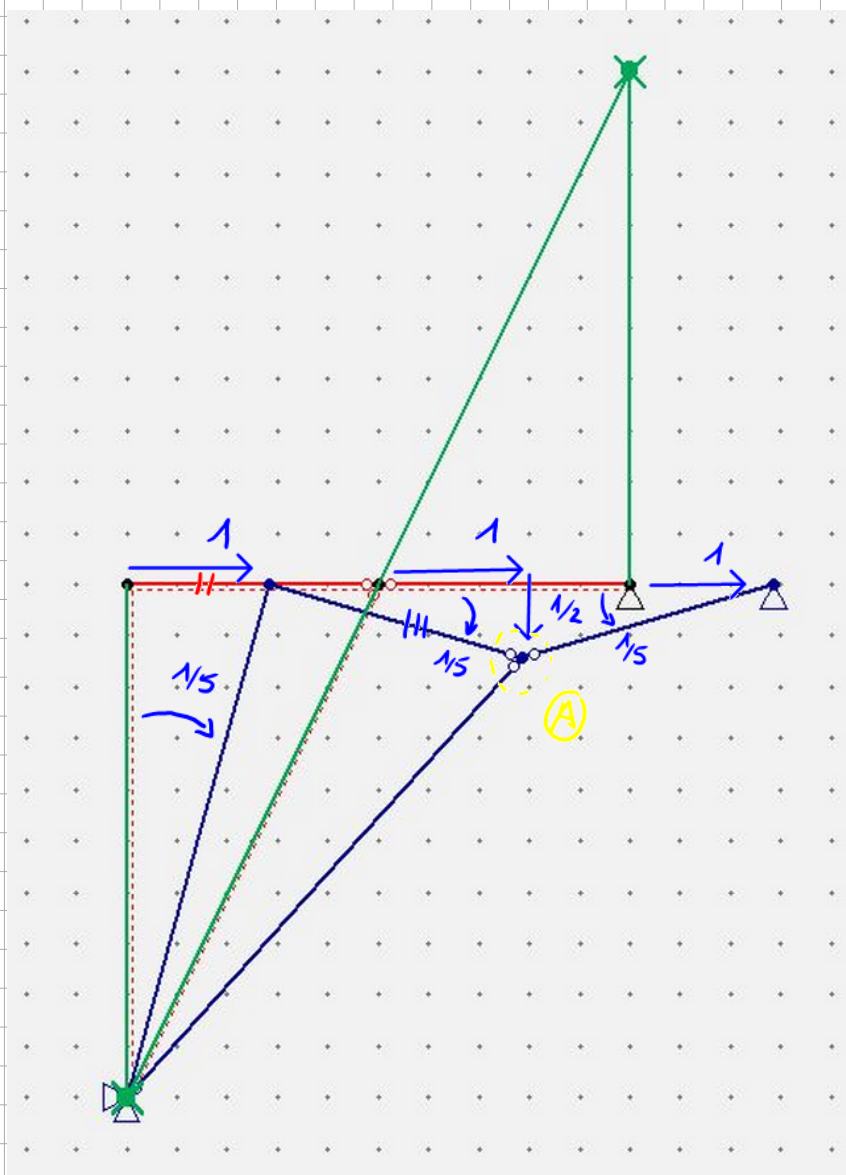
$$EA_{1,2,3} \rightarrow \infty$$

$$EI_4 \rightarrow \infty$$

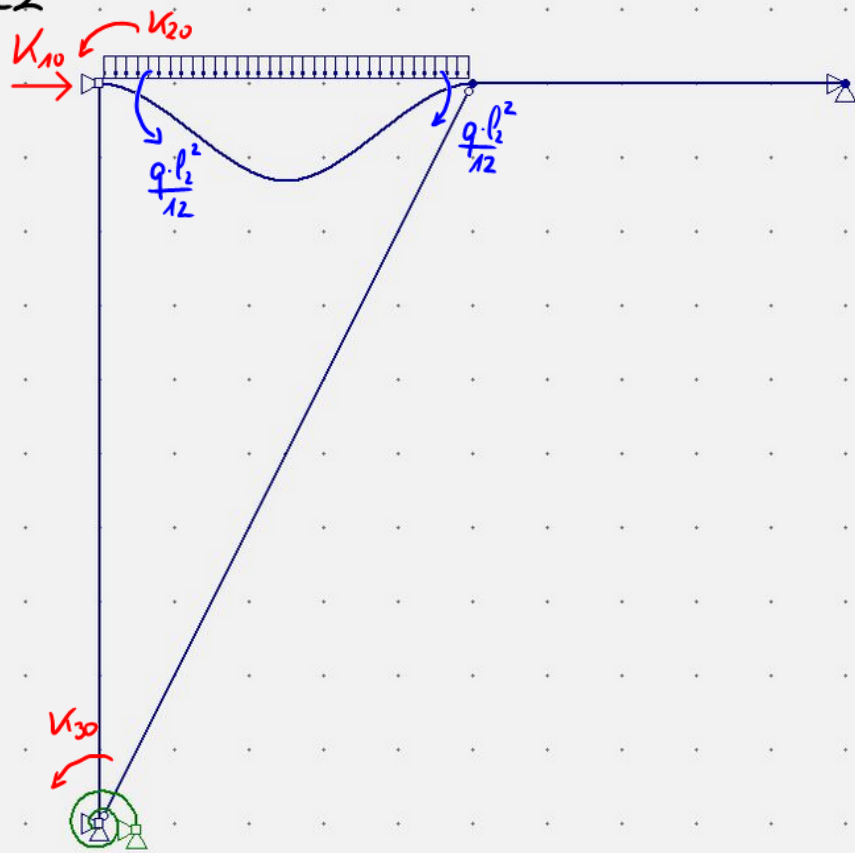
$$EA_4 = 1000$$

$$C_\varphi = 500$$

Gelenkfigur



L2

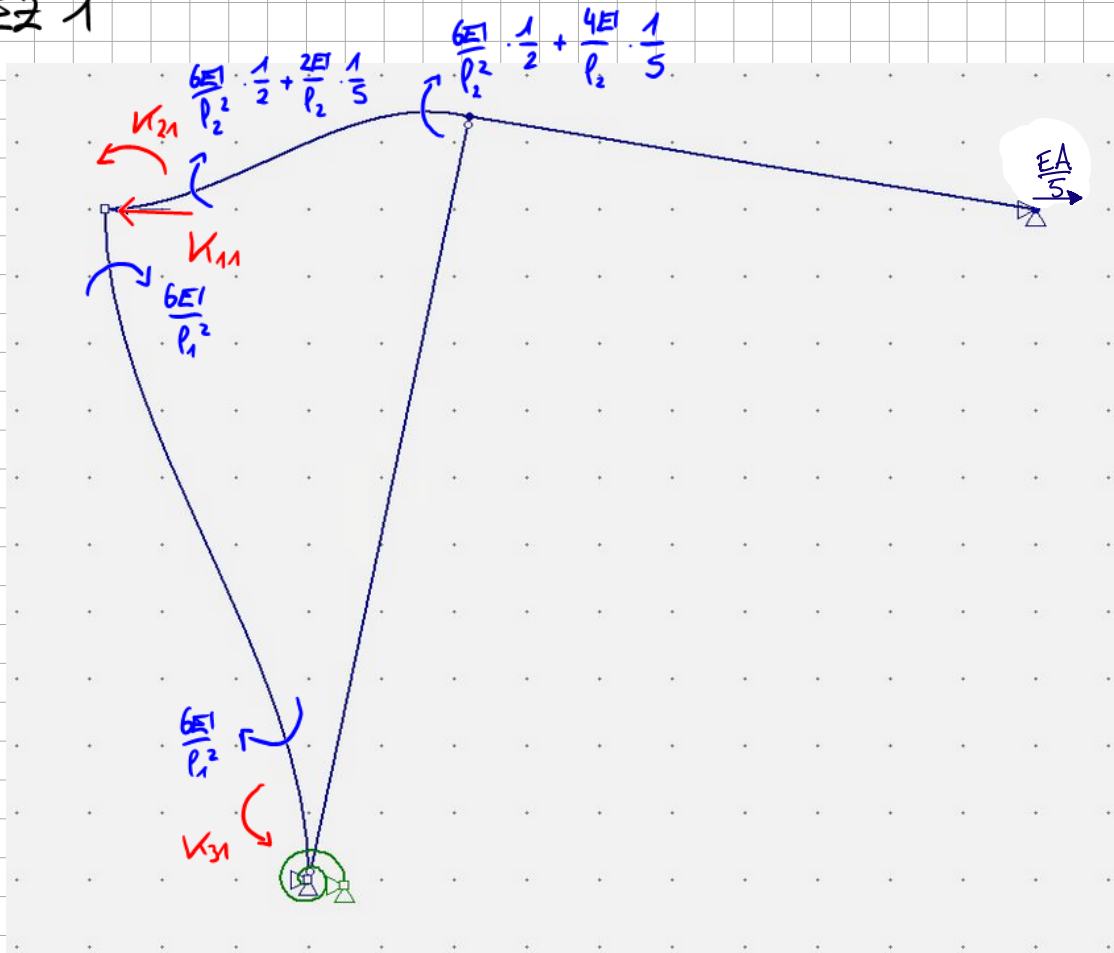


$$\begin{aligned} \text{P.V: } K_{20} \cdot \bar{1} &= -\frac{q \cdot l_1^2}{12} \cdot \frac{1}{5} + \frac{q \cdot l_1^2}{12} \cdot \frac{2}{5} + q \cdot l_1 \cdot \frac{1}{4} = \\ &= -1,042 + 2,08 + 6,25 = \\ &= 7,29 \end{aligned}$$

$$K_{20} = \frac{q \cdot l_1^2}{12} = 5,21$$

$$K_{30} = 0$$

E2 1

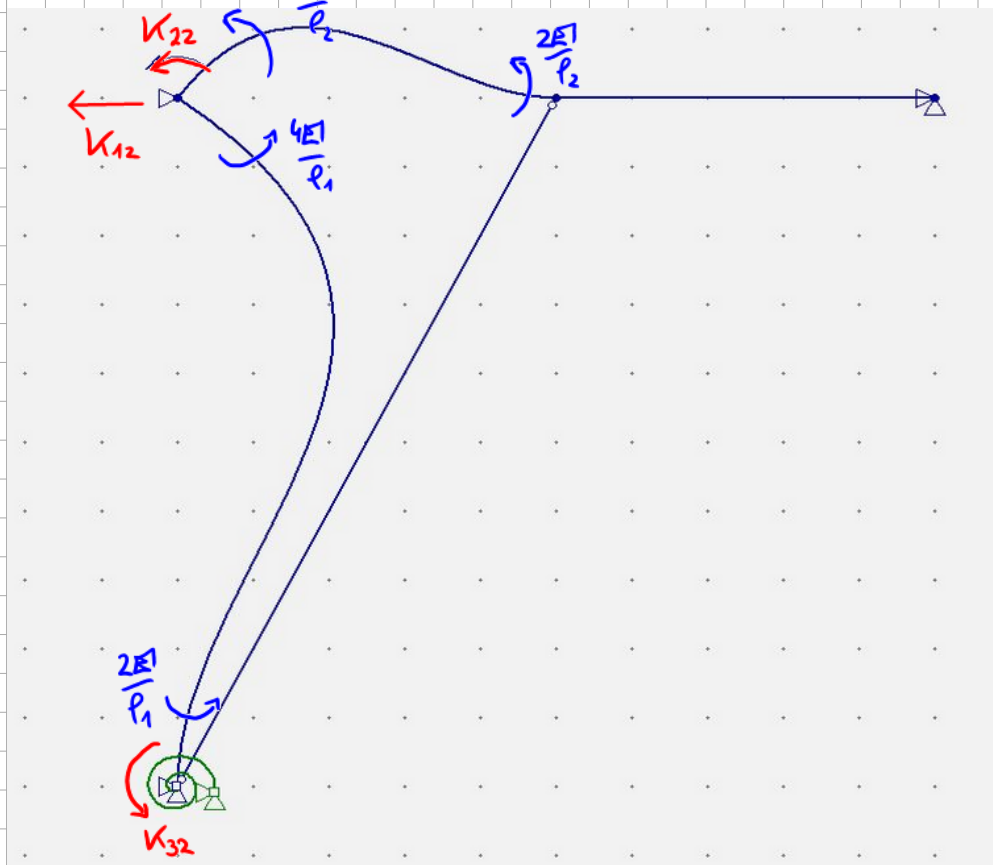


$$\begin{aligned} \text{P.V: } K_{11} \cdot \bar{1} &= \frac{6EI}{l_1^2} \cdot \frac{1}{5} \cdot 2 + \frac{EA}{l_1} \cdot \frac{1}{5} \\ &+ \frac{1}{5} \cdot \left(\frac{6EI}{l_1^2} \cdot \frac{1}{2} + \frac{2EI}{l_2} \cdot \frac{1}{5} \right) + \\ &+ \frac{2}{5} \cdot \left(\frac{6EI}{l_1^2} \cdot \frac{1}{2} + \frac{4EI}{l_2} \cdot \frac{1}{5} \right) = \\ &= 96 + 400 + 128 + 320 = \\ &= 944 \end{aligned}$$

$$\begin{aligned} \text{G.G: } K_{21} &= -\frac{6EI}{5^2} - \frac{6EI}{2,5^2} \cdot \frac{1}{2} - \frac{2EI}{2,5} \cdot \frac{1}{5} = \\ &= -240 - 480 - 160 = \\ &= -880 \end{aligned}$$

$$K_{31} = -\frac{6EI}{5^2} = -240$$

E2 2

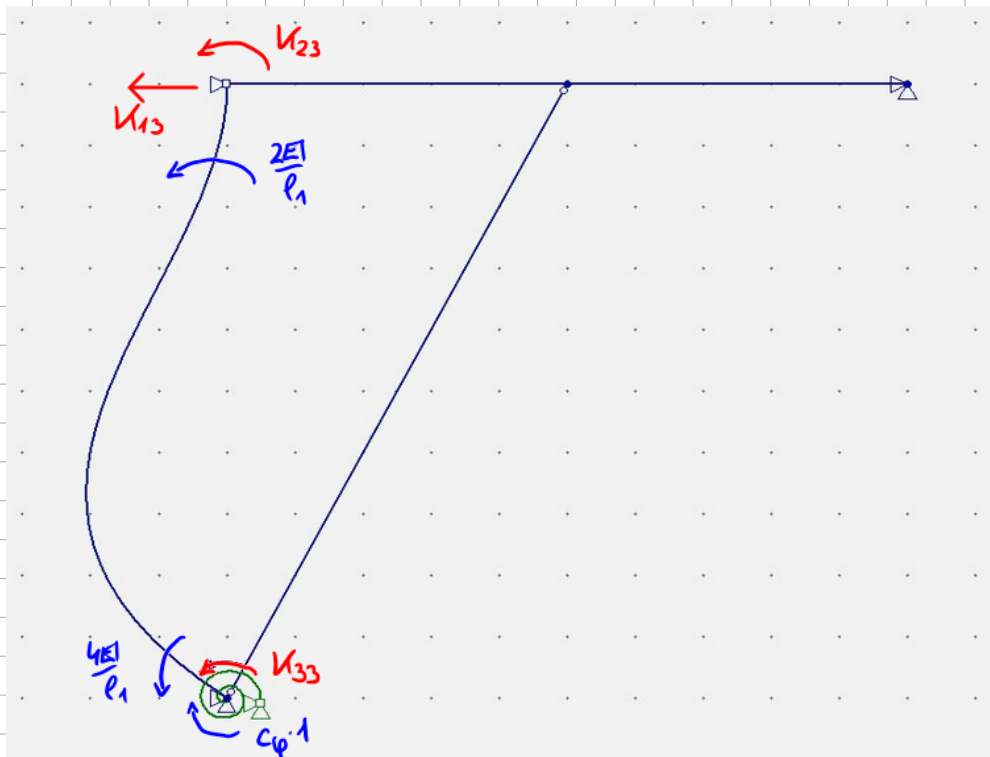


$$\begin{aligned} \text{P.V: } K_{12} \cdot \bar{1} &= -\frac{2EI}{l_1} \cdot \frac{1}{5} - \frac{4EI}{l_1} \cdot \frac{1}{5} - \\ &\quad - \frac{4EI}{l_2} \cdot \frac{1}{5} - \frac{2EI}{l_2} \cdot \frac{2}{5} = \\ &= -80 - 160 - 320 - 320 = \\ &= -880 \end{aligned}$$

$$\text{G.G: } K_{22} = \frac{4EI}{l_1} + \frac{4EI}{l_2} = 800 + 1600 = 2400$$

$$K_{32} = \frac{2EI}{l_1} = 400$$

E2 3



$$\begin{aligned} \text{P.V: } K_{13} \cdot \bar{1} &= -\frac{2EI}{l_1} \cdot \frac{1}{5} - \frac{4EI}{l_1} \cdot \frac{1}{5} = \\ &= -80 - 160 = -240 \end{aligned}$$

$$\text{G.G: } K_{23} = \frac{2EI}{l_1} = 400$$

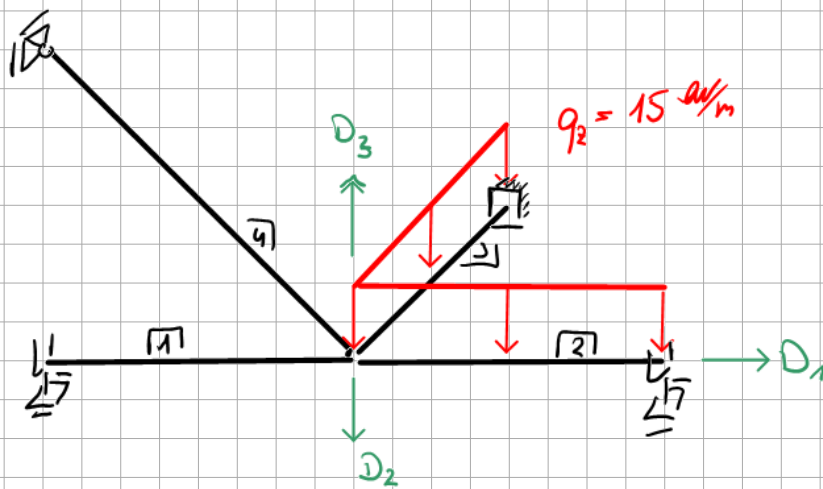
$$K_{33} = \frac{4EI}{l_1} + c_{\varphi} = 800 + 500 = 1300$$

$$\underline{\underline{K}} = \begin{bmatrix} 944 & -880 & -240 \\ -880 & 2400 & 400 \\ -240 & 400 & 1300 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} -7,23 \\ -5,21 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \quad \Rightarrow \quad \underline{\underline{u}} = \begin{bmatrix} -0,01477 \text{ m} \\ -7,519 \cdot 10^{-3} \text{ rad} \\ -4,139 \cdot 10^{-4} \text{ rad} \end{bmatrix}$$

Probeklausur 4 - Aufgabe 3



$$EI = 10\,000$$

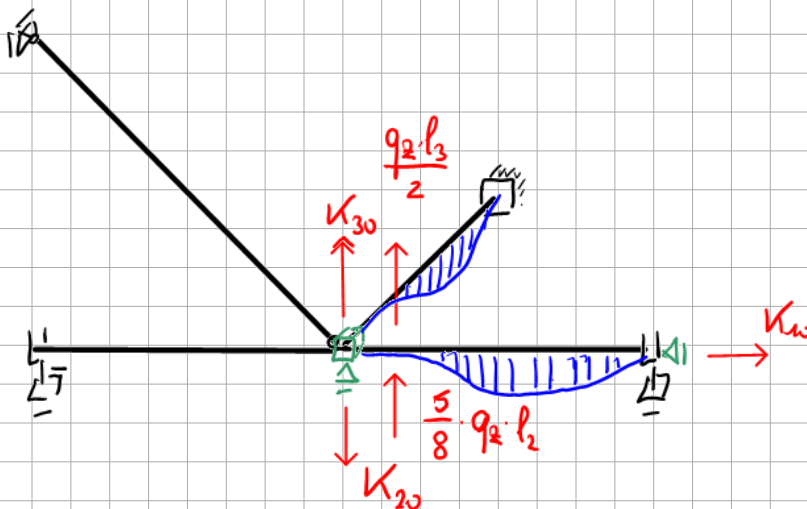
$$EA_{1,2,3} \rightarrow \infty$$

$$EA_4 = 100\,000$$

$$GI_{T,1,2,3} \rightarrow \infty$$

$$q_2 = 15 \text{ kN/m}$$

L2



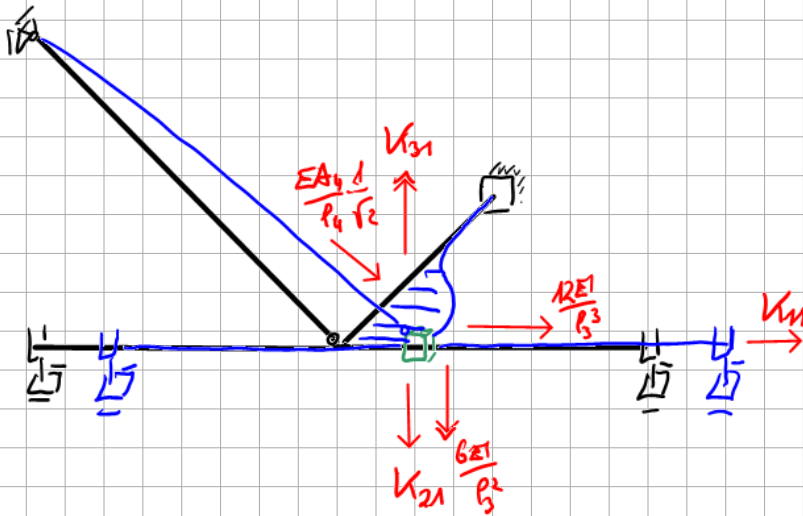
$$K_{10} = 0$$

$$K_{20} = -\frac{5}{8} \cdot q_2 \cdot l_2 - \frac{q_2 \cdot l_2}{2}$$

$$= -37,5 - 30 = -67,5$$

$$K_{30} = 0$$

E2 1



$$K_{11} = \frac{12EI}{l^3} + \frac{EA_4}{l_4} \cdot \frac{1}{2} =$$

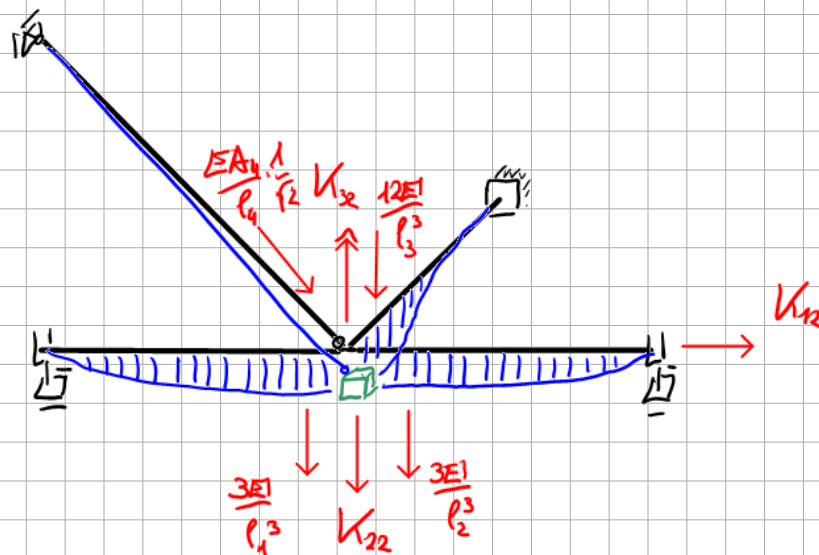
$$= 1875 + 8838,83 =$$

$$= 10713,83$$

$$K_{21} = \frac{EA_4}{l_4} \cdot \frac{1}{2} = 8838,83$$

$$K_{31} = -\frac{6EI}{l^2} = -3750$$

E2 2



$$K_{12} = \frac{EA_4}{l_4} \cdot \frac{1}{2} = 8838,83$$

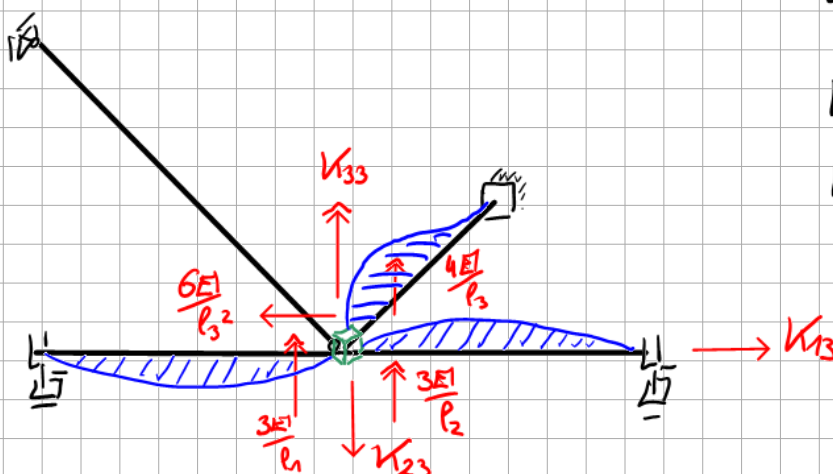
$$K_{22} = \frac{3EI}{l_1^3} + \frac{3EI}{l_2^3} + \frac{12EI}{l_3^3} + \frac{EA_4}{l_4} \cdot \frac{1}{2} =$$

$$= 468,75 + 468,75 + 1875 +$$

$$+ 8838,83 = 11651,33$$

$$K_{32} = 0$$

E2 3



$$K_{13} = -\frac{6EI}{l^2} = -3750$$

$$K_{23} = 0$$

$$K_{33} = \frac{3EI}{l_1} + \frac{3EI}{l_2} + \frac{4EI}{l_3} = 7500 + 7500 +$$

$$+ 10000 = 25000$$

$$\underline{\underline{K}} = \begin{bmatrix} 10\,713,83 & 8\,838,83 & -3\,750 \\ 8\,838,83 & 11\,651,33 & 0 \\ -3\,750 & 0 & 25\,000 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 0 \\ 67,5 \\ 0 \end{bmatrix} \quad \underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \Rightarrow \underline{\underline{u}} = \begin{bmatrix} -0,01486 \text{ m} \\ 0,01707 \text{ m} \\ -2,229 \cdot 10^{-3} \text{ rad} \end{bmatrix}$$

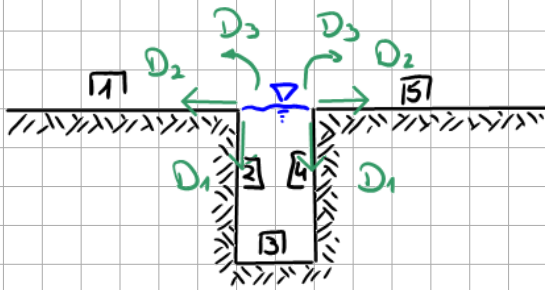
(b) M_T in Stab 3 aufgrund unendlichem $G I_T$ nicht ermittelbar
 \rightarrow über $G I_T$ aus M_y in Stab 2 berechnen

$$M_{T,3} = - \frac{\cancel{q_2} \cdot l_2^2}{8} + 0 + 0 + 0 = -30 \text{ Nm}$$

$\cancel{L_2} \quad E I_1 \quad E I_2 \quad E I_3$

\uparrow
 M_y in Stab 1 und in Stab 2 haben sich im Knotengleichgewicht auf!

Probeklausur 4 - Aufgabe 4



$$EI_{1,5} = 200$$

$$EI_{2,4} = 100$$

$$EI_3 \rightarrow \infty$$

$$EA \rightarrow \infty$$

$$R = 10 \text{ m}$$

$$q_w = 10 \text{ m/m}^2$$

$$\lambda_{1,5} = \sqrt[4]{\frac{10 \cdot 100}{4 \cdot 200}} = 1,880$$

$$\lambda_{1,5} \cdot l_{1,5} = 1,88 \cdot 3 = 5,64 > \pi$$

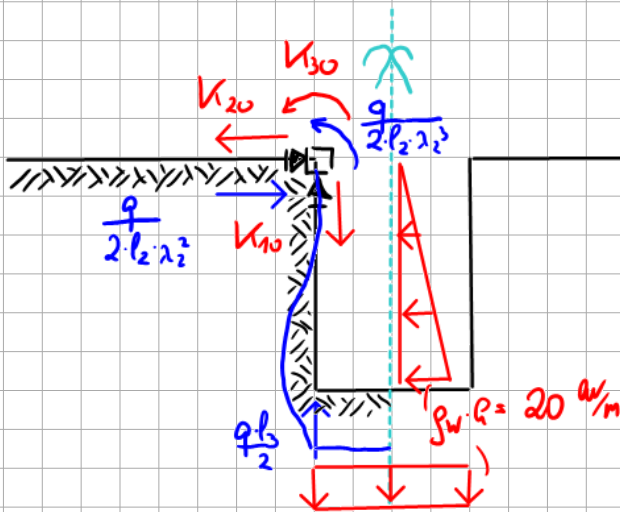
$\rightarrow \infty$ lange Balken

$$\lambda_{2,4} = \sqrt[4]{\frac{10 \cdot 100}{4 \cdot 100}} = 2,236$$

$$\lambda_{2,4} \cdot l_{2,4} = 2,236 \cdot 2 = 4,472$$

$\rightarrow \infty$ lange Balken

L2

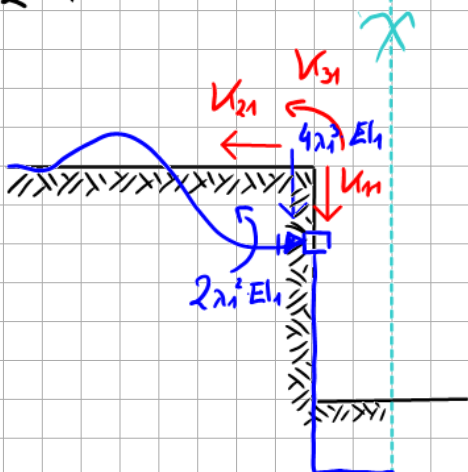


$$V_{10} = -\frac{20 \cdot 1}{2} = -10$$

$$V_{20} = \frac{-20}{2 \cdot 2 \cdot 2,236^2} = -1$$

$$V_{30} = \frac{20}{2 \cdot 2 \cdot 2,236^3} = 0,447$$

E2 1

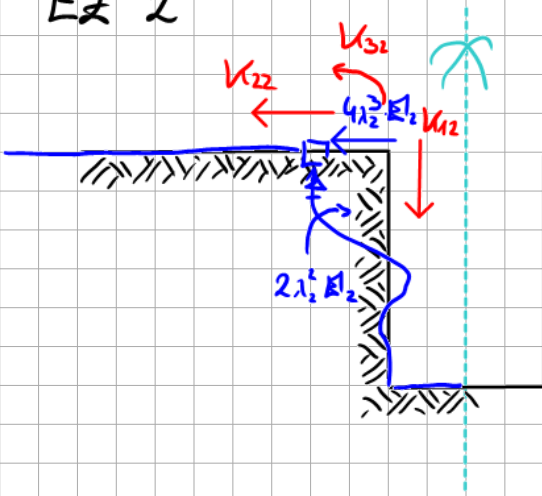


$$M_{11} = 4 \lambda_1^3 \cdot EI_1 = 5318,3$$

$$V_{21} = 0$$

$$M_{31} = 2 \lambda_1^2 \cdot EI_1 = 1414,2$$

Ex 2

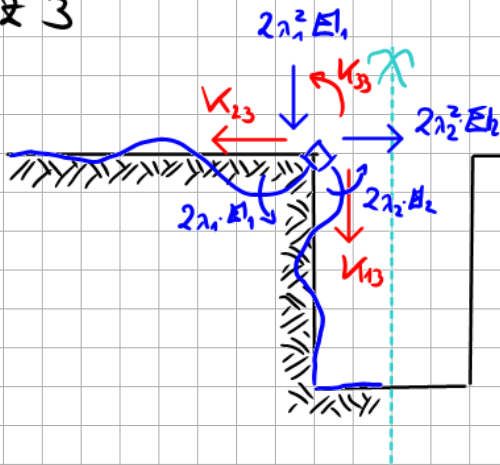


$$K_{12} = 0$$

$$K_{22} = 4 \cdot \lambda_2^3 \cdot E I_2 = 4472,1$$

$$K_{32} = -2 \lambda_2^3 \cdot E I_2 = -1000$$

Ex 3



$$K_{13} = 2 \lambda_1^2 E I_1 = 1414,2$$

$$K_{23} = -2 \lambda_1^2 E I_1 = -1000$$

$$K_{33} = 2 \lambda_1 E I_1 + 2 \lambda_2 E I_2 = 1199,3$$

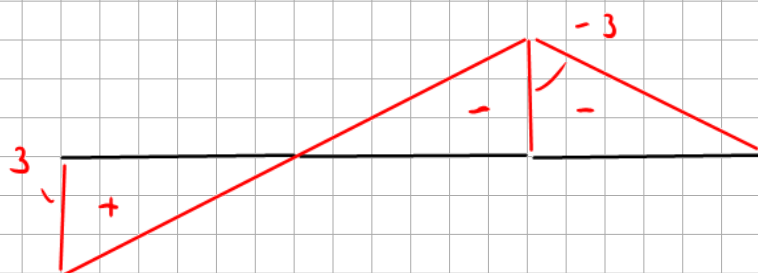
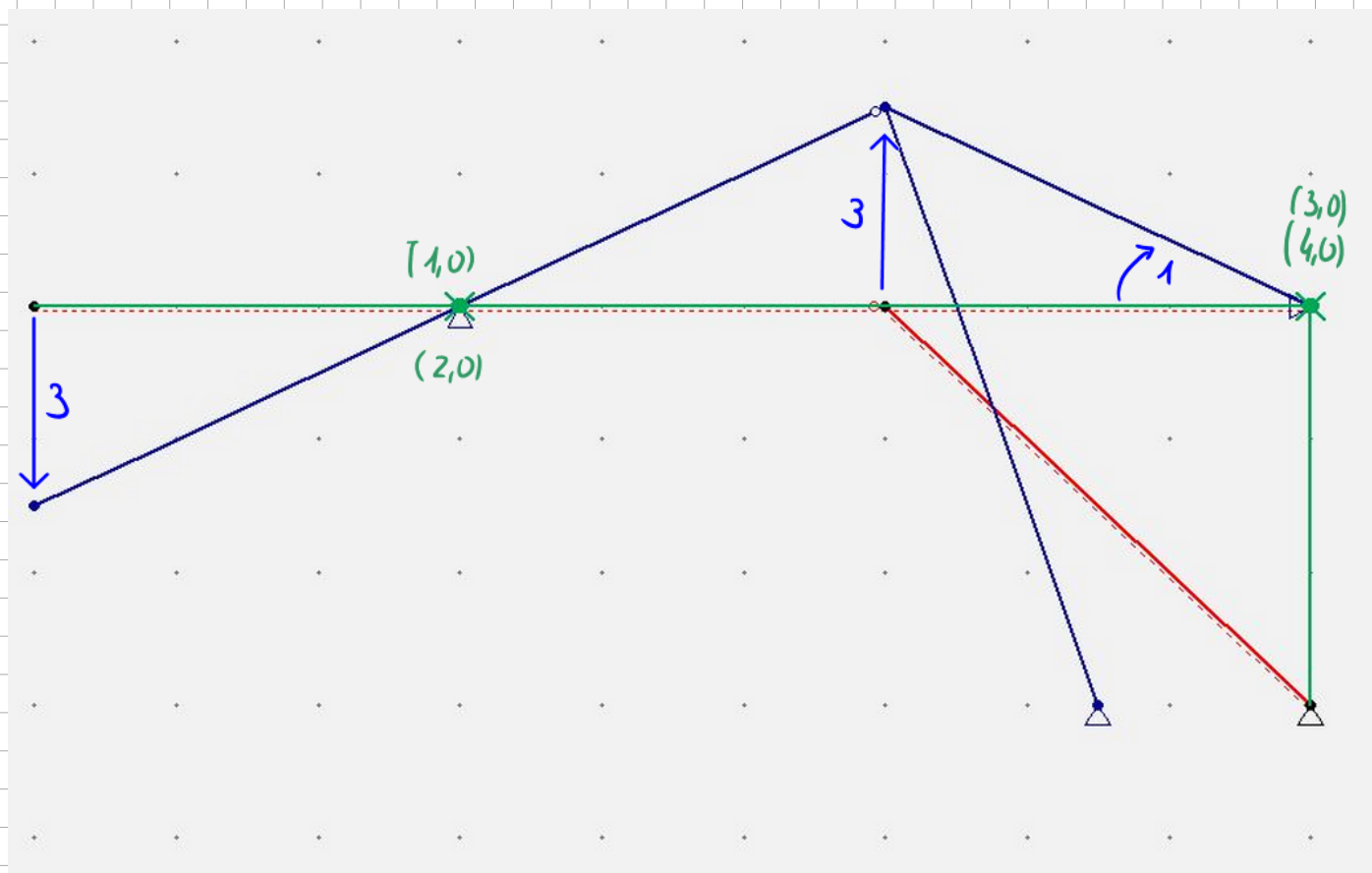
$$\underline{\underline{K}} = \begin{bmatrix} 5318,3 & 0 & 1414,2 \\ 0 & 4472,1 & -1000 \\ 1414,2 & -1000 & 1199,3 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 10 \\ 1 \\ -0,447 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \Rightarrow \underline{\underline{u}} = \begin{bmatrix} 3,16 \cdot 10^{-3} \text{ m} \\ -8,51 \cdot 10^{-4} \text{ m} \\ -4,31 \cdot 10^{-3} \text{ rad} \end{bmatrix}$$

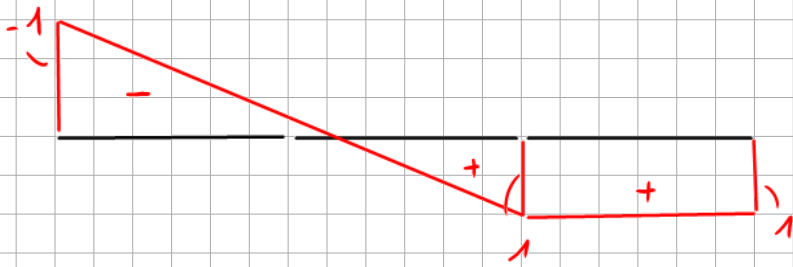
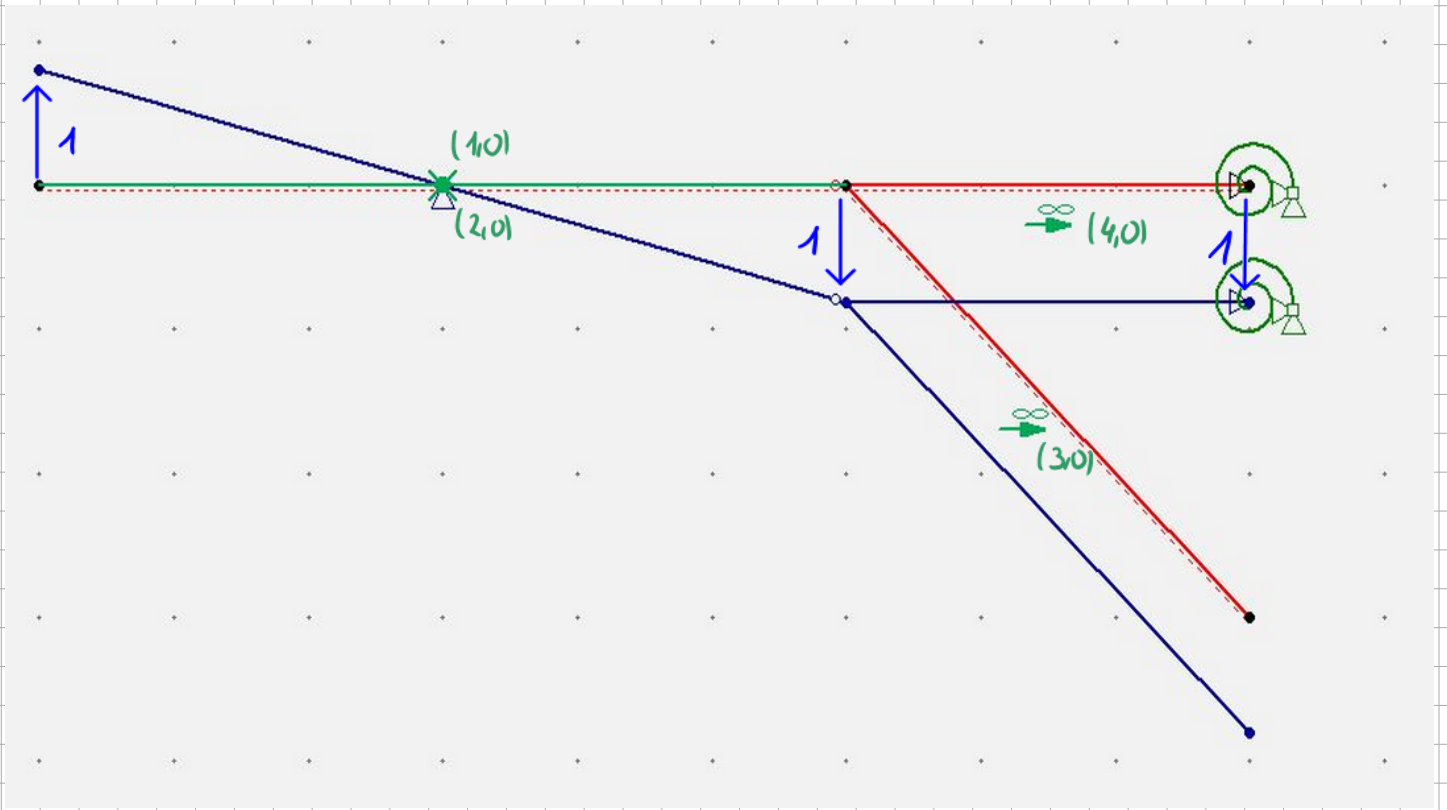
Probeklausur 4 - Aufgabe 5

(a)

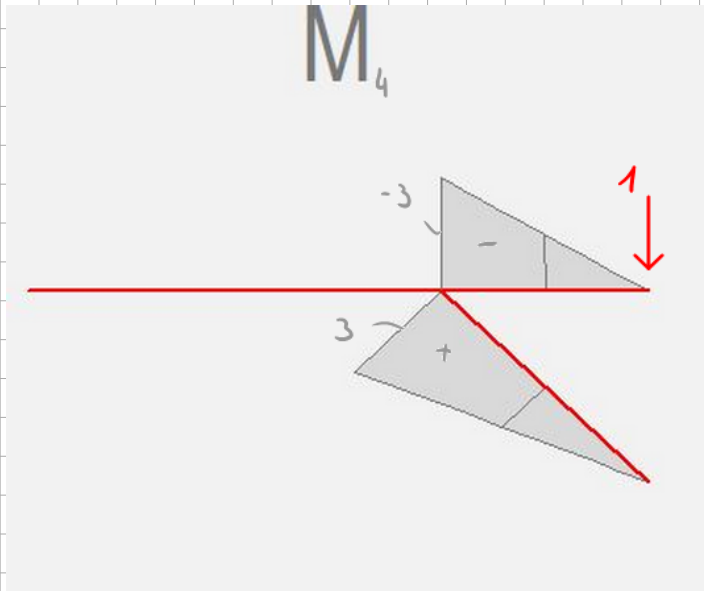
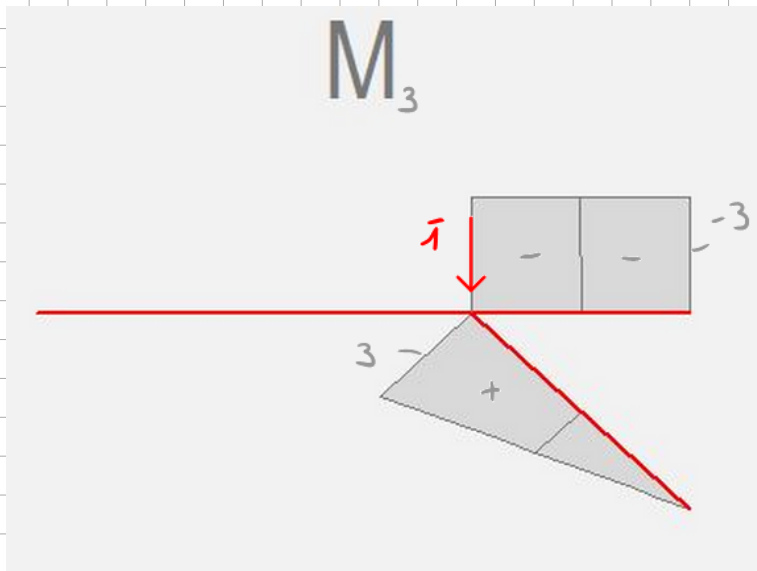


$$\begin{aligned}\eta(x_1) &= 3 - x_1 \\ \eta(x_2) &= -x_2 \\ \eta(x_3) &= -3 + x_3\end{aligned}$$

(b)

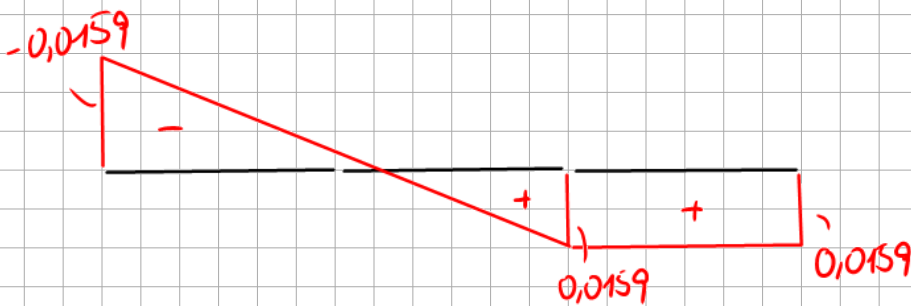


(c)



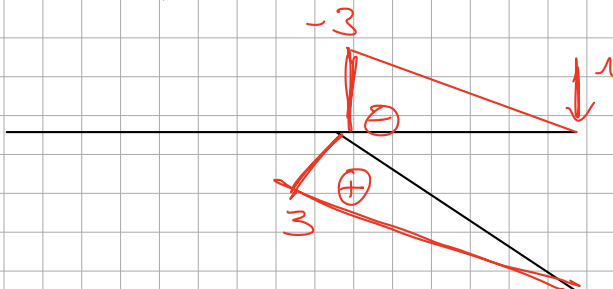
$$W_4 = \frac{1}{3} \cdot 3^2 \cdot \frac{3\sqrt{2}}{EI_3} + \frac{1}{3} \cdot (-3)^2 \cdot \frac{3}{EI_4} = 0,0159$$

$$W_3 = \frac{1}{3} \cdot 3^2 \cdot \frac{3\sqrt{2}}{EI_3} + \frac{1}{2} \cdot (-3)^2 \cdot \frac{3}{EI_4} = 0,0159$$

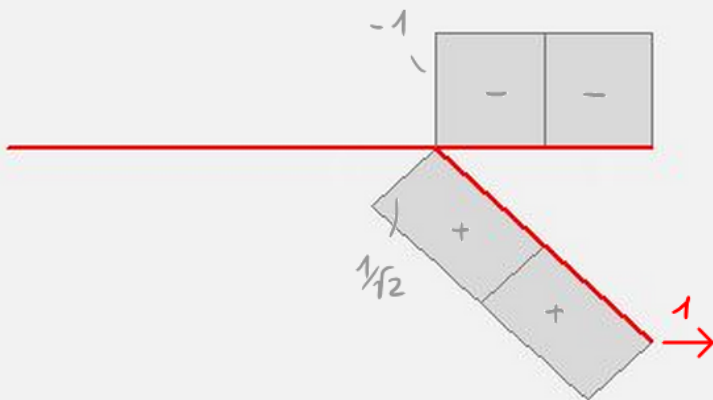


d)

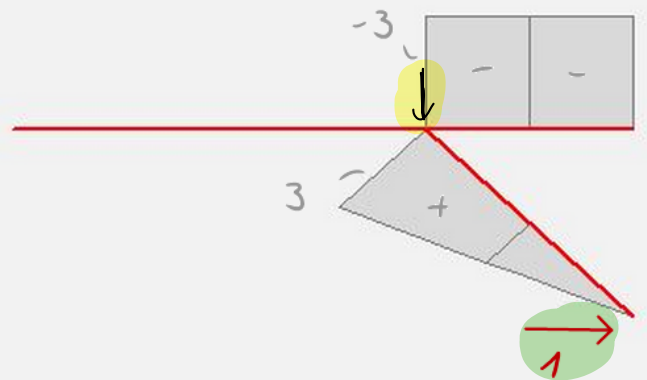
M_4



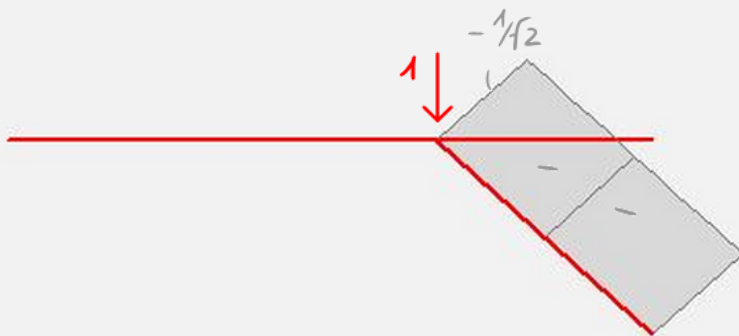
N_5



M_5 / M_3



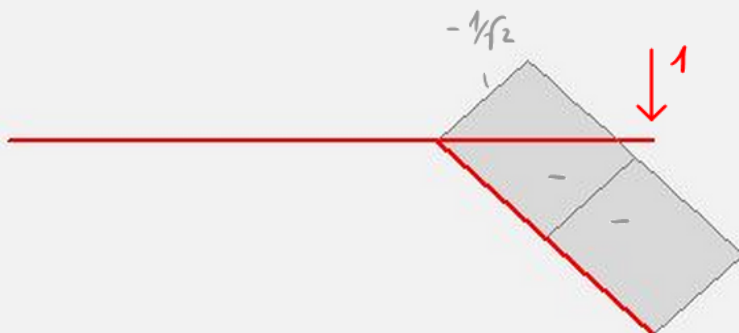
N_3



$$W_3 = \frac{1}{3} \cdot 3^2 \cdot \frac{3\sqrt{2}}{EI} - \left(\frac{1}{\sqrt{2}}\right)^2 \cdot \frac{3\sqrt{2}}{EA} + \frac{3 \cdot 3}{1000}$$

$$= 0,0159 - 2,12 \times 10^{-3} + 9 \times 10^{-3} = \underline{\underline{0,0228 \text{ m}}}$$

N_4

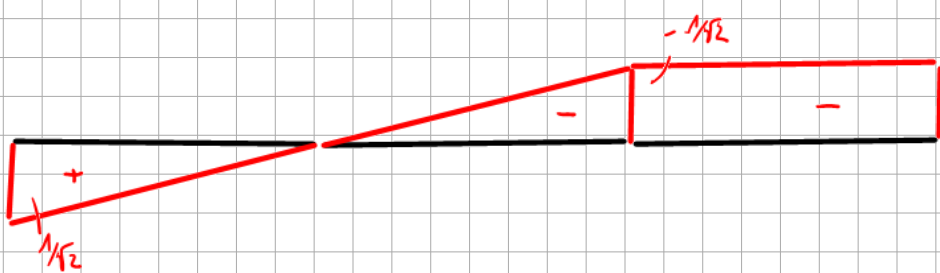
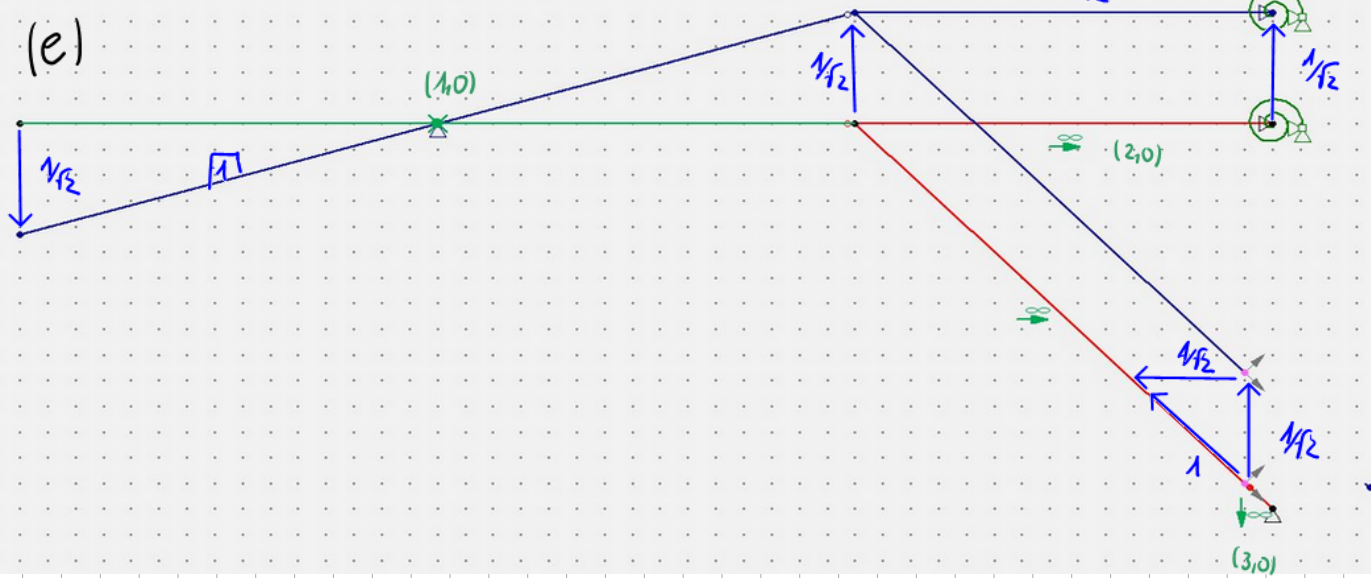


$$W_4 = \frac{1}{3} \cdot 3^2 \cdot \frac{3\sqrt{2}}{EI} + \frac{1}{\sqrt{2}} \cdot \left(-\frac{1}{\sqrt{2}}\right) \cdot \frac{3\sqrt{2}}{EA}$$

$$= 0,0159 - 2,12 \cdot 10^{-3}$$

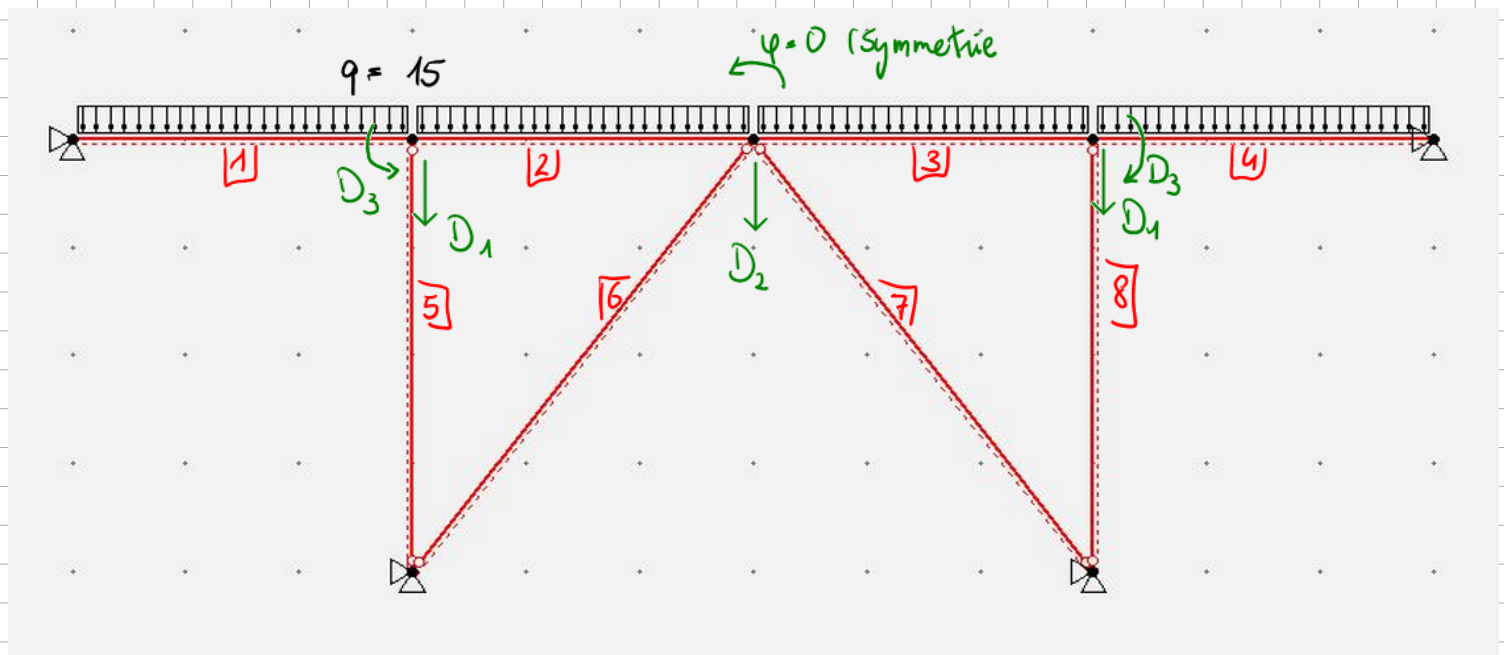
$$= 0,0138 \text{ m}$$





Statik Musterlösungen

Probeklausur 5 - Aufgabe 1



$$q = 15 \text{ kN/m}$$

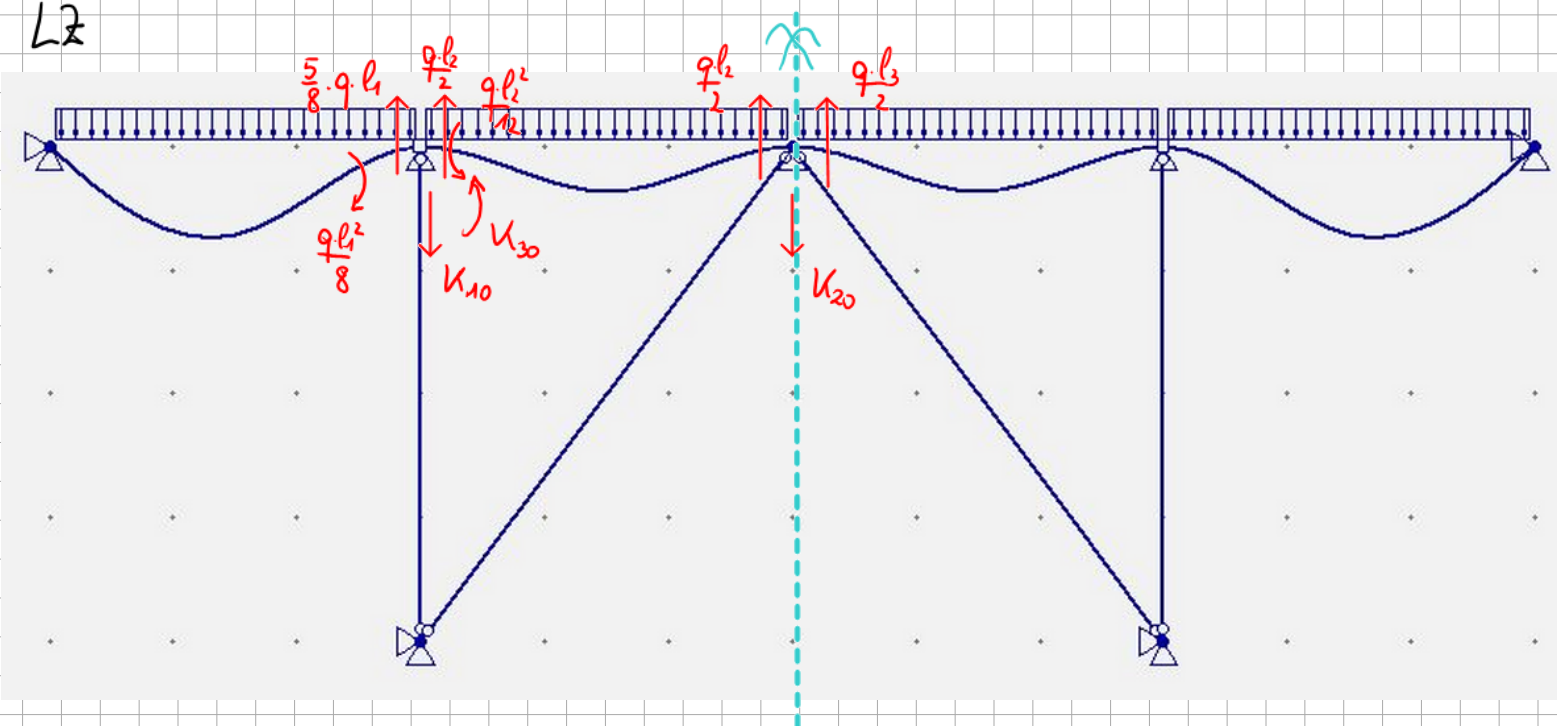
$$EI = 2000 \text{ kNm}^2$$

$$EA_{1,2,3,4} \rightarrow \infty$$

$$EA_{5,8} = 10000 \text{ kN}$$

$$EA_{6,7} = 5000 \text{ kN}$$

L2

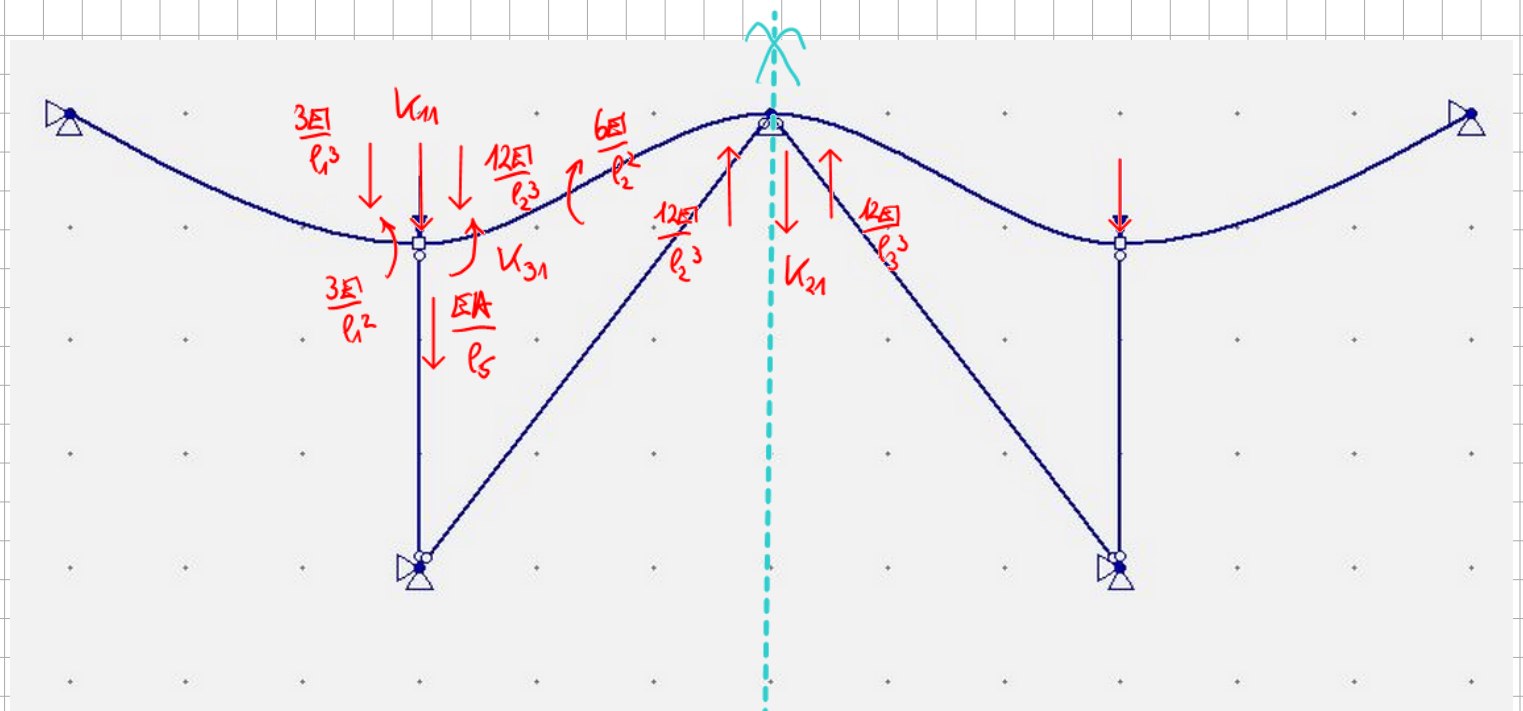


$$K_{10} = -\left(\frac{5}{8} \cdot q \cdot l_1 + \frac{q \cdot l_2^2}{2}\right) \cdot 2 = -101,25$$

$$K_{20} = -\frac{q \cdot l_2}{2} - \frac{q \cdot l_3}{2} = -45$$

$$K_{30} = \left(\frac{q \cdot l_2^2}{12} - \frac{q \cdot l_1^2}{8}\right) \cdot 2 = -11,25$$

E2 1

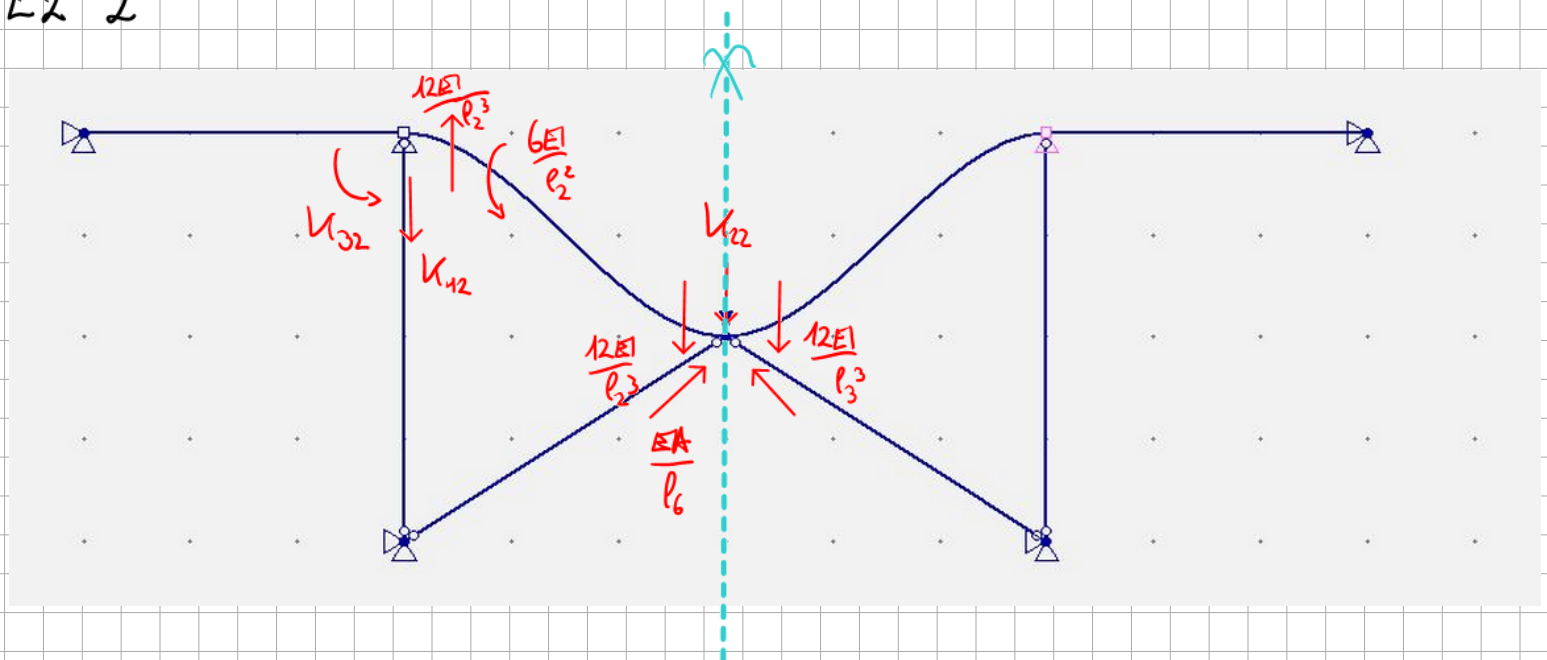


$$K_{11} = \left(\frac{3EI}{l_1^3} + \frac{12EI}{l_2^3} + \frac{EA}{l_5} \right) \cdot 2 = (222,2 + 888,8 + 2500) \cdot 2 = 7222,2$$

$$K_{21} = -\frac{12EI}{l_2^3} \cdot 2 = -1777,7$$

$$K_{31} = \left(\frac{3EI}{l_1^3} - \frac{6EI}{l_2^3} \right) \cdot 2 = (666,6 - 1333,3) \cdot 2 = -1333,3$$

E2 2

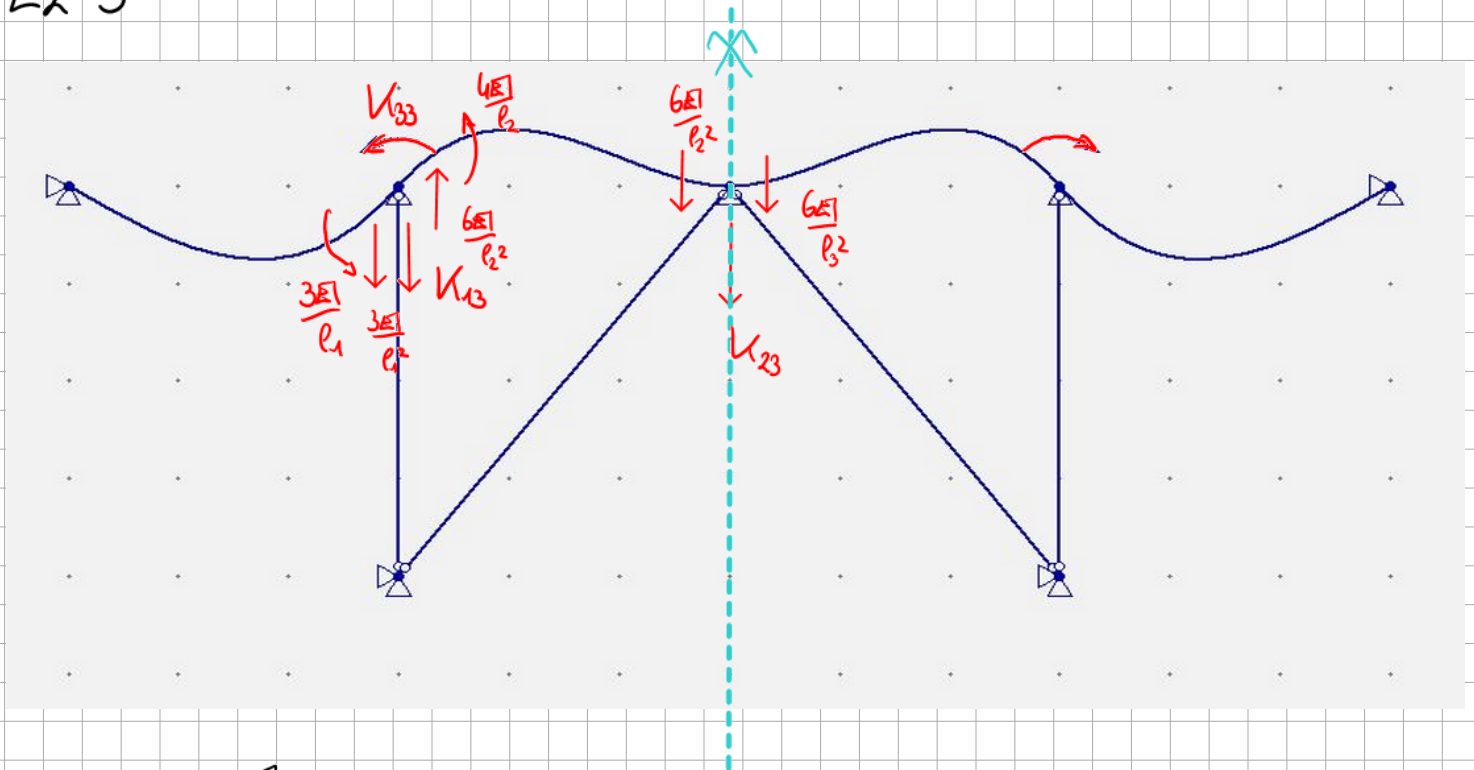


$$K_{12} = -\frac{12EI}{l_2^3} \cdot 2 = -1777,7$$

$$K_{22} = \left(\frac{12EI}{l_2^3} \cdot 2 + \frac{EA}{l_6} \cdot \frac{4}{5} \cdot \frac{4}{5} \right) \cdot 2 = (888,8 + 640) \cdot 2 = 3057,7$$

$$K_{32} = \frac{6EI}{l_2^3} \cdot 2 = 2666,6$$

E2 3



$$K_{13} = \left(\frac{3EI}{l_1^2} - \frac{6EI}{l_2^2} \right) \cdot 2 = (666,6 - 1333,3) \cdot 2 = -1333,3$$

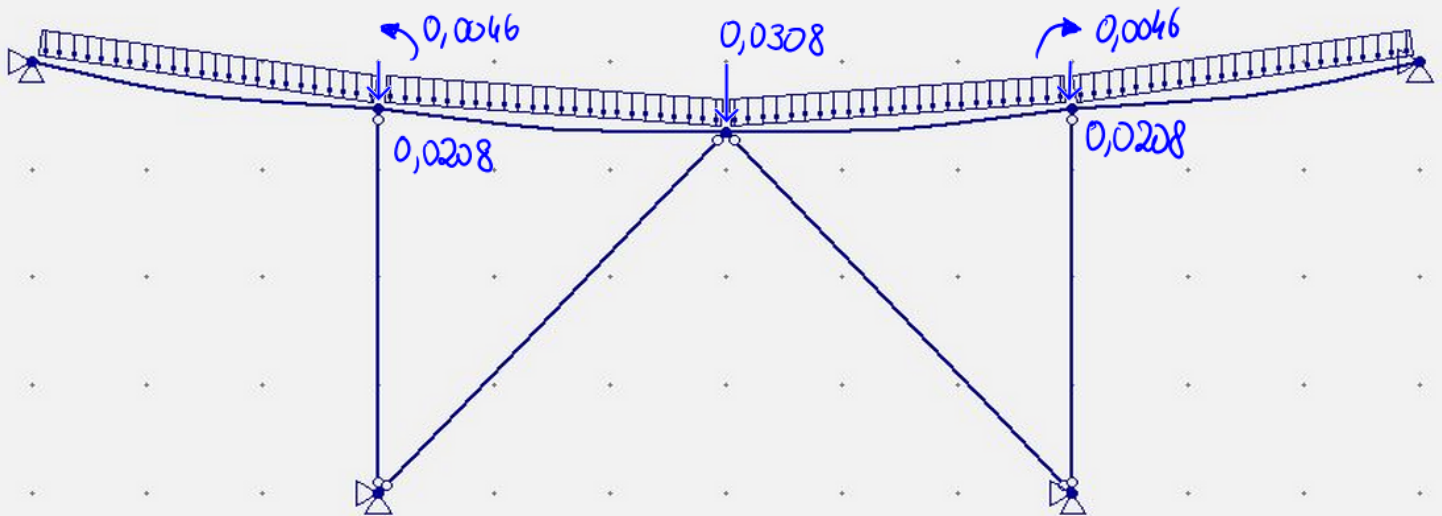
$$K_{23} = \frac{6EI}{l_2^2} \cdot 2 = 2666,6$$

$$K_{33} = \left(\frac{3EI}{l_1} + \frac{4EI}{l_2} \right) \cdot 2 = (2000 + 2666,6) \cdot 2 = 9333,3$$

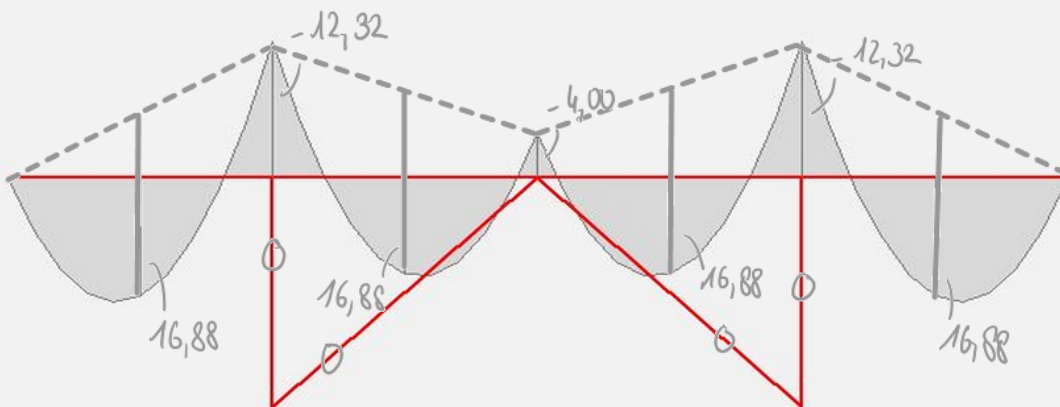
$$\underline{K} = \begin{bmatrix} 7222,2 & -1777,7 & -1333,3 \\ -1777,7 & 3057,7 & 2666,6 \\ -1333,3 & 2666,6 & 9333,3 \end{bmatrix} \quad \underline{F} = -\underline{k}_0 = \begin{bmatrix} 101,25 \\ 45 \\ 11,25 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \Rightarrow \underline{u} = \begin{bmatrix} 0,02075 \text{ m} \\ 0,03083 \text{ m} \\ 0,00464 \text{ rad} \end{bmatrix}$$

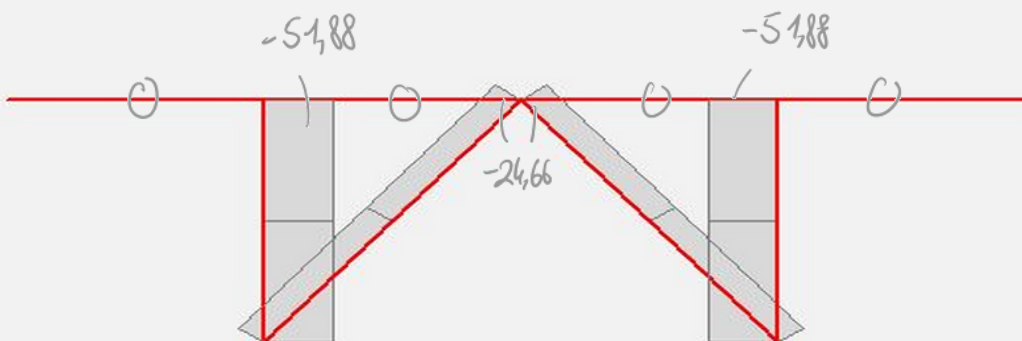
(b) Verformung



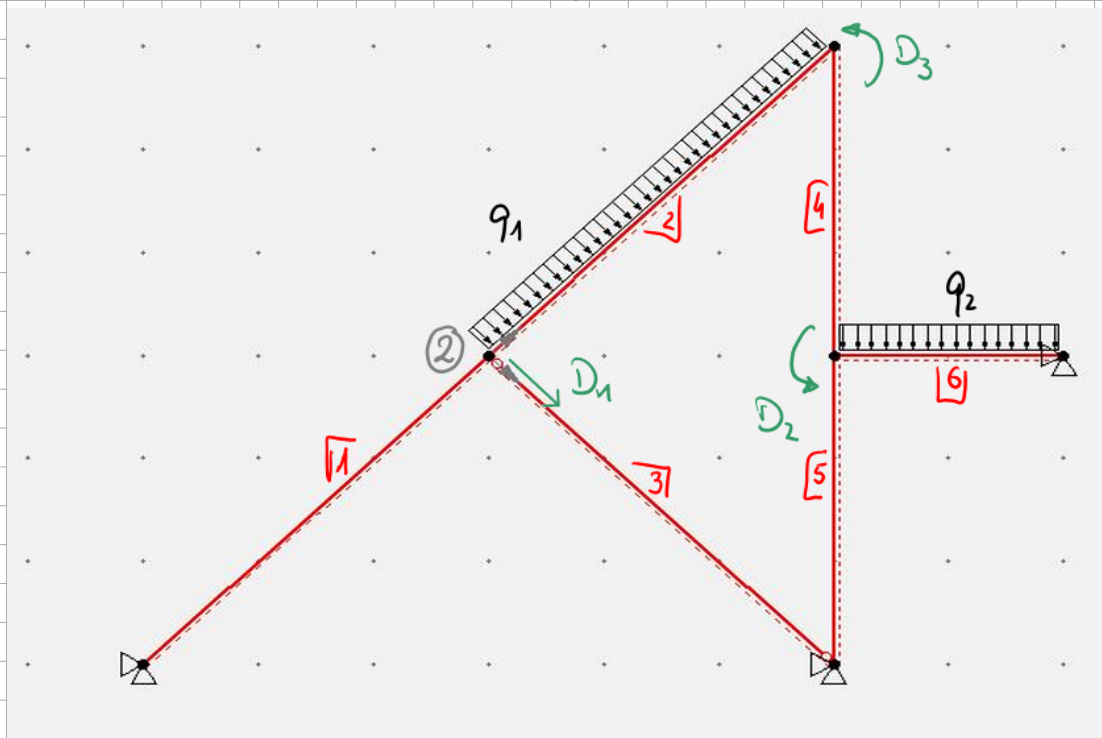
Momentenverlauf M



Normalkraftverlauf N



Probeklausur 5 - Aufgabe 2



$$q_1 = 15 \text{ kN/m}$$

$$q_2 = 30 \text{ kN/m}$$

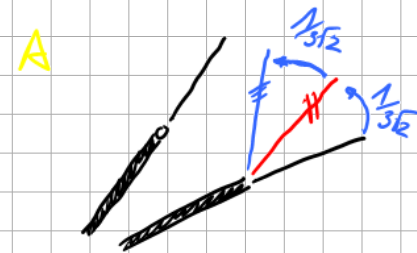
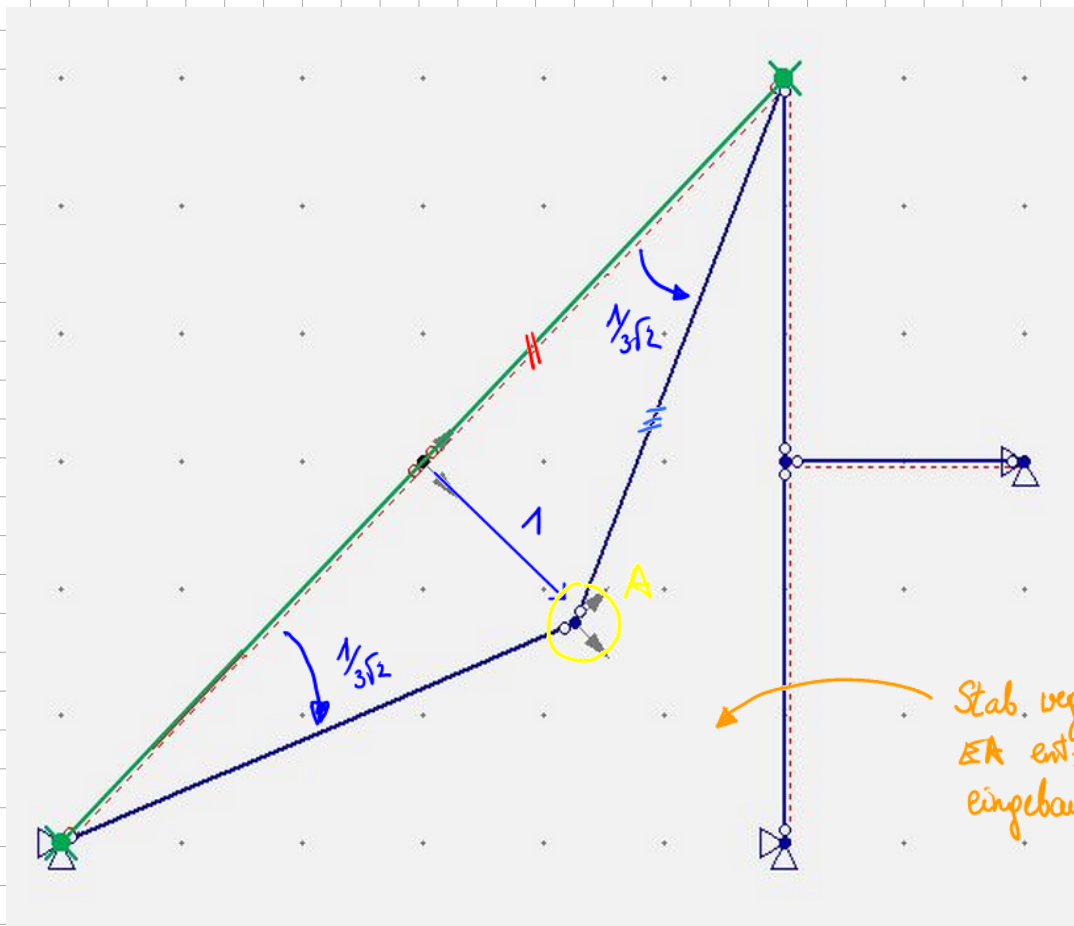
$$EI_1 \rightarrow \infty$$

$$EI_{\text{rest}} = 20 \text{ kNm}^2$$

$$EA_3 = 30 \text{ kN}$$

$$EA_{\text{rest}} \rightarrow \infty$$

Gelenkfigur



Stab wegen endlichem EA entfernt ($\hat{=}$ Normaldruckgelenk eingebaut)

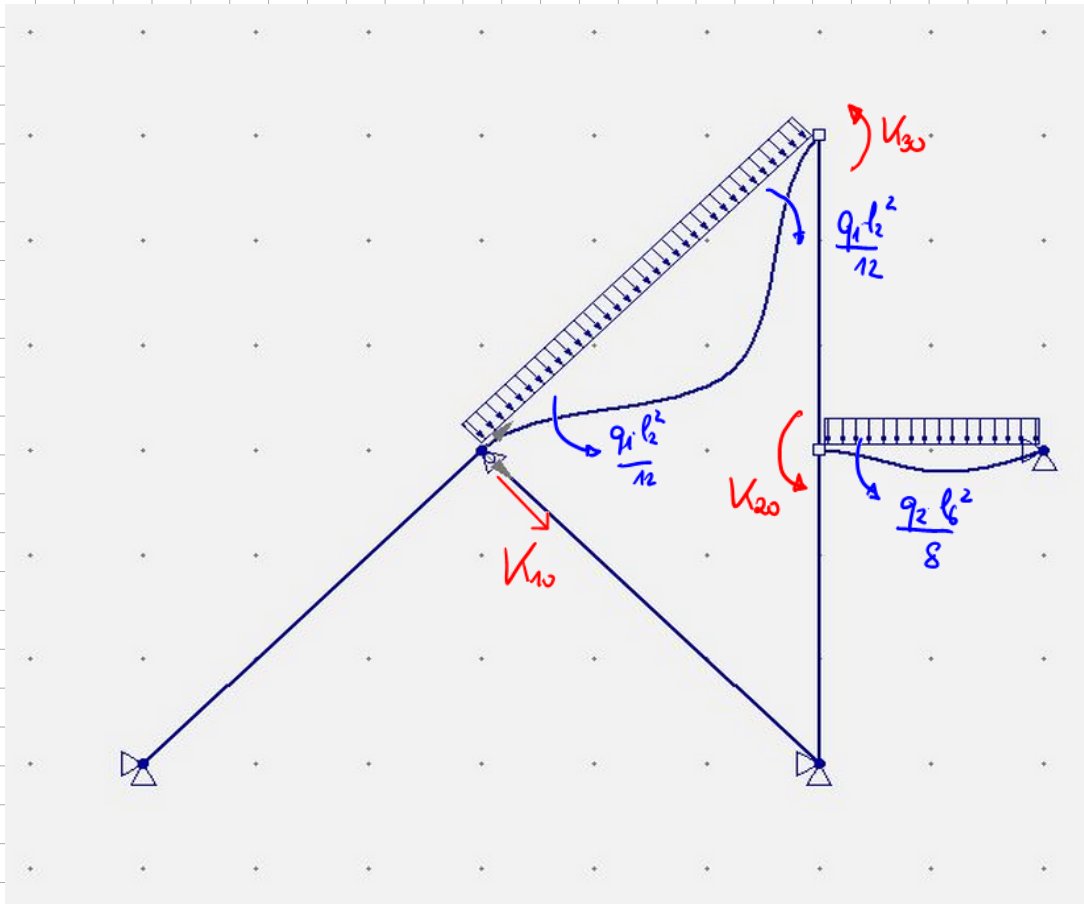
(a) kinematische Abhängigkeit: $\varphi_1 = \frac{D_1}{3\sqrt{2}}$

$$u_2 = \frac{D_2}{f_2}$$

$$w_2 = \frac{D_2}{f_2}$$

(u_2 & w_2 global)

L2

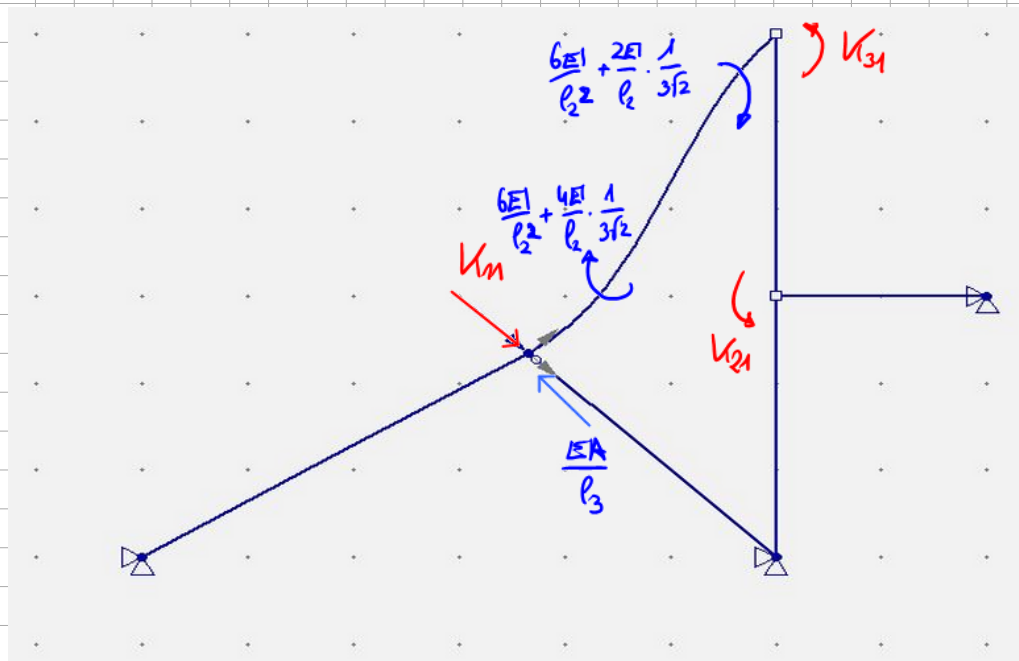


$$GG: K_{20} = \frac{q_2 \cdot l^2}{8} = 15$$

$$K_{30} = -\frac{q_1 \cdot l^2}{12} = -22,5$$

$$PV: K_{10} \cdot \bar{1} = \frac{q_1 \cdot l^2}{12} \cdot \frac{1}{3\sqrt{2}} - \frac{q_1 \cdot l^2}{12} \cdot \frac{2}{3\sqrt{2}} - q_1 \cdot l \cdot \frac{1}{2} = 5,303 - 10,606 - 31,820 = -37,123$$

Ex 1



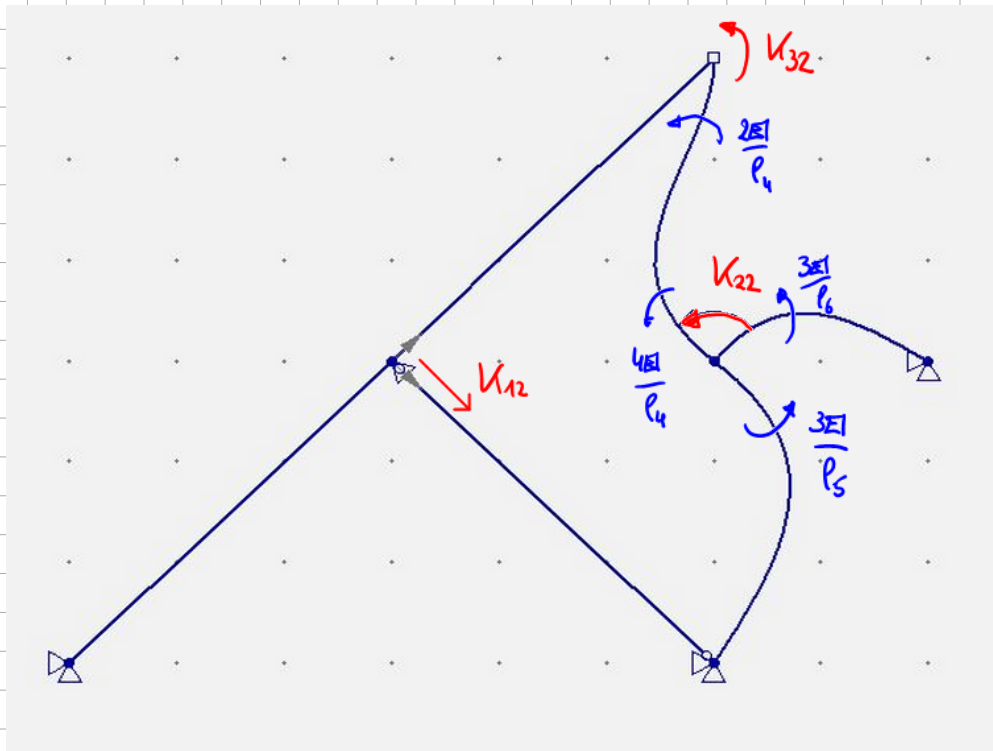
$$GG: K_{21} = 0$$

$$K_{31} = -\frac{6EI}{l^2} - \frac{2EI}{l} \cdot \frac{1}{3\sqrt{2}} = -6666,6 - 2222,2 = -8888,8$$

$$PV: K_{11} \cdot \bar{1} = \left(\frac{6EI}{l^2} + \frac{4EI}{l} \cdot \frac{1}{3\sqrt{2}} \right) \cdot \frac{2}{3\sqrt{2}} + \left(\frac{6EI}{l^2} + \frac{2EI}{l} \cdot \frac{1}{3\sqrt{2}} \right) \cdot \frac{1}{3\sqrt{2}} + \frac{EA}{l} \cdot \bar{1} =$$

$$= (6666,6 + 4444,4) \cdot \frac{2}{3\sqrt{2}} + (6666,6 + 2222,2) \cdot \frac{1}{3\sqrt{2}} + 7071,08 \cdot \bar{1} = 14404,03$$

E2 2



$$GG: K_{22} = \frac{4EI}{l_4} + \frac{3EI}{l_6} + \frac{3EI}{l_5} =$$

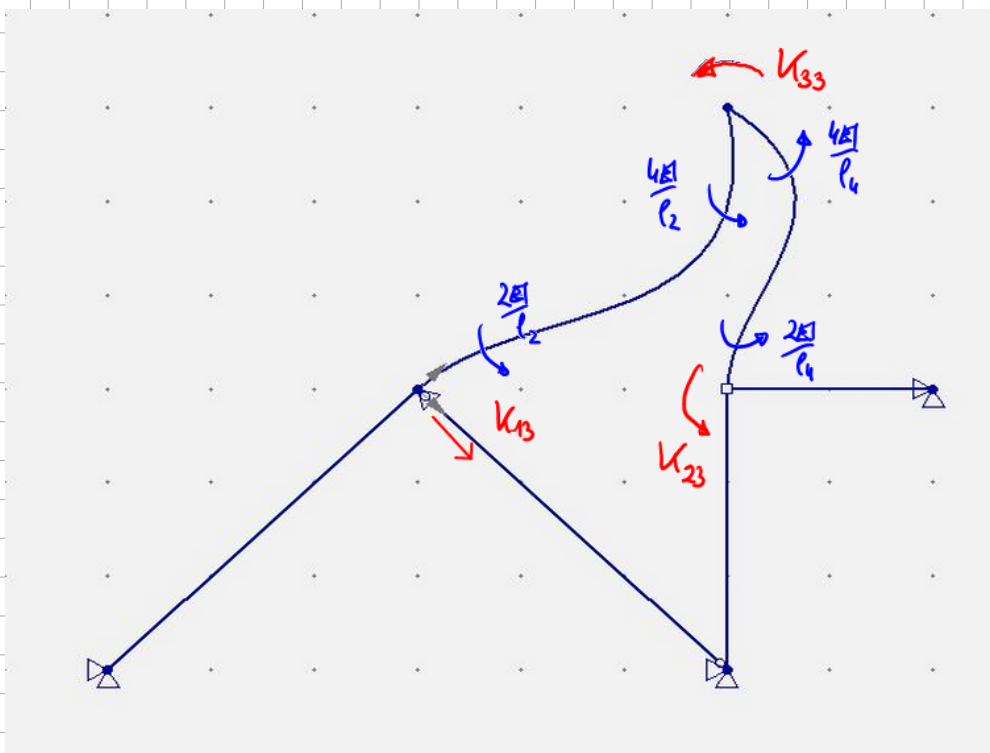
$$= 26\,666,6 + 30\,000 + 20\,000 =$$

$$= 76\,666,6$$

$$K_{32} = \frac{2EI}{l_4} = 13\,333,3$$

$$PV: K_{12} \cdot \bar{\Delta} = 0$$

E2 3



$$GG: K_{23} = \frac{2EI}{l_4} = 13\,333,3$$

$$K_{33} = \frac{4EI}{l_2} + \frac{4EI}{l_4} =$$

$$= 18\,856,18 + 26\,666,6 =$$

$$= 45\,522,85$$

$$PV: K_{13} \cdot \bar{\Delta} = -\frac{2EI}{l_2} \cdot \frac{2}{3\sqrt{2}} - \frac{4EI}{l_2} \cdot \frac{1}{3\sqrt{2}} = -4\,444,4 - 4\,444,4 = -8\,888,8$$

$$\underline{\underline{K}} = \begin{bmatrix} 14\,404,03 & 0 & -8\,888,8 \\ 0 & 76\,666,6 & 13\,333,3 \\ -8\,888,8 & 13\,333,3 & 45\,522,85 \end{bmatrix}$$

$$\underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 37,123 \\ -15 \\ 22,5 \end{bmatrix}$$

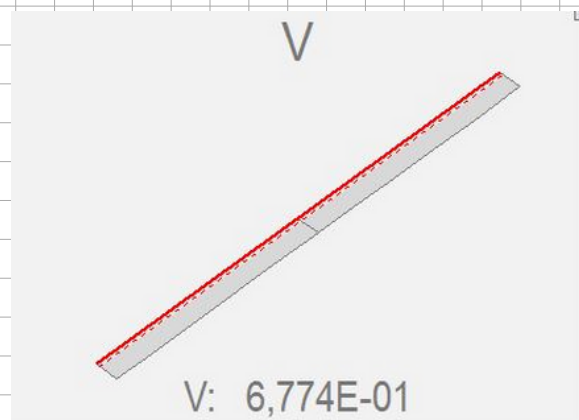
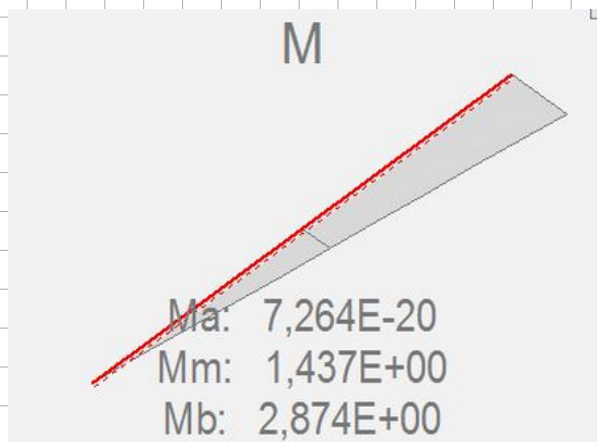
$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \Rightarrow \underline{\underline{u}} = \begin{bmatrix} 3,36 \cdot 10^{-3} \text{ m} \\ -4,17 \cdot 10^{-4} \text{ rad} \\ 1,27 \cdot 10^{-2} \text{ rad} \end{bmatrix}$$

(c)

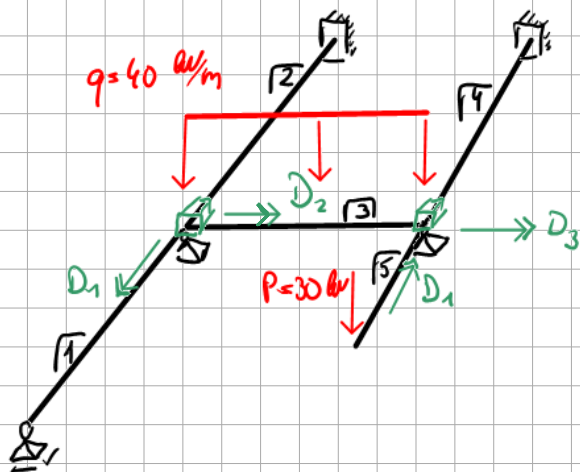
$$M = -\frac{q_1 \cdot l_2^2}{12} + 3,36 \cdot 10^{-3} \cdot \left(\frac{6EI}{l_2^2} + \frac{4EI}{l_2} \cdot \frac{1}{3\sqrt{2}} \right) + 1,27 \cdot 10^{-2} \cdot \left(-\frac{2EI}{l_2} \right) =$$

$$= -22,5 + 37,3 - 11,97 = 2,86 \text{ kNm}$$

$$V = \frac{M}{l_1} = \frac{2,86}{3\sqrt{2}} = 0,67 \text{ kN}$$



Probeklausur 5 - Aufgabe 3



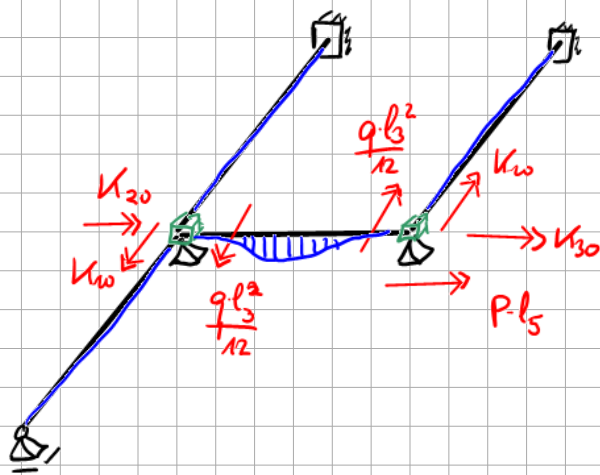
$$EI = 1000 \text{ kNm}^2$$

$$GI_T = 500 \text{ kNm}^2$$

$$q = 40 \text{ kN/m}$$

$$P = 30 \text{ kN}$$

L2

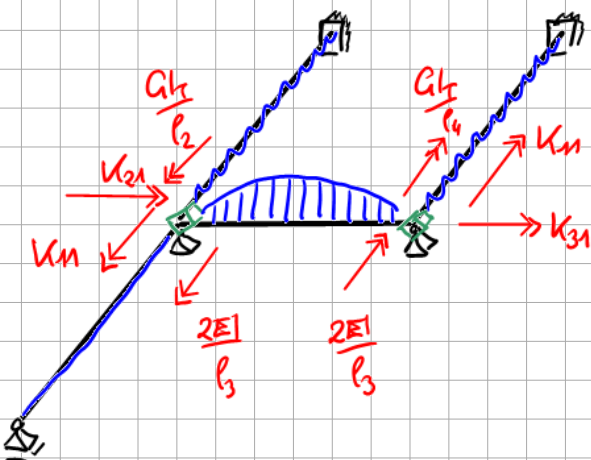


$$K_{10} = \frac{q l_3^2}{12} = 13,3$$

$$K_{20} = 0$$

$$K_{30} = P l_5 = 30$$

E2 1

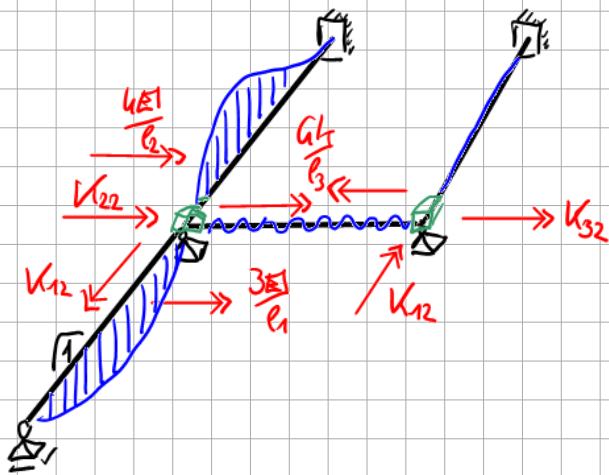


$$K_{11} = \frac{2EI}{l_3} + \frac{GI_T}{l_2} = 1000 + 250 = 1250$$

$$K_{21} = 0$$

$$K_{31} = 0$$

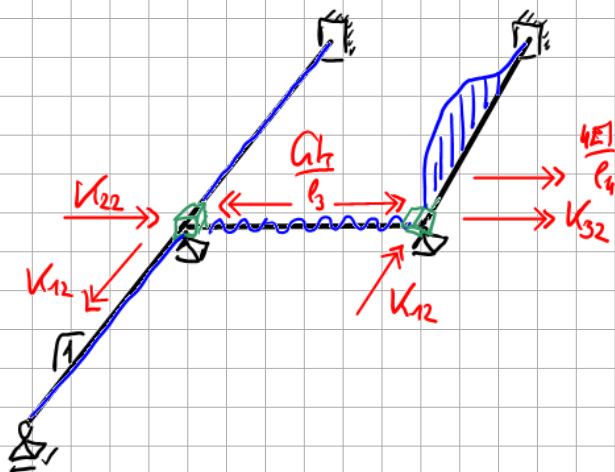
E2 2



$$K_{12} = 0$$

$$K_{22} = \frac{4EI}{l_2} + \frac{3EI}{l_1} = 2000 + 1500 + 250 = 3750$$

$$K_{32} = -\frac{4EI}{l_3} = -250$$



$$K_{12} = 0$$

$$K_{22} = -\frac{4EI}{l_3} = -250$$

$$K_{32} = \frac{4EI}{l_3} + \frac{4EI}{l_4} = 250 + 2000 = 2250$$

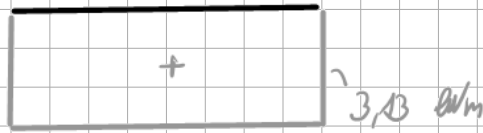
$$\underline{K} = \begin{bmatrix} 1250 & 0 & 0 \\ 0 & 3750 & -250 \\ 0 & -250 & 2250 \end{bmatrix} \quad \underline{F} = -\underline{K}_0 = \begin{bmatrix} -13,3 \\ 0 \\ 30 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \quad \Rightarrow \quad \underline{u} = \begin{bmatrix} -0,0106 \text{ rad} \\ 8,95 \cdot 10^{-4} \text{ rad} \\ 0,0134 \text{ rad} \end{bmatrix}$$

$$M_T = 0 + 0 + 8,95 \cdot 10^{-4} \cdot \left(-\frac{q_L}{b_3}\right) + 0,024 \cdot \frac{q_L}{b_3} =$$

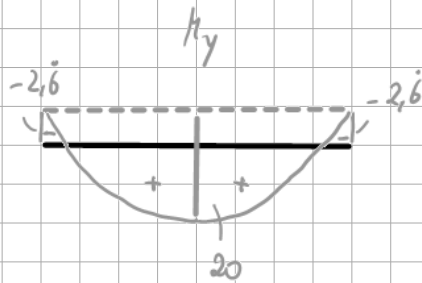
$$= -0,22 + 3,35 = 3,13 \text{ kNm}$$

M_T



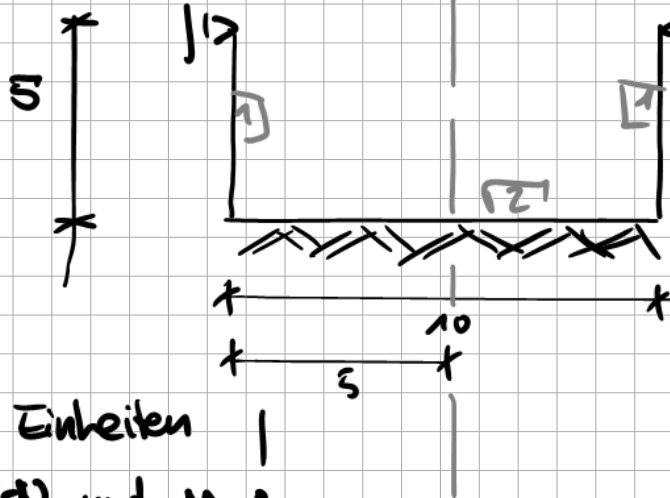
$$M_y = -\frac{q \cdot l_3^2}{12} + (-0,0106) \cdot \left(-\frac{2EI}{l_3}\right) + 0 + 0 =$$

$$= -13,3 + 10,6 = -2,6$$



Probeklausur 3 - Aufgabe 4

$$40 \cdot 0.2 = 8$$



Alle Einheiten
kN und m.

$$EI_1 = 5000$$

$$EI_2 = 10000$$

$$EA \rightarrow \infty$$

$$k = 1000$$

$$\lambda = \left(\frac{1000}{4 \cdot 10000} \right)^{1/4} = 0.398$$

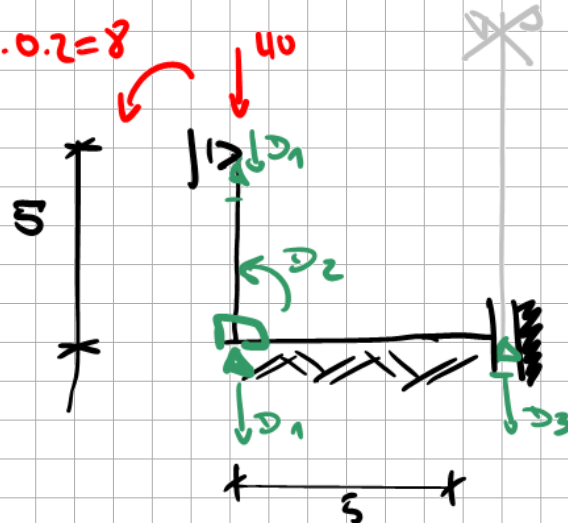
- Kompletter Balken : $\lambda \cdot 10 = 3.98 > \pi \rightarrow$ genäherte Werte
- halber Balken : $\lambda \cdot 5 = 1.99 < \pi \rightarrow$ exakte Werte

→ Problem:

Rückrechnung bis in Mitte nicht einfach, da nicht ∞ -lang bis in Mitte.

→ Lösung:

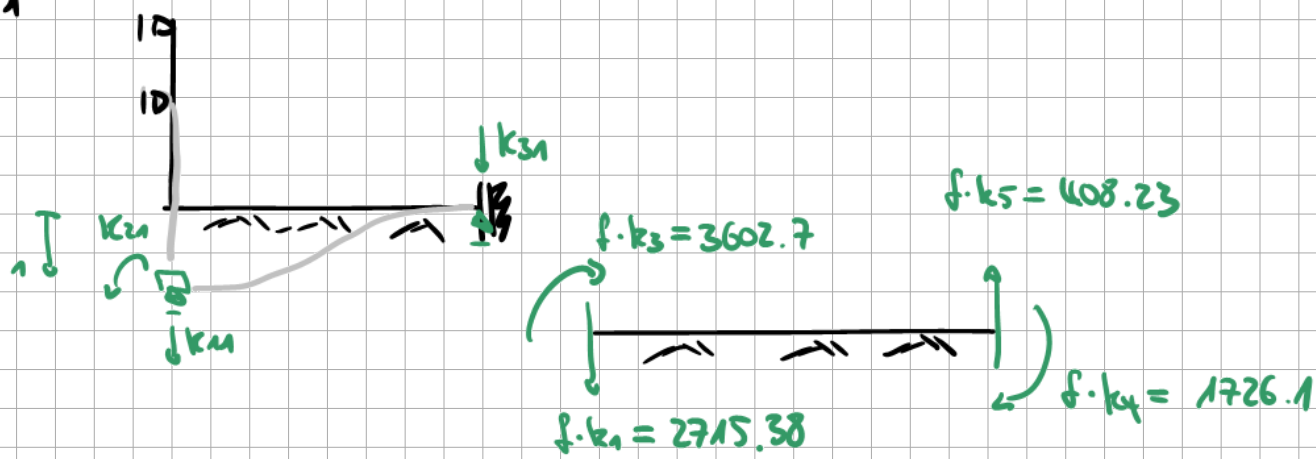
$$40 \cdot 0.2 = 8$$



$$\lambda \cdot l = \lambda \cdot 5 = 1.99$$

Wir nutzen die Symmetrie und rechnen direkt am halben System
→ Nachlaufrechnung der Zustandsgrößen einfach aus Ets.

E21 $\mathcal{D}_1 = 1$

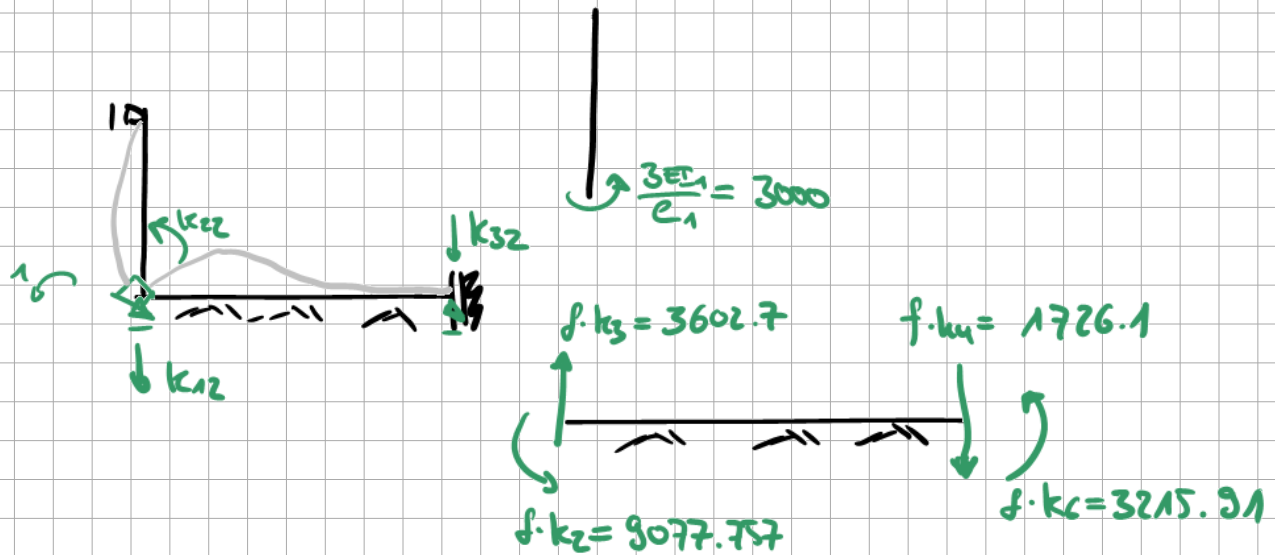


$$k_{11} = 2715.38$$

$$k_{21} = -3602.7$$

$$k_{31} = -408.23$$

E22 $\mathcal{D}_2 = 1$

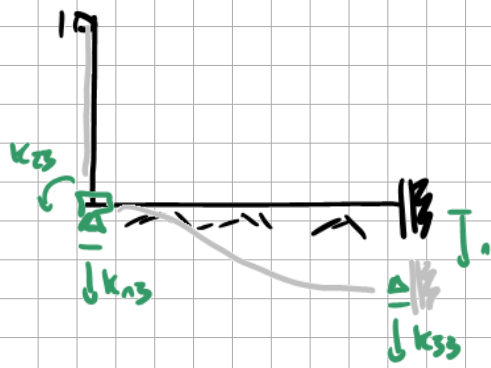


$$k_{12} = -3602.7 = k_{21} \checkmark$$

$$k_{22} = 9077.757 + 3000 = 12077.757$$

$$k_{32} = 1726.1$$

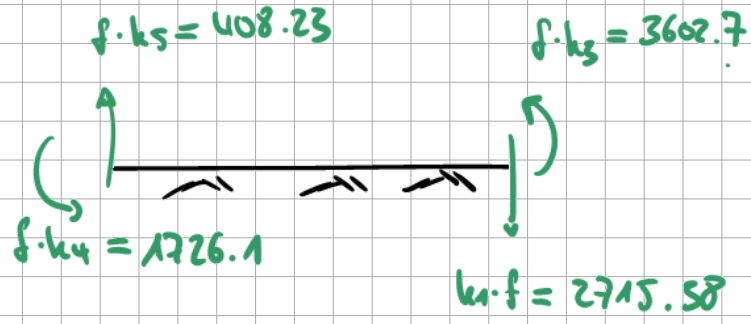
E23 $D_3 = 1$



$$k_{13} = -408.23 = k_{31} \quad \checkmark$$

$$k_{23} = 1726.1 = k_{32} \quad \checkmark$$

$$k_{33} = 2715.38$$

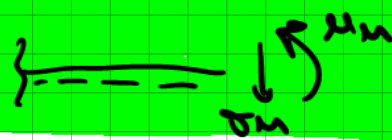


System:

$$\begin{bmatrix} 2715.38 & -3602.7 & -408.23 \\ -3602.7 & 12077.757 & 1726.1 \\ -408.23 & 1726.1 & 2715.38 \end{bmatrix} \cdot \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = -\underline{k_0} = \underline{F}$$

$$\rightarrow \underline{D}^T = [0.0237; 6.853 \cdot 10^{-3}; -7.925 \cdot 10^{-4}]$$

M+Q in Feldmitte



Darzeichen!
pos. SM

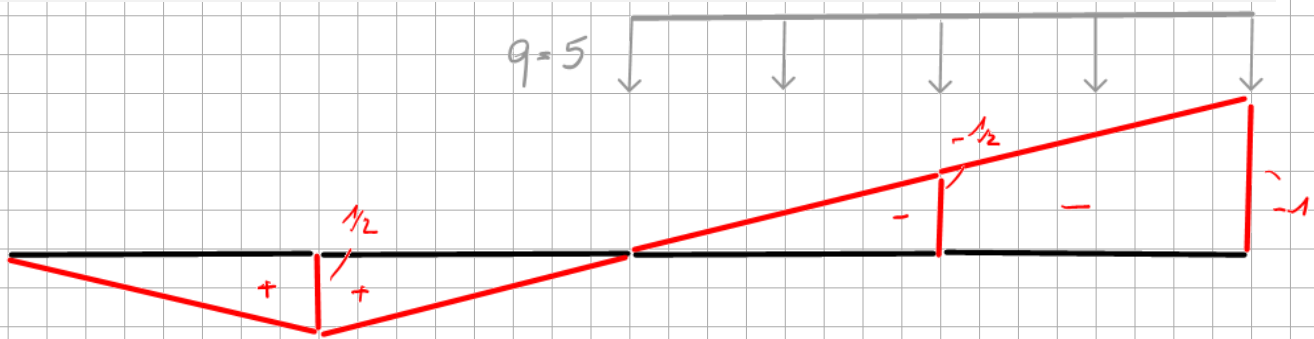
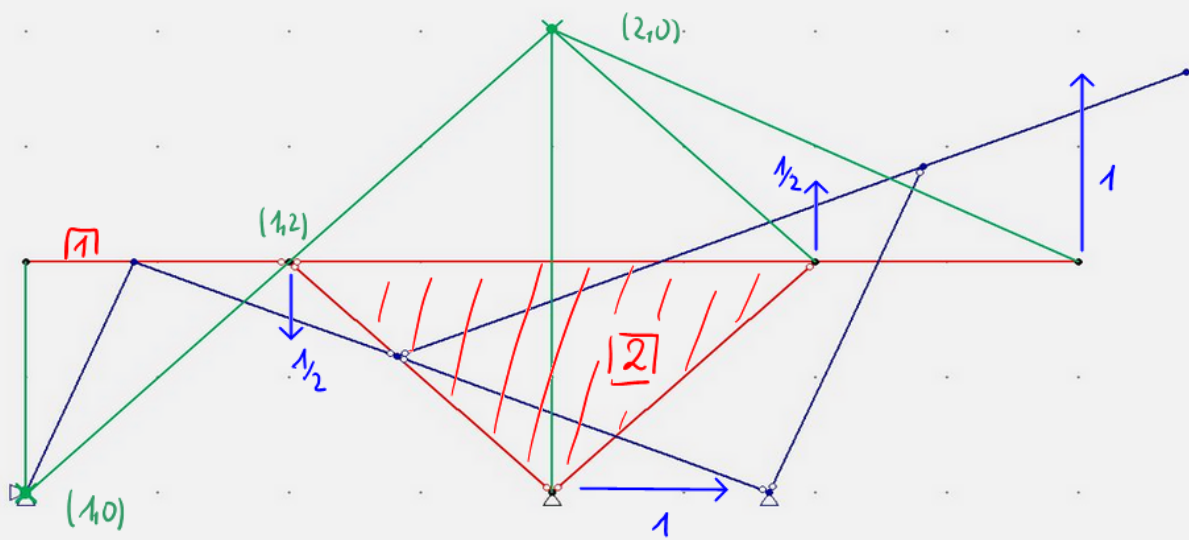
$$\begin{aligned} \cdot Q_m &= -408.23 \cdot D_1 + 17726.1 \cdot D_2 + 2715.38 \cdot D_3 \\ &= 1.574 \cdot 10^3 \approx 0 \end{aligned}$$

→ war aus last + System sym. zu erwarten

$$\begin{aligned} \cdot M_m &= -1726.1 \cdot D_1 + 3215.91 \cdot D_2 + 3602.7 \cdot D_3 \\ &= -21.725 \end{aligned}$$

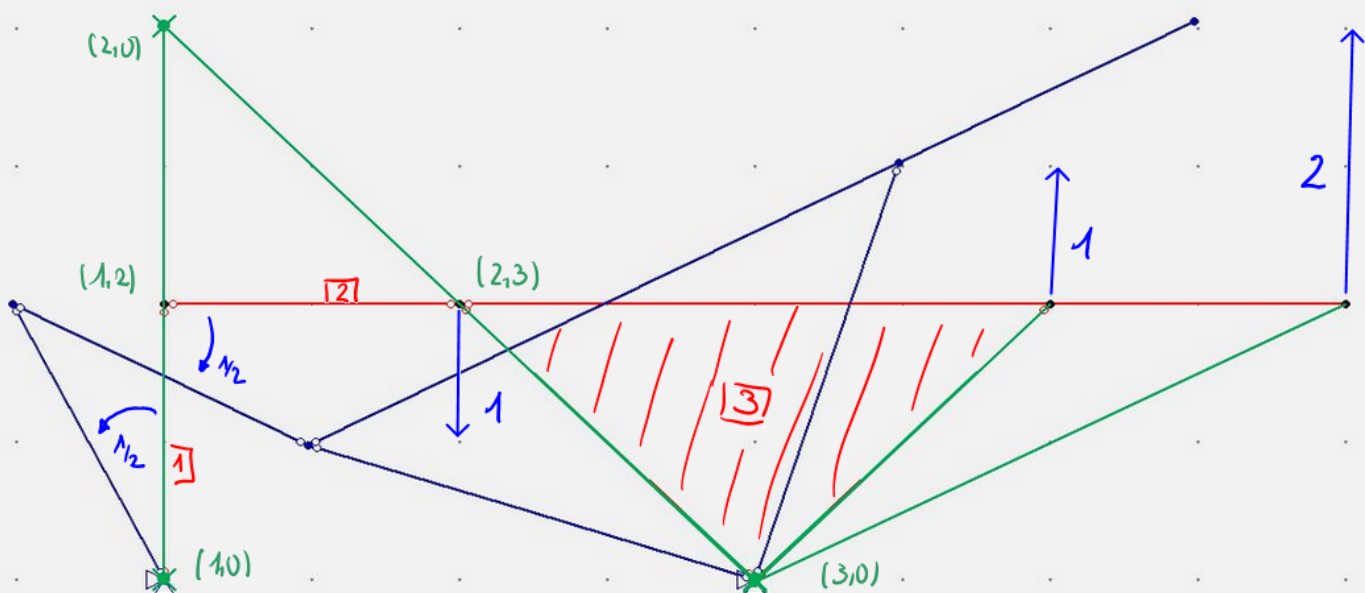
Probeklausur 5 - Aufgabe 5

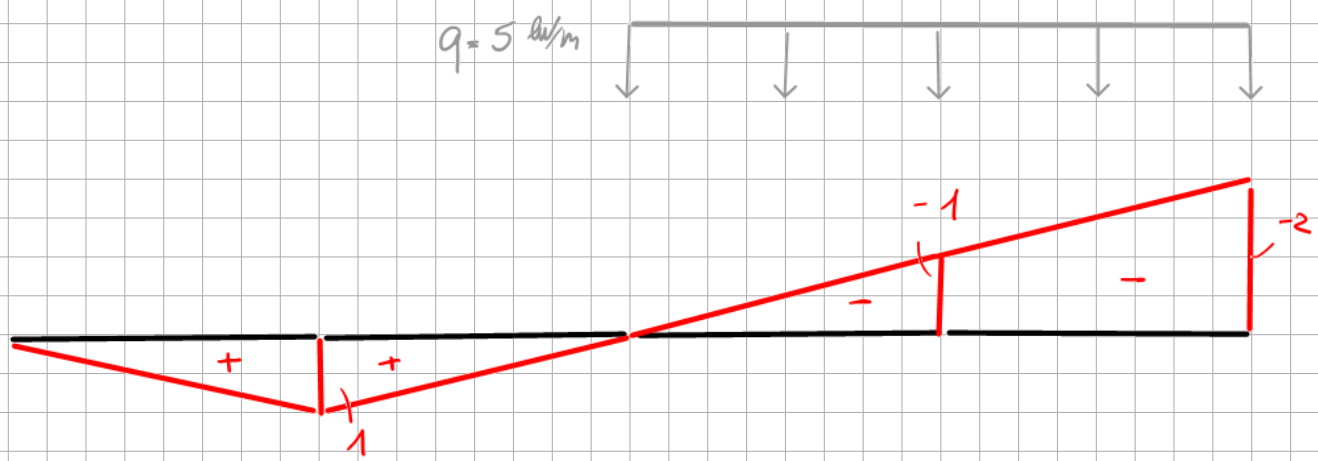
(a)



$$B_H = \int q(x) \cdot \eta(x) dx = 5 \frac{\text{m}}{\text{m}} \cdot 4 \text{m} \cdot \left(-\frac{1}{2}\right) = -10 \text{ m}$$

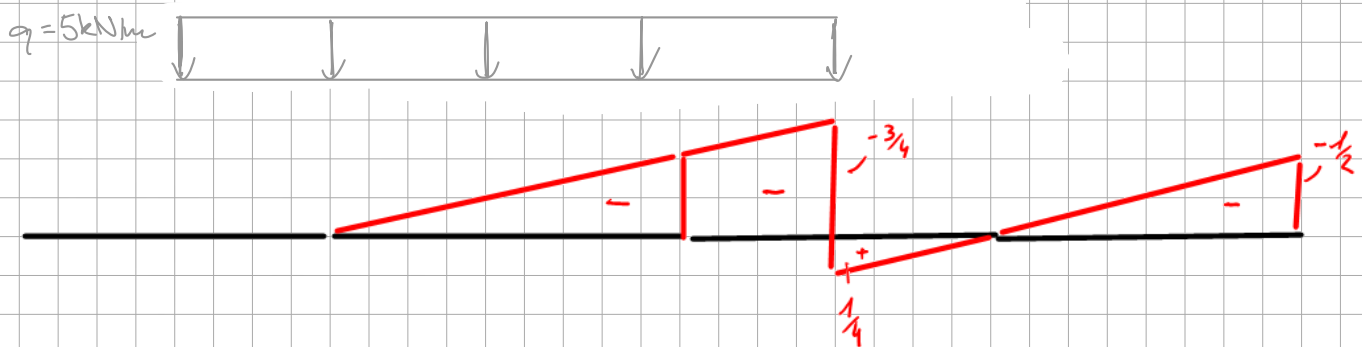
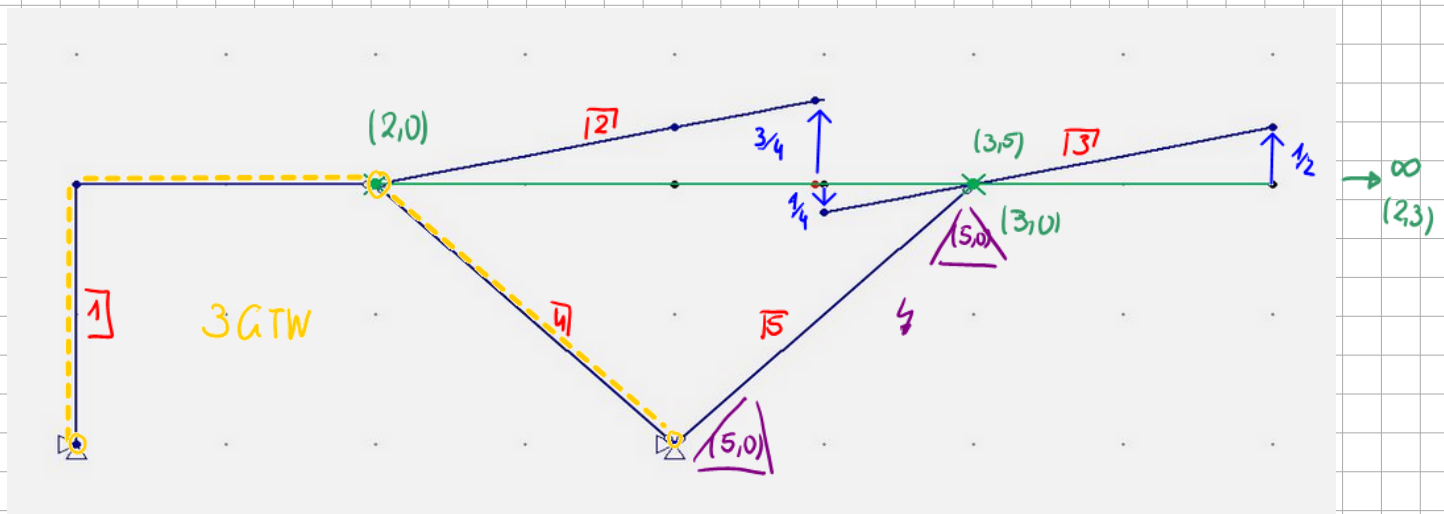
(b)





$$M_i = \int_l q(x) \eta(x) dx = 5 \cdot 4 \cdot 1 = 20 \text{ kNm}$$

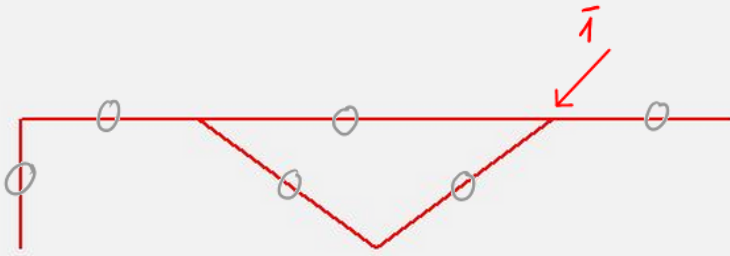
(c)



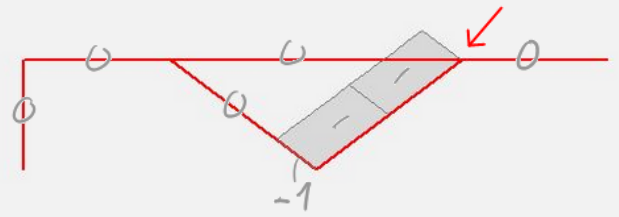
$$V_k = \int_l q(x) \cdot \eta(x) dx = 5 \cdot 3 \cdot \left(-\frac{3}{8}\right) = -5,625 \text{ kN}$$

(d)

M

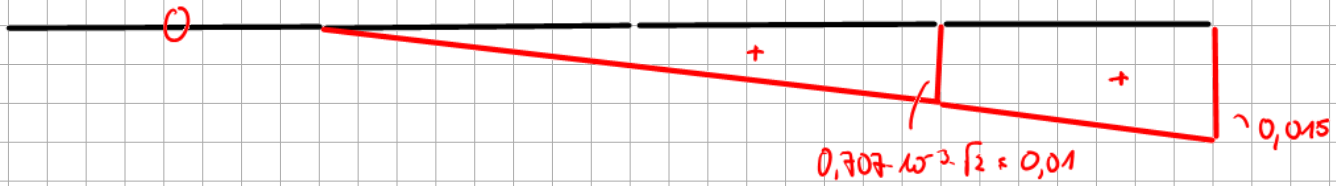


N



$$W = (-1)^2 \cdot \frac{2\sqrt{2}}{\Delta A} = 7,07 \cdot 10^{-3}$$

$$q = 5 \text{ N/m}$$

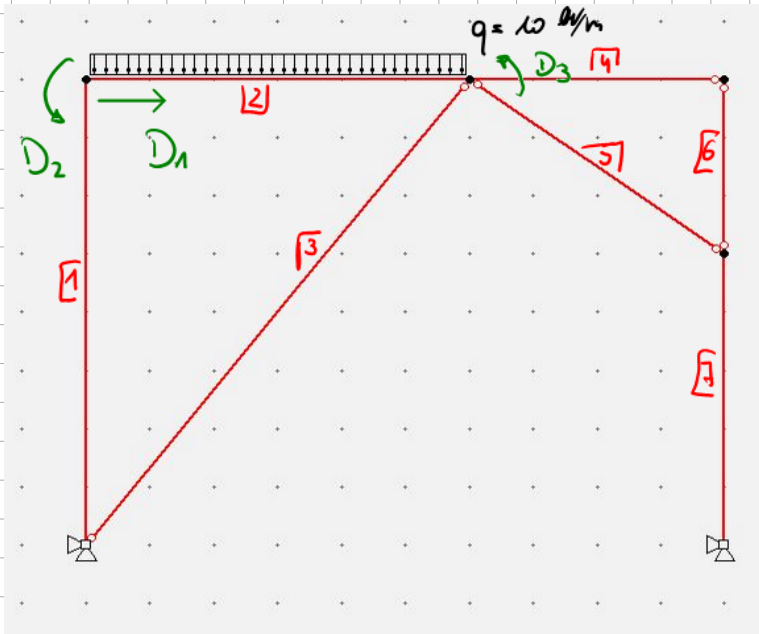


$$W = \int_0^l q(x) \cdot \eta(x) = 5 \cdot 4 \cdot 0,05 + \frac{1}{2} \cdot 5 \cdot 4 \cdot 0,01 = 1,1$$

$\square \times \square$ $\square \times \triangle$

Statik Musterlösungen

Probeklausur G - Aufgabe 1



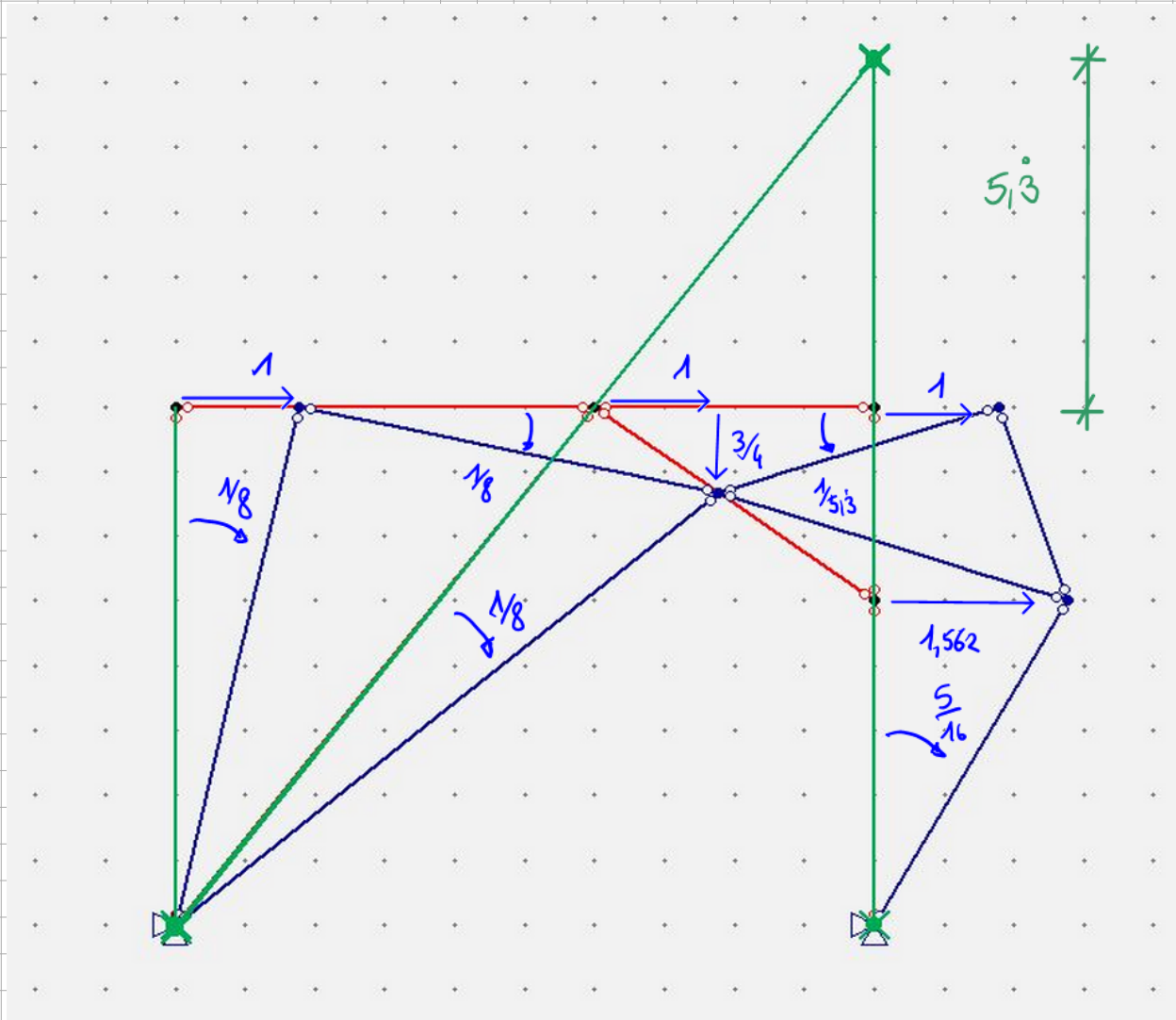
$$EI = 1000$$

$$EA \rightarrow \infty$$

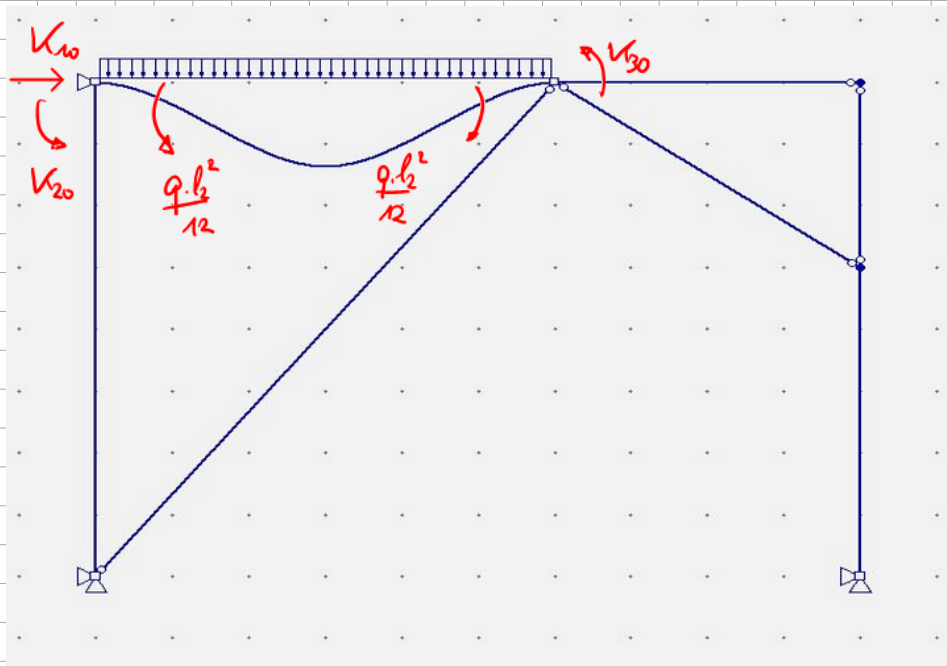
$$q = 10 \text{ kN/m}$$

$$\Rightarrow n_g = 3$$

Gelenkfiguren



L2

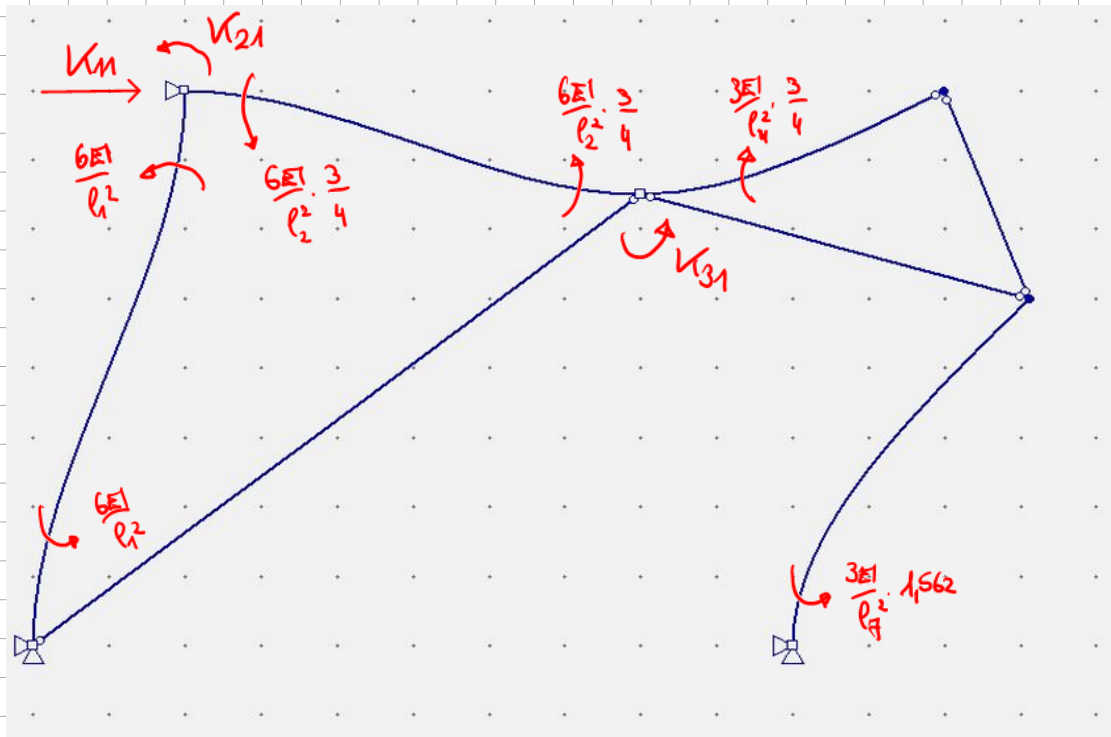


$$K_{10} \cdot \bar{1} = \frac{q \cdot l^2}{12} \cdot \frac{1}{8} - \frac{q \cdot l^2}{12} \cdot \frac{1}{8} - q \cdot l \cdot \frac{5}{8} = 30 - 30 - 22,5 = -22,5$$

$$K_{20} = \frac{q \cdot l^2}{12} = 30$$

$$K_{30} = -\frac{q \cdot l^2}{12} = -30$$

E2 1



AG:

$$K_{21} = \frac{6EI}{l^2} + \frac{6EI}{l^2} \cdot \frac{3}{4} = 93,75 + 125 = 218,75$$

$$K_{31} = \frac{6EI}{l^2} \cdot \frac{3}{4} - \frac{3EI}{l^2} \cdot \frac{3}{4} = 125 - 140,63 = -15,63$$

$$K_{11} \cdot \bar{1} = \frac{6EI}{l^2} \cdot \frac{1}{8} \cdot 2 + \frac{6EI}{l^2} \cdot \frac{3}{4} \cdot \frac{1}{8} \cdot 2 + \frac{3EI}{l^2} \cdot \frac{3}{4} \cdot \frac{1}{5,3} + \frac{3EI}{l^2} \cdot 1,562 \cdot \frac{5}{16} =$$

$$= 23,48 + 31,25 + 26,37 + 58,58 = 139,68$$

Diagram of a frame structure with stiffness values:

- Member 1-2: k_{12} (horizontal), k_{22} (vertical), $\frac{4EI}{l_2}$ (rotation at node 2)
- Member 2-3: $\frac{2EI}{l_2}$ (rotation at node 2), k_{32} (rotation at node 3)
- Member 3-4: $\frac{2EI}{l_1}$ (rotation at node 3)

$$K_{22} = \frac{4E}{l_1} + \frac{4E}{l_2} = 500 + 666,6 = 1166,6$$

$$K_{12} \cdot \bar{1} = \frac{451}{l_1} \cdot \frac{\bar{1}}{8} + \frac{251}{l_1} \cdot \frac{\bar{1}}{8} + \frac{451}{l_2} \cdot \frac{\bar{1}}{8} + \frac{251}{l_2} \cdot \frac{\bar{1}}{8} = 62,5 + 31,25 + 83,3 + 41,6 = 218,75 = K_{21} \checkmark$$

$$K_{23} = \frac{2A}{b_2} = 333,3$$

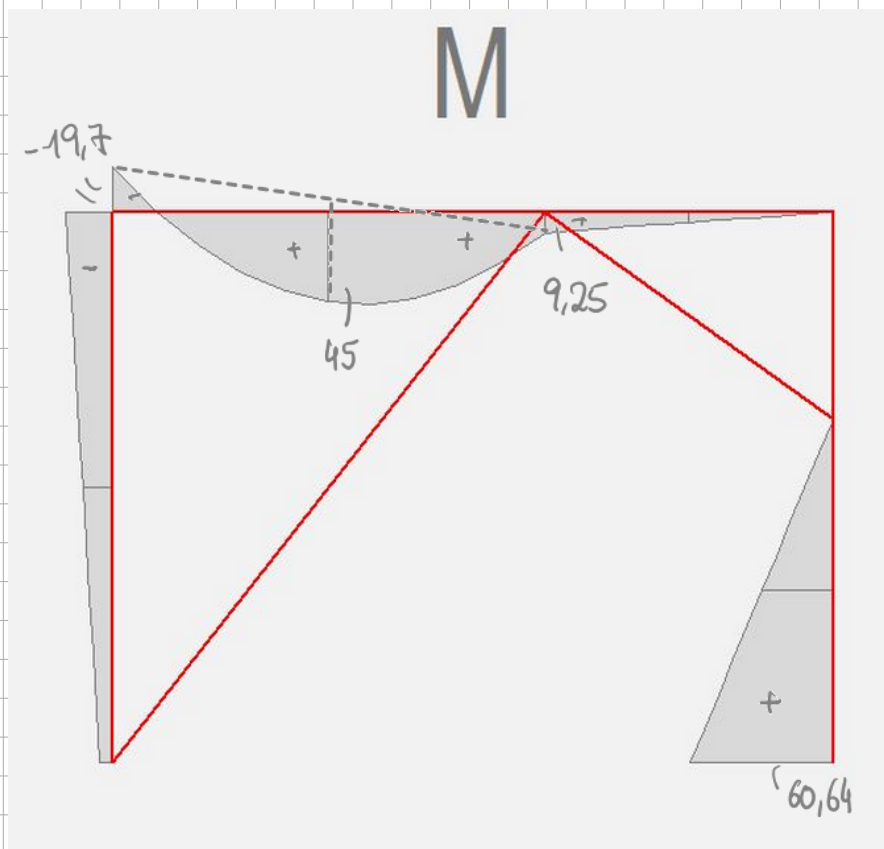
$$V_{33} = \frac{4E}{l_2} + \frac{3E}{l_4} = 666,6 + 750 = 1416,6$$

$$K_{13} \cdot \bar{1} = \frac{4\cancel{2}1}{b_3} \cdot \frac{\bar{1}}{8} + \frac{2\cancel{2}1}{b_6} \cdot \frac{\bar{1}}{8} - \frac{3\cancel{2}1}{b_4} \cdot \frac{\bar{1}}{5\cancel{3}} = 83,3 + 44,6 - 140,635 = -15,63$$

$$\underline{\underline{K}} = \begin{bmatrix} 139,65 & 218,75 & -15,63 \\ 218,75 & 1166,6 & 333,3 \\ -15,63 & 333,3 & 1416,6 \end{bmatrix} \quad \underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 22,5 \\ -30 \\ 30 \end{bmatrix}$$

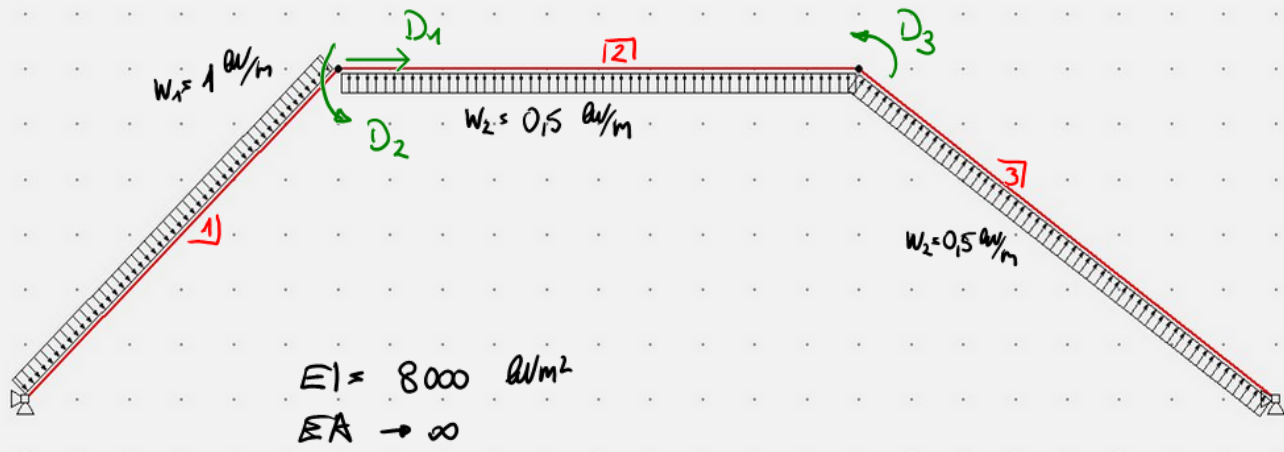
$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \Rightarrow \underline{\underline{u}} = \begin{bmatrix} 0,3234 \text{ m} \\ -0,1002 \text{ rad} \\ 0,0483 \text{ rad} \end{bmatrix}$$

Momentenverlauf



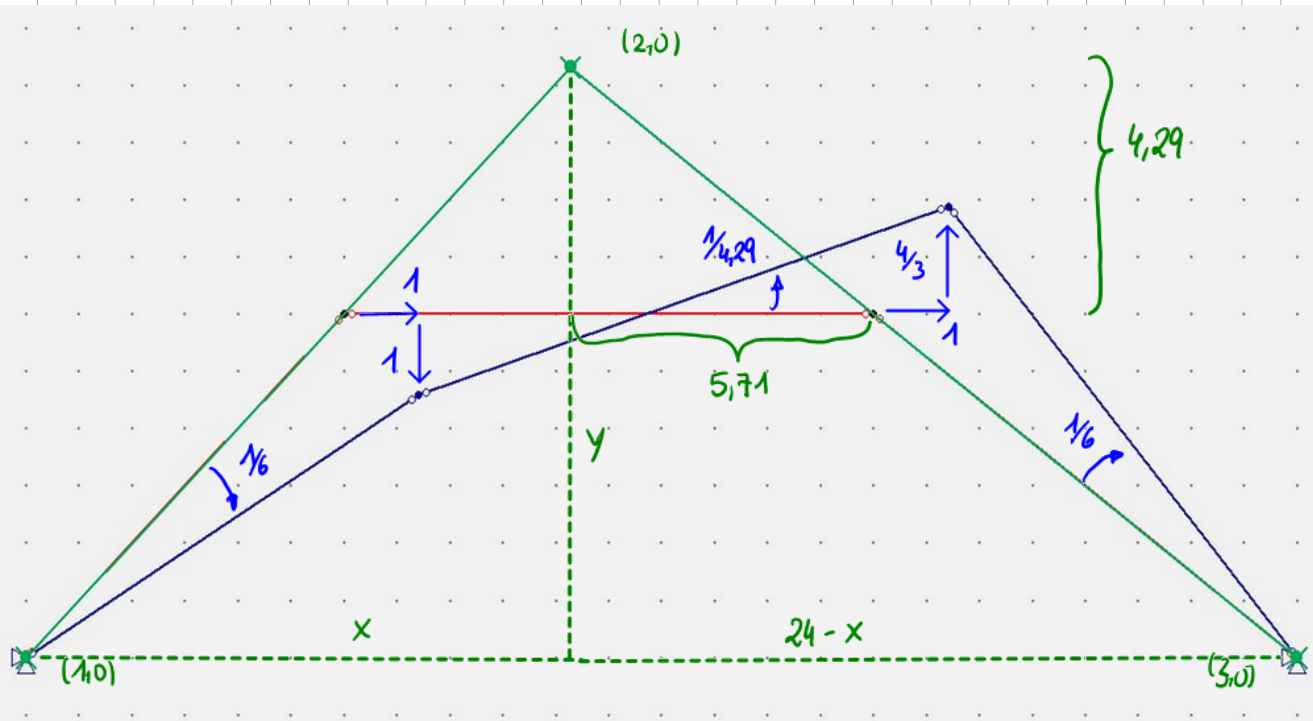
Probeklausur 6 - Aufgabe 2

(a)



Gelenkfigur

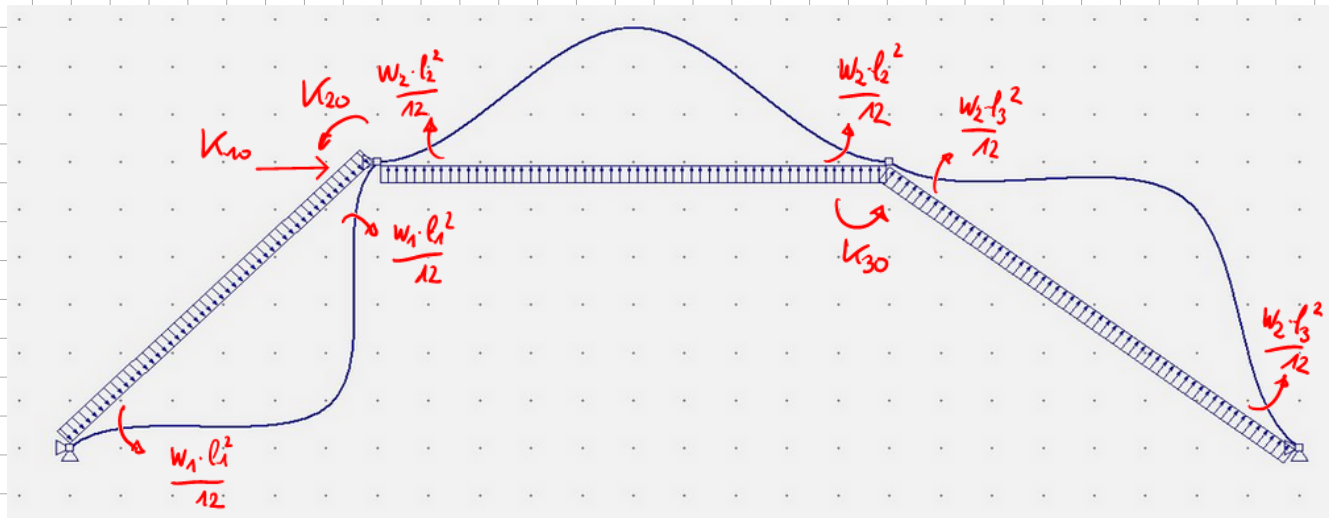
$n_g = 3$



$$\begin{aligned} \text{Stab 1: } y &= x \cdot \frac{6}{6} = x \\ \text{Stab 2: } y &= (24 - x) \cdot \frac{6}{8} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Stab 1: } y &= x \cdot \frac{6}{6} = x \\ \text{Stab 2: } y &= (24 - x) \cdot \frac{6}{8} \end{aligned}} \right\} \rightarrow x = (24 - x) \cdot \frac{6}{8} \rightarrow x = 10,29$$

$$y = 10,29$$

(b) L2

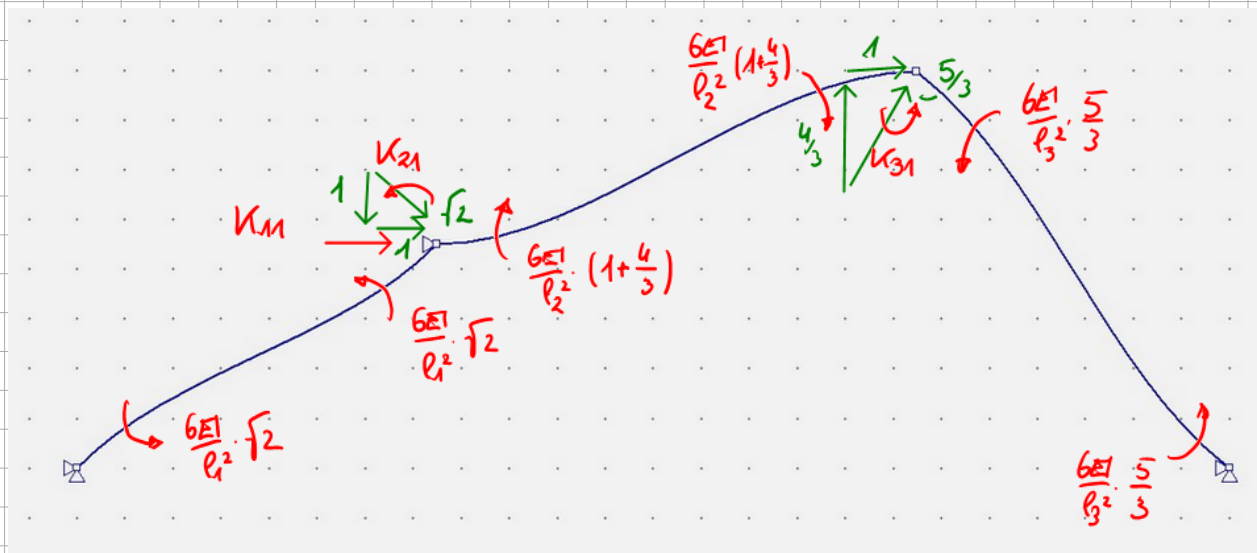


$$PvV: K_{10} \cdot \bar{1} = \frac{w_1 l_1^2}{12} \cdot \left(\frac{\bar{1}}{6} - \frac{\bar{1}}{6} \right) + \frac{w_2 l_2^2}{12} \cdot \left(\frac{\bar{1}}{4,29} - \frac{\bar{1}}{4,29} \right) + \frac{w_3 l_3^2}{12} \cdot \left(\frac{\bar{1}}{6} - \frac{\bar{1}}{6} \right) - w_1 l_1 \cdot \frac{\bar{1}}{2} \cdot \sqrt{2} - w_2 l_2 \cdot \left(\frac{\bar{4}}{3} - \bar{1} \right) \cdot \frac{1}{2} - w_3 l_3 \cdot \frac{\bar{5}}{3} \cdot \frac{1}{2} = -6 - 0,83 - 4,16 = -11$$

$$GG: K_{20} = -\frac{w_1 l_1^2}{12} - \frac{w_2 l_2^2}{12} = -6 - 4,16 = -10,16$$

$$K_{30} = \frac{w_2 l_2^2}{12} - \frac{w_3 l_3^2}{12} = 0$$

E2 1



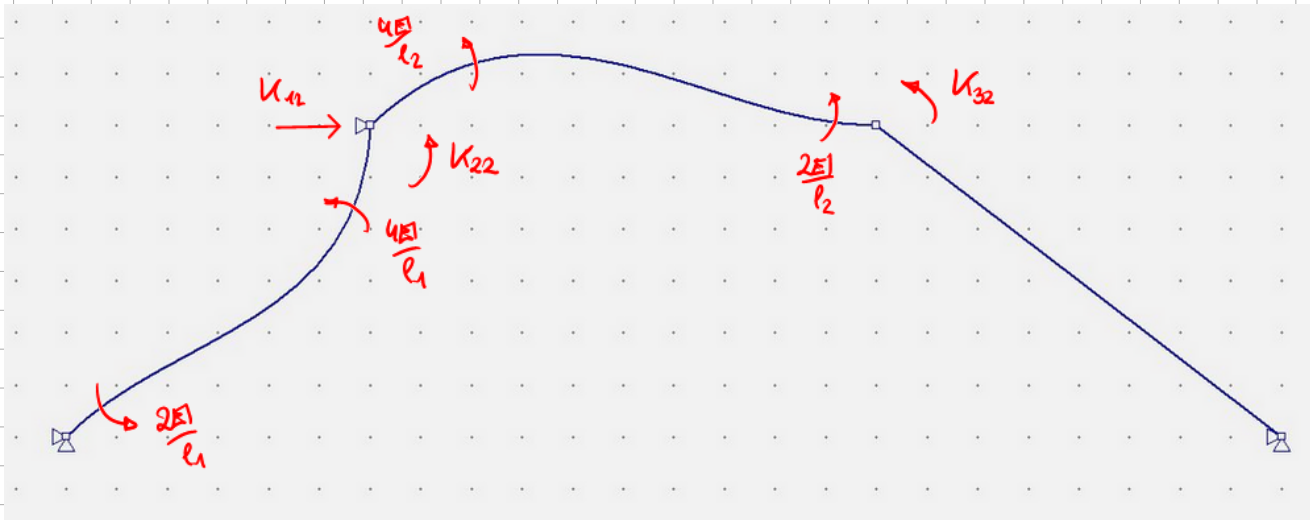
$$PvV: K_{11} \cdot \bar{1} = \frac{6EI}{l_1^2} \cdot \sqrt{2} \cdot \frac{\bar{1}}{6} \cdot 2 + \frac{6EI}{l_2^2} \cdot \left(1 + \frac{4}{3} \right) \cdot \frac{\bar{1}}{4,29} \cdot 2 + \frac{6EI}{l_3^2} \cdot \frac{5}{3} \cdot \frac{\bar{1}}{6} \cdot 2 =$$

$$= 314,27 + 522,14 + 266,6 = 1103,08$$

$$GG: K_{21} = \frac{6EI}{l_1^2} \cdot \sqrt{2} - \frac{6EI}{l_2^2} \cdot \left(1 + \frac{4}{3} \right) = 942,81 - 1120 = -177,19$$

$$K_{31} = \frac{6EI}{l_3^2} \cdot \frac{5}{3} - \frac{6EI}{l_2^2} \cdot \left(1 + \frac{4}{3} \right) = 800 - 1120 = -320$$

E2 2



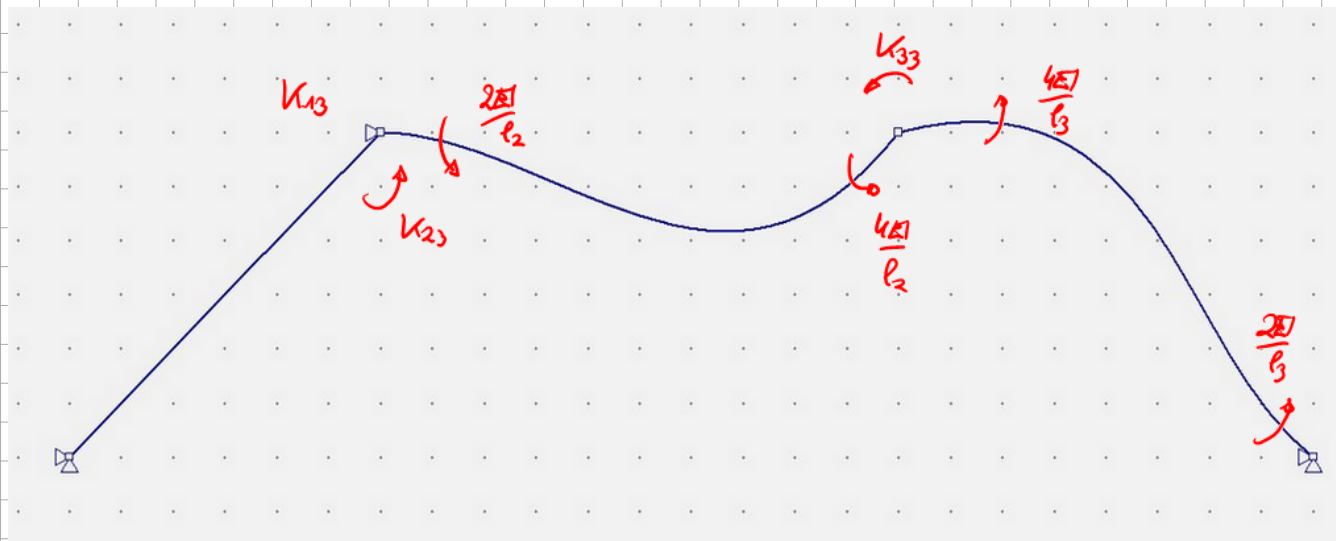
$$P_v V: K_{12} \cdot \bar{1} = \left(\frac{4EI}{l_1} + \frac{2EI}{l_1} \right) \cdot \frac{1}{6} - \left(\frac{4EI}{l_2} + \frac{2EI}{l_2} \right) \cdot \frac{1}{4,29} = 942,81 - 1118,88 = -176,07$$

(Abweichung von K_{21} wegen
Rundung in q_2)

$$GG: K_{22} = \frac{4EI}{l_1} + \frac{4EI}{l_2} = 3771,24 + 3200 = 6971,24$$

$$K_{32} = \frac{2EI}{l_2} = 1600$$

E2 3



$$P_v V: K_{13} \cdot \bar{1} = -\left(\frac{4EI}{l_2} + \frac{2EI}{l_2} \right) \cdot \frac{1}{4,29} + \left(\frac{4EI}{l_3} + \frac{2EI}{l_3} \right) \cdot \frac{1}{6} = -1118,88 + 800 = -318,88 \quad \text{so.}$$

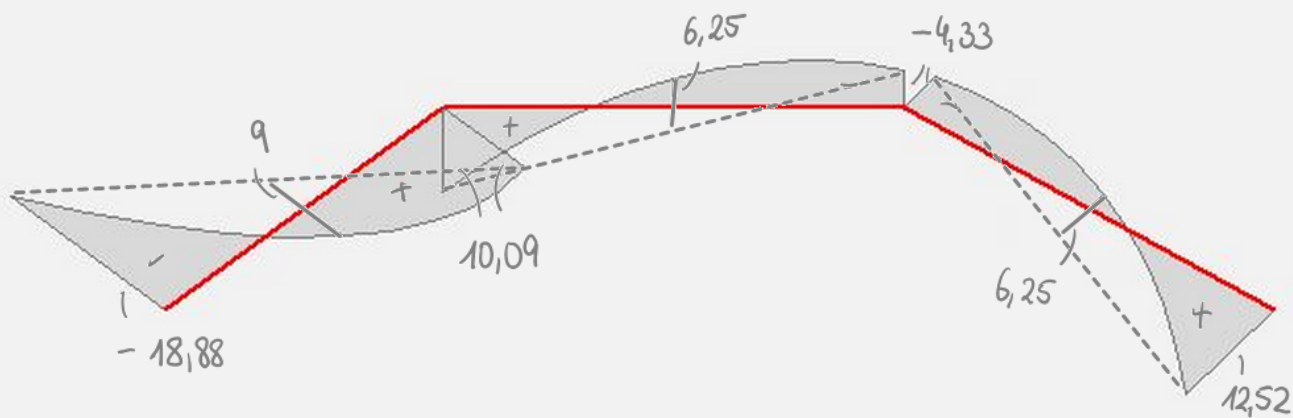
$$GG: K_{23} = \frac{2EI}{l_2} = 1600$$

$$K_{33} = \frac{4EI}{l_2} + \frac{4EI}{l_3} = 3200 + 3200 = 6400$$

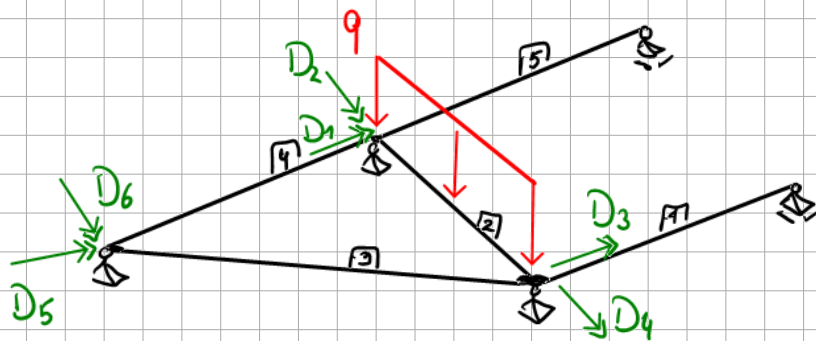
$$\underline{\underline{K}} = \begin{bmatrix} 1103,08 & -177,19 & -320 \\ -177,19 & 6971,24 & 1600 \\ -320 & 1600 & 6400 \end{bmatrix} \quad \underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 11 \\ 10,16 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \quad \Rightarrow \quad \underline{\underline{u}} = \begin{bmatrix} 0,0103 \text{ m} \\ 1,699 \cdot 10^{-3} \text{ rad} \\ 8,877 \cdot 10^{-5} \text{ rad} \end{bmatrix}$$

M



Probeklausur 6 - Aufgabe 3

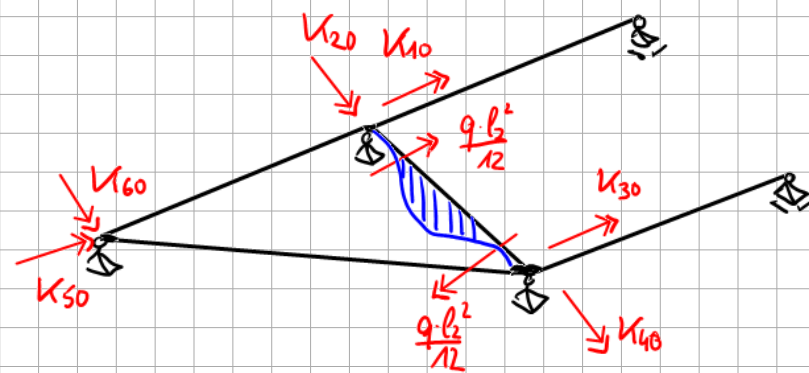


$$EI = 800 \text{ kNm}^2$$

$$GI_T = 400 \text{ kNm}^2$$

$$q = 30 \text{ kN/m}$$

L2

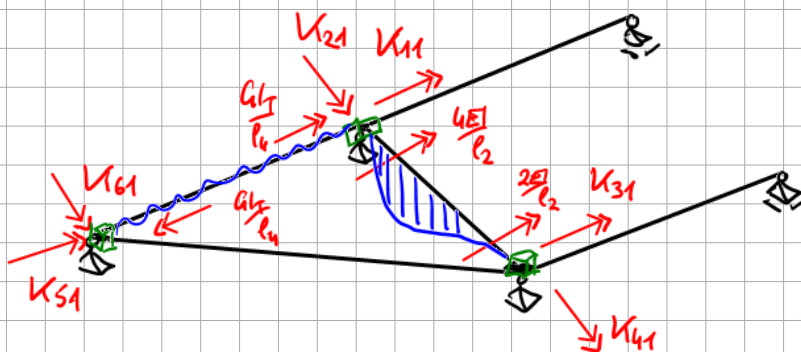


$$K_{10} = \frac{q \cdot l_2^2}{12} = 40$$

$$K_{30} = -\frac{q \cdot l_2^2}{12} = -40$$

$$K_{20} = K_{40} = K_{50} = K_{60} = 0$$

E2 1



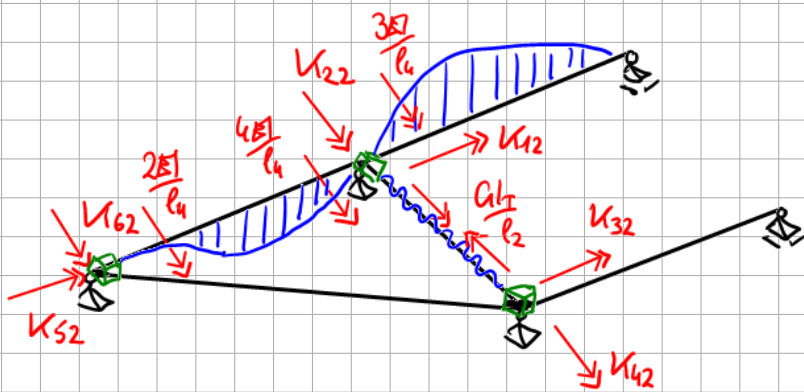
$$K_{11} = \frac{4EI}{l_2} + \frac{GI_T}{l_4} = 800 + 100 = 900$$

$$K_{31} = \frac{2EI}{l_2} = 400$$

$$K_{51} = -\frac{GI_T}{l_4} = -100$$

$$K_{21} = K_{41} = K_{61} = 0$$

E2 2



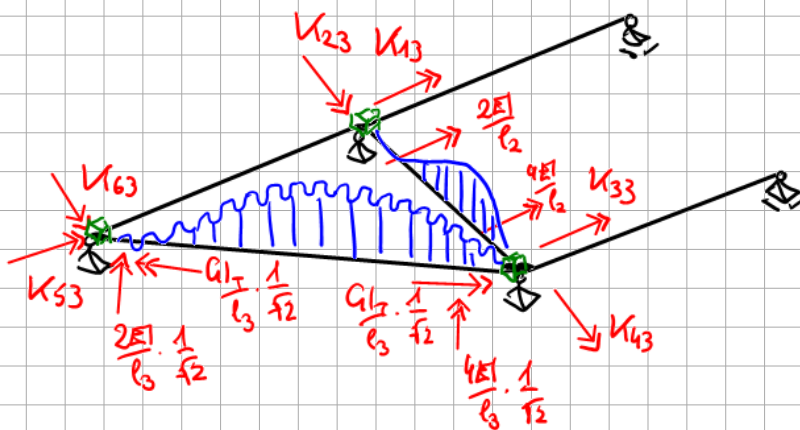
$$K_{22} = \frac{GI_T}{l_2} + \frac{4EI}{l_4} + \frac{3EI}{l_5} = 100 + 800 + 600 = 1500$$

$$K_{42} = -\frac{GI_T}{l_2} = -100$$

$$K_{62} = \frac{2EI}{l_4} = 400$$

$$K_{12} = K_{32} = K_{52} = 0$$

E2 3



$$K_{13} = \frac{2EI}{l_2} = 400$$

$$K_{23} = 0$$

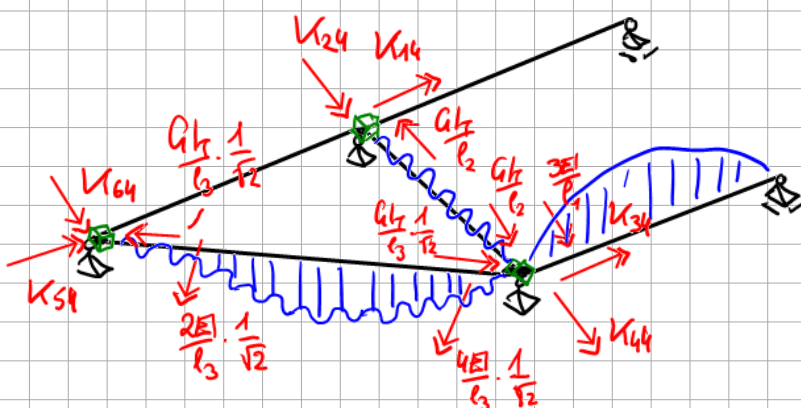
$$K_{33} = \frac{4EI}{l_2} + \frac{GI_T}{l_3} \cdot \frac{1}{2} + \frac{4EI}{l_3} \cdot \frac{1}{2} = 800 + 35,36 + 282,84 = 1118,2$$

$$K_{43} = \frac{GI_T}{l_3} \cdot \frac{1}{2} - \frac{4EI}{l_3} \cdot \frac{1}{2} = 35,36 - 282,84 = -247,48$$

$$K_{53} = \frac{2EI}{l_3} \cdot \frac{1}{2} - \frac{GI_T}{l_3} \cdot \frac{1}{2} = 141,42 - 35,36 = 106,06$$

$$K_{63} = -\frac{2EI}{l_3} \cdot \frac{1}{2} - \frac{GI_T}{l_3} \cdot \frac{1}{2} = -141,42 - 35,36 = -176,78$$

E2 4



$$K_{14} = 0$$

$$K_{24} = -\frac{GI_T}{l_2} = -100$$

$$K_{34} = \frac{GI_T}{l_3} \cdot \frac{1}{2} - \frac{4EI}{l_3} \cdot \frac{1}{2} = 35,36 - 282,84 = -247,48$$

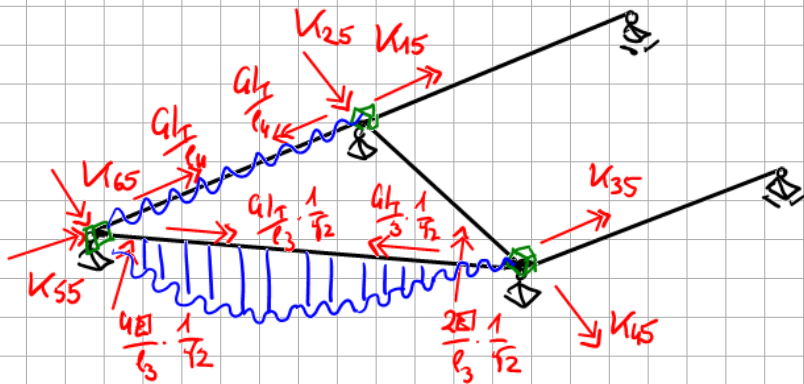
$$K_{44} = \frac{GI_T}{l_2} \cdot \frac{1}{2} + \frac{3EI}{l_1} \cdot \frac{1}{2} + \frac{GI_T}{l_3} \cdot \frac{1}{2} + \frac{4EI}{l_3} \cdot \frac{1}{2}$$

$$100 + 600 + 35,36 + 282,84 = 1018,2$$

$$K_{54} = -\frac{GI_T}{l_3} \cdot \frac{1}{2} - \frac{2EI}{l_3} \cdot \frac{1}{2} = -35,36 - 141,42 = -176,78$$

$$K_{64} = \frac{2EI}{l_3} \cdot \frac{1}{2} - \frac{GI_T}{l_3} = 141,42 - 35,36 = 106,06$$

E2 5



$$K_{15} = -\frac{G I_T}{\rho_h} = -100$$

$$K_{25} \approx 0$$

$$K_{35} = \frac{2E}{l_3} \cdot \frac{1}{2} - \frac{G I_T}{l_3} \cdot \frac{1}{2} = 141,42 - 35,36 = 106,06$$

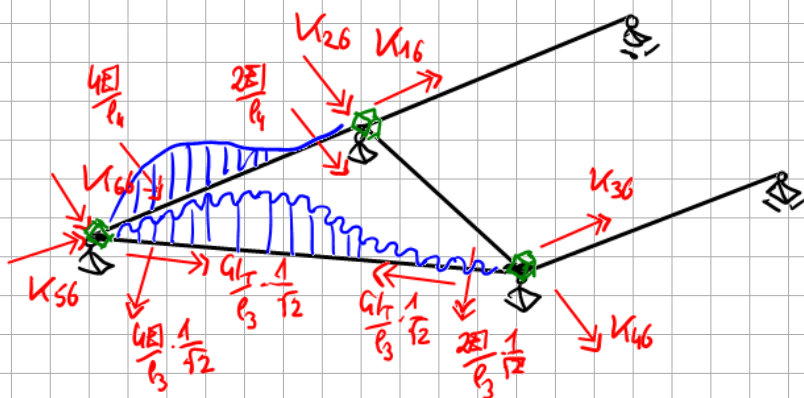
$$K_{45} = -\frac{2EI}{l_3} \cdot \frac{1}{2} - \frac{ql_1}{l_3} \cdot \frac{1}{2} = -141,42 - 35,36 =$$

$$= -176,78$$

$$L_{55} = \frac{G_1}{l_3} \cdot \frac{1}{2} + \frac{G_2}{l_3} \cdot \frac{1}{2} + \frac{G_4}{l_4} = 35,36 + 22,84 + 10 = 68,2$$

$$K_{GS} = \frac{G_{LT}}{b} \cdot \frac{1}{2} - \frac{K_{ET}}{b} \cdot \frac{1}{2} = 35,36 - 282,84 = -247,48$$

Ex 6



$$K_{16} = 0$$

$$K_{26} = \frac{287}{f_4} = 400$$

$$L_{36} = -\frac{0,4}{b_3} \cdot \frac{1}{2} - \frac{28}{b_3} \cdot \frac{1}{2} = -35,36 - 14,42 = -176,78$$

$$V_{46} = \frac{2E}{l_3} \cdot \frac{1}{2} - \frac{q_l}{l_3} \cdot \frac{1}{2} = 141,42 - 35,36 = 106,06$$

$$V_{S6} = \frac{q_L}{\epsilon_3} \cdot \frac{1}{2} - \frac{q_{E1}}{\epsilon_3} \cdot \frac{1}{2} = 35,36 - 282,84 = -247,48$$

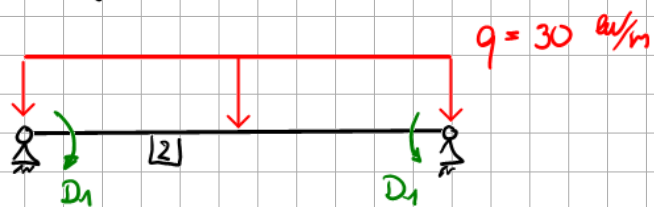
$$K_{GS} = \frac{G_I}{l_3} \cdot \frac{1}{2} + \frac{4EI}{l_3} \cdot \frac{1}{2} + \frac{4EI}{l_4} =$$

$$\approx 35,36 + 282,84 + 800 = 1118,2$$

$$K = \begin{bmatrix} 900 & 0 & 400 & 0 & -100 & 0 \\ 0 & 1500 & 0 & -100 & 0 & 400 \\ 400 & 0 & 1118,2 & -247,48 & 106,06 & -176,78 \\ 0 & -100 & -247,48 & 1018,2 & -176,78 & 106,06 \\ -100 & 0 & 106,06 & -176,78 & 418,2 & -247,48 \\ 0 & 400 & -176,78 & 106,06 & -247,48 & 1118,2 \end{bmatrix}$$

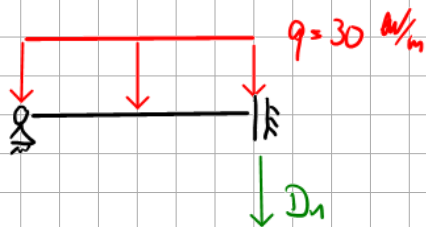
$$\underline{F} = -\underline{K}_0 \underline{x} = \begin{bmatrix} -40 \\ 0 \\ 40 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) neues System:



$n_g = 1$ (Symmetrie)

Durchsenkung in Feldmitte am halben System berechnen:

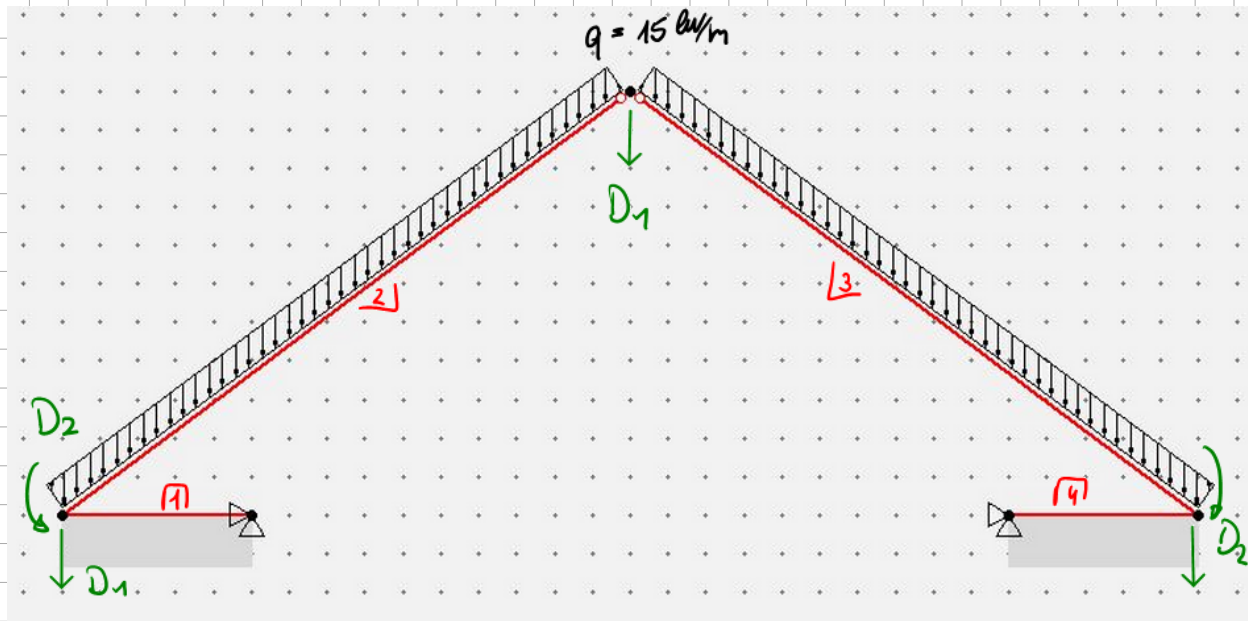


$$K_n = \frac{3EI}{(l/2)^3} = 300$$

$$K_w = -\frac{5}{8} \cdot q \cdot \left(\frac{l}{2}\right)^2 = -37,5$$

$$\Rightarrow D_1 = \frac{-K_w}{K_n} = 0,125 \text{ m}$$

Probeklausur 6 - Aufgabe 4



$$EI_{14} = 7800 \text{ kNm}^2$$

$$EI_{23} = 2300 \text{ kNm}^2$$

$$EA \rightarrow \infty$$

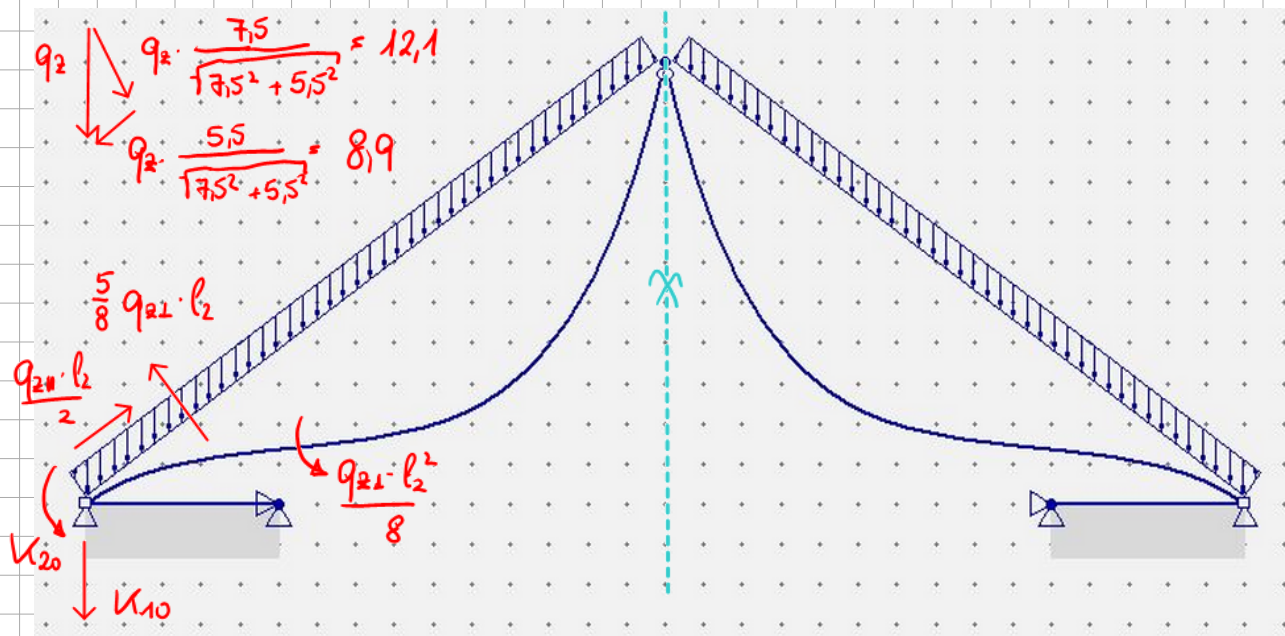
$$l = 400000 \text{ kN/m}$$

$$q = 15 \text{ kN/m}$$

$$\lambda = \sqrt[4]{\frac{400000}{4 \cdot 7800}} = 1,892$$

$$\lambda \cdot l = 4,73 > \pi \rightarrow \infty \text{ langer Balken}$$

L2



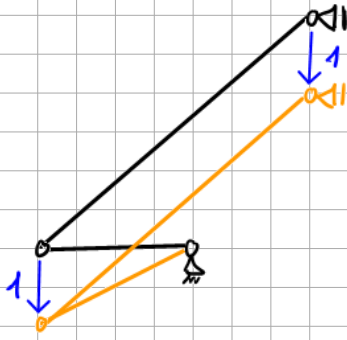
$$K_{10} = -\frac{5}{8} \cdot q_{21} \cdot l_2 \cdot \frac{7,5}{l_2} - \frac{q_{21} \cdot l_2}{2} \cdot \frac{5,5}{l_2} - \frac{3}{8} \cdot q_{21} \cdot l_2 \cdot \frac{7,5}{l_2} - \frac{q_{21} \cdot l_2}{2} \cdot \frac{5,5}{l_2} =$$

$$= -56,7 - 24,39 - 34,02 - 24,39 = -139,5$$

oben Ende

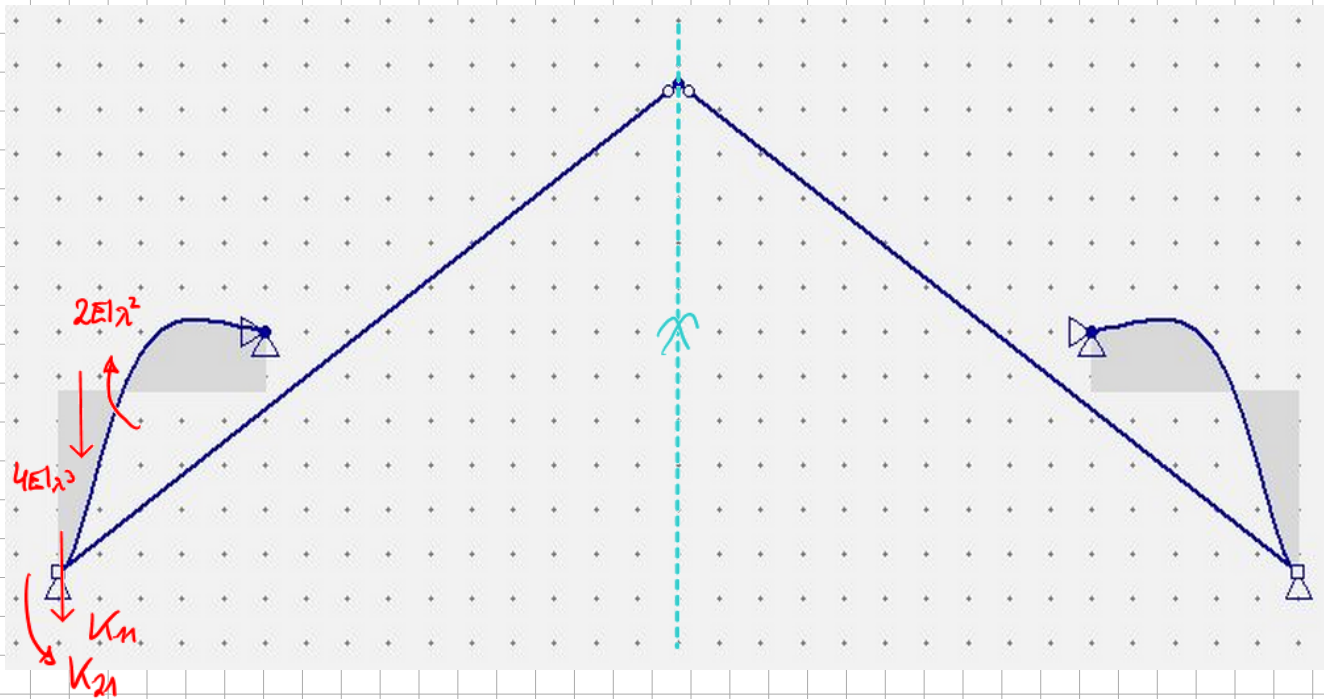
$$K_{20} = \frac{q_{21} \cdot l_2^2}{8} = 130,83$$

einsetzen mit PV:



$$K_{10} \cdot \bar{\Delta} = -q_2 \cdot l_2 \cdot \bar{\Delta} = -139,5$$

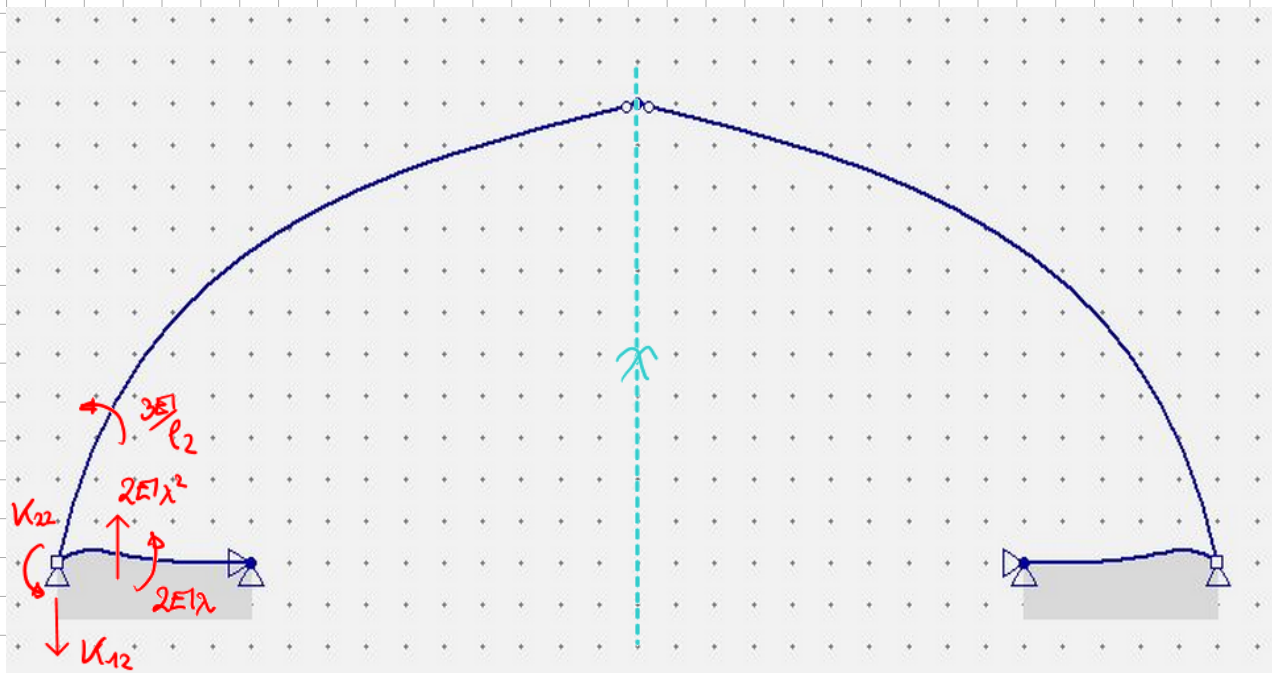
E2 1



$$K_{11} = 4EI\lambda^3 = 211\,389,61$$

$$K_{21} = -2EI\lambda^2 = -55\,856,96$$

E2 2

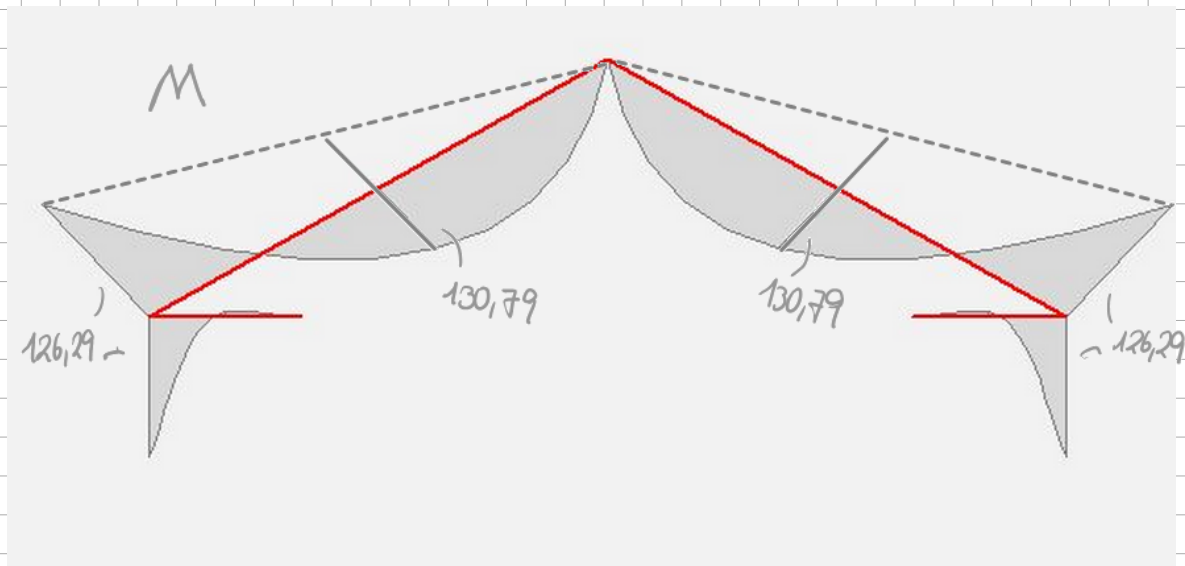


$$K_{12} = -2EI\lambda^2 = -55\,856,96$$

$$K_{22} = 2EI\lambda + \frac{3EI}{l_2} = 29\,518,95 + 741,89 = 30\,260,84$$

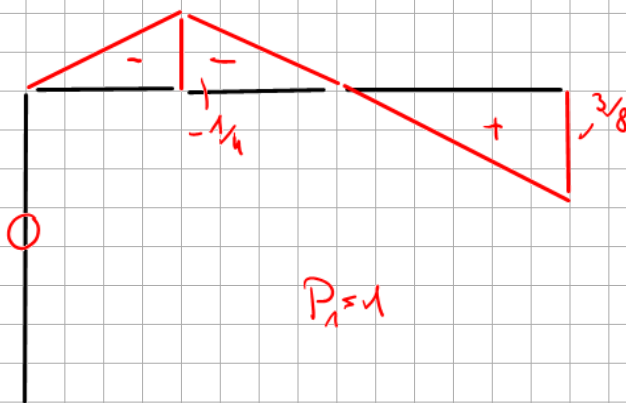
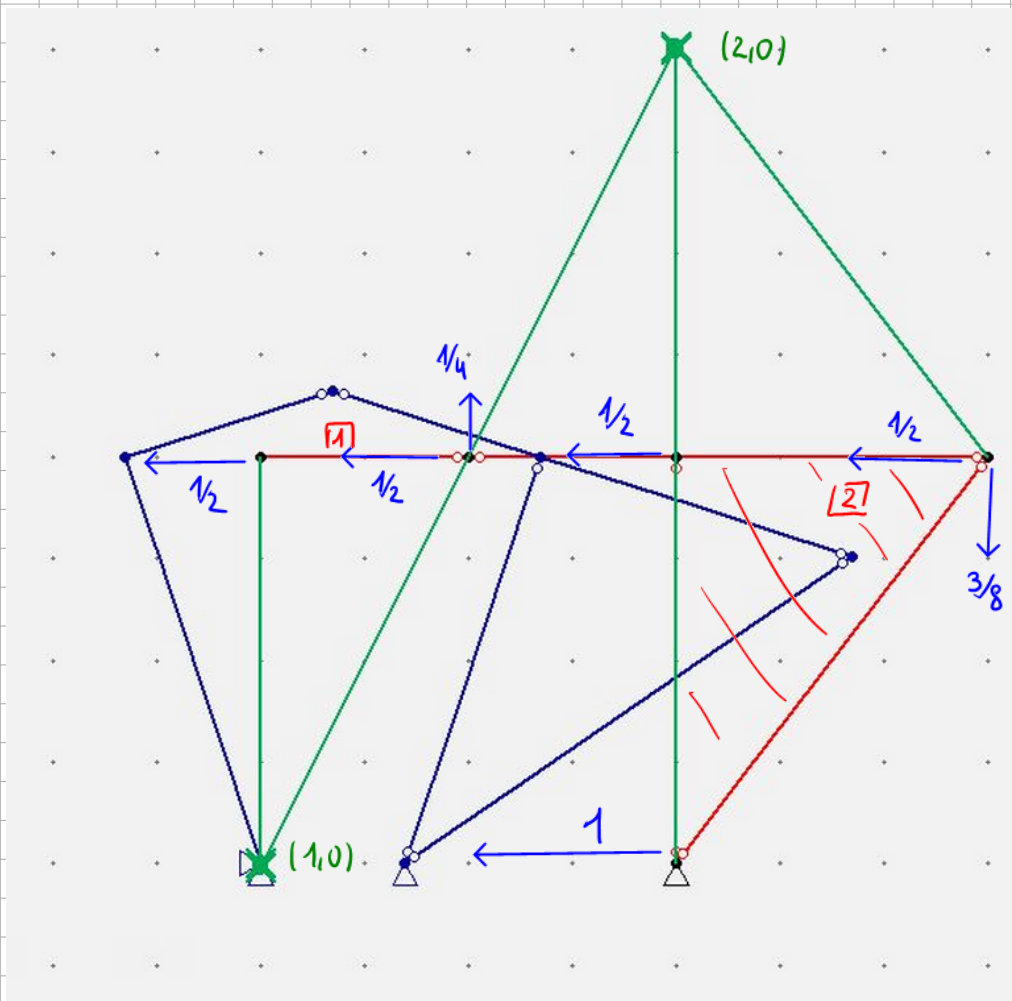
$$\underline{\underline{K}} = \begin{bmatrix} 211\,389,61 & -55\,856,96 \\ -55\,856,96 & 30\,260,84 \end{bmatrix} \quad \underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 139,5 \\ -130,83 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \quad \Rightarrow \quad \underline{\underline{u}} = \begin{bmatrix} -9,419 \cdot 10^{-4} \text{ m} \\ -6,062 \cdot 10^{-3} \text{ rad} \end{bmatrix}$$

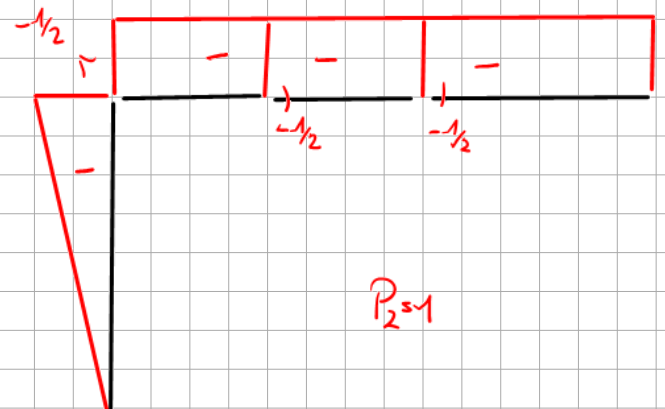


Probeklausur 6 - Aufgabe 5

(a)



$P_1 = 1$



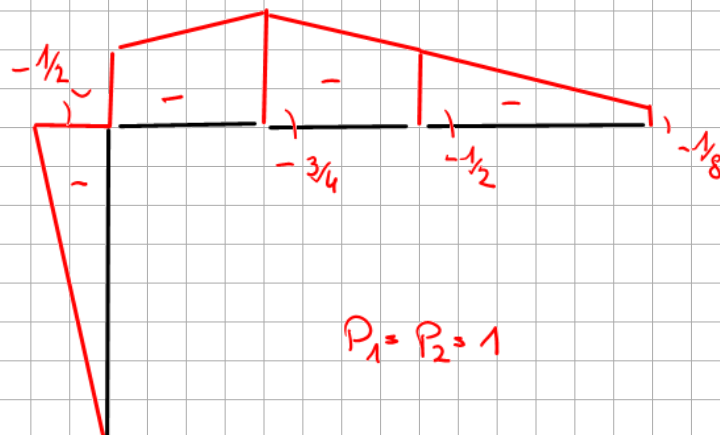
$P_2 = 1$

$$Q(x_1) = -\frac{1}{2} \cdot \frac{x_1}{4}$$

$$Q(x_2) = -\frac{1}{2} - \frac{1}{4} \cdot \frac{x_2}{2}$$

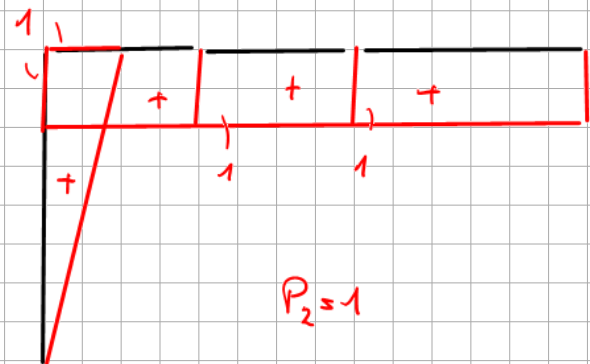
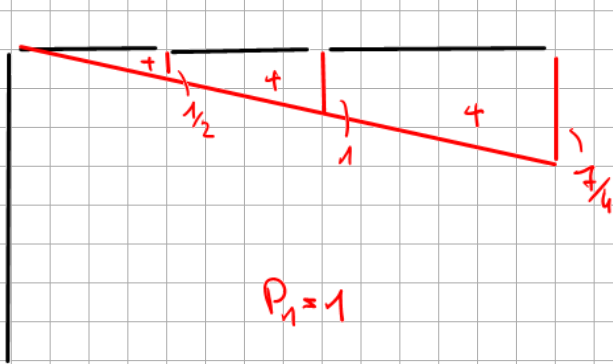
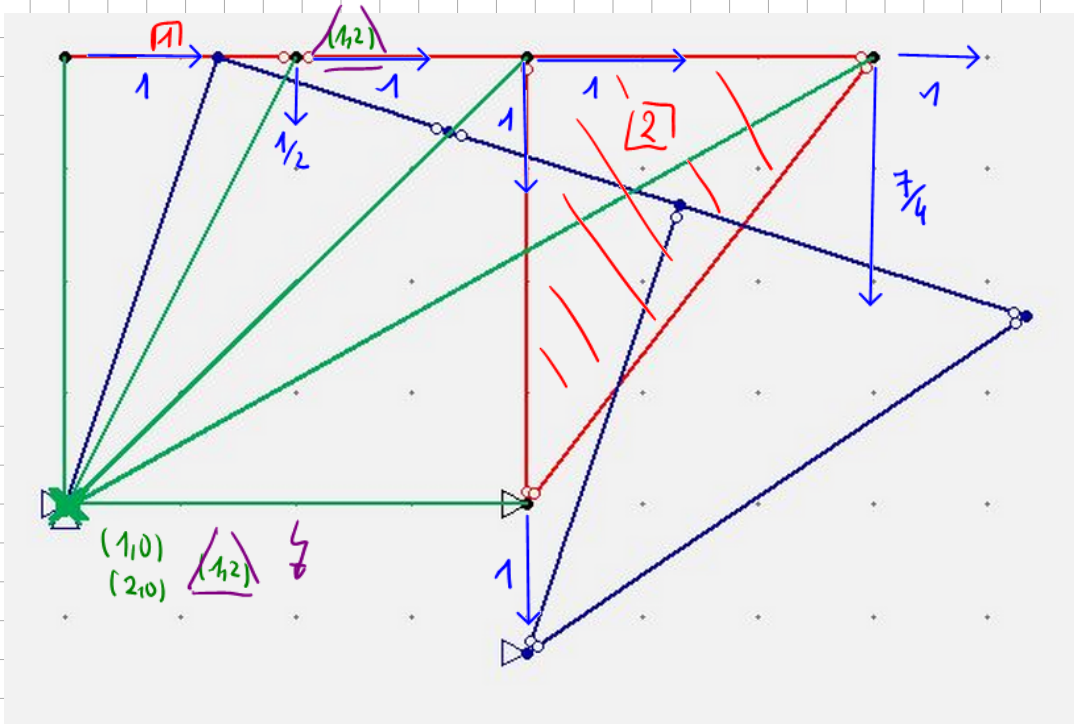
$$Q(x_3) = -\frac{3}{4} + \frac{1}{4} \cdot \frac{x_3}{2}$$

$$Q(x_4) = -\frac{1}{2} + \frac{3}{8} \cdot \frac{x_4}{3}$$



$P_1 = P_2 = 1$

(b)

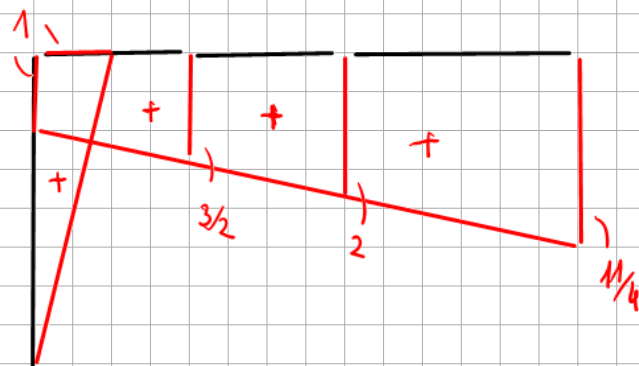


$$p(x_1) = \frac{x_1}{4}$$

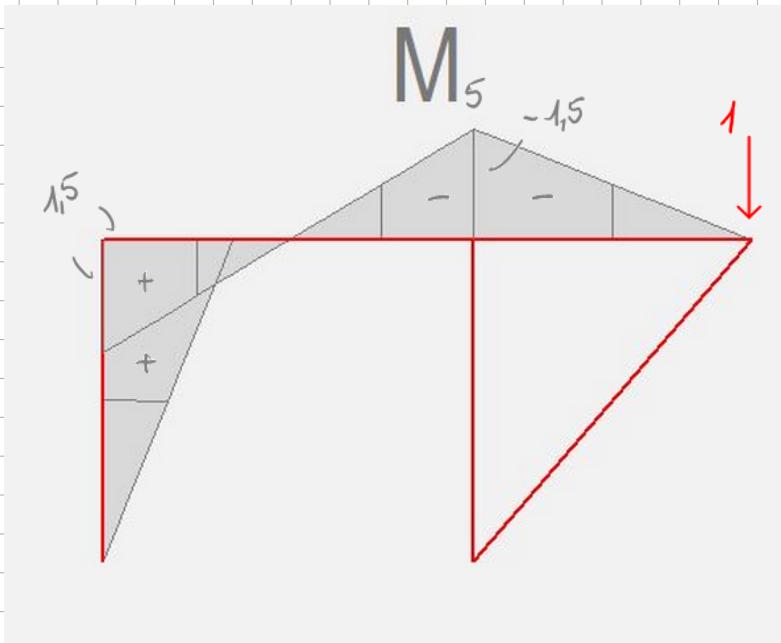
$$p(x_2) = 1 + \frac{1}{2} \cdot \frac{x_2}{2}$$

$$h(x_3) = \frac{3}{2} + \frac{1}{2} \cdot \frac{x_3}{2}$$

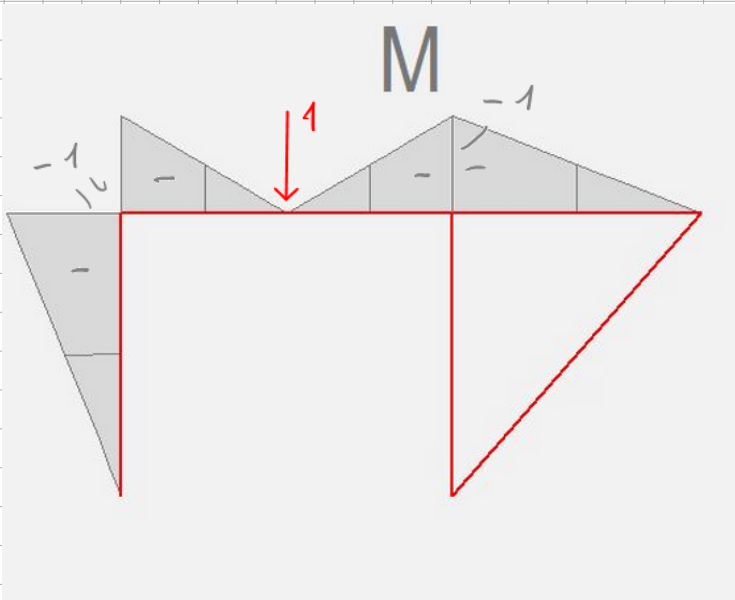
$$Q(x_4) = 2 + \frac{3}{4} \cdot \frac{x_4}{3}$$



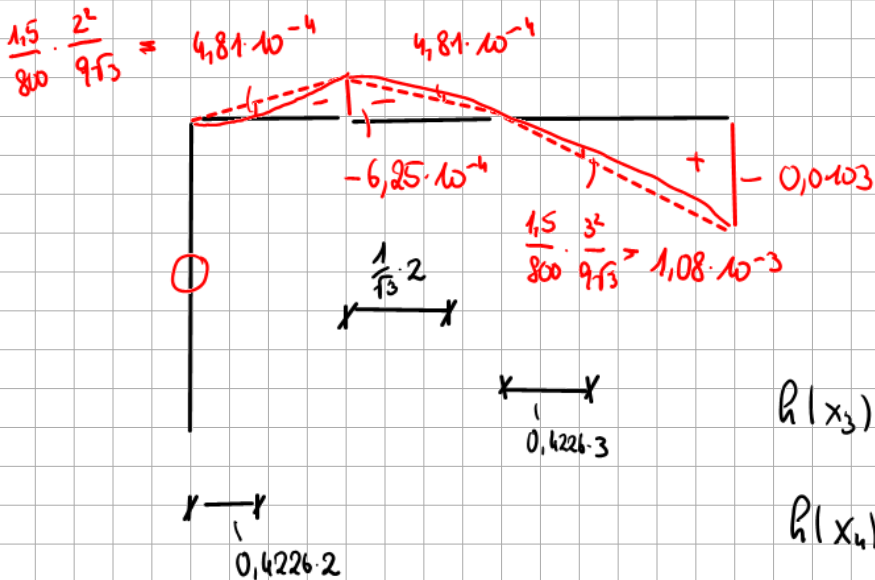
(C)



$$W_5 = \frac{1}{3} \cdot 4,5^2 \cdot \left(\frac{4+2+2+3}{57} \right) = 0,0103 \text{ m}$$



$$W_2 = -\frac{1}{3} \cdot 1,45 \cdot \frac{4}{EI} + \frac{1}{3} \cdot 1,45 \cdot \frac{3}{EI} s$$
$$= -6,25 \cdot 10^{-4} m$$



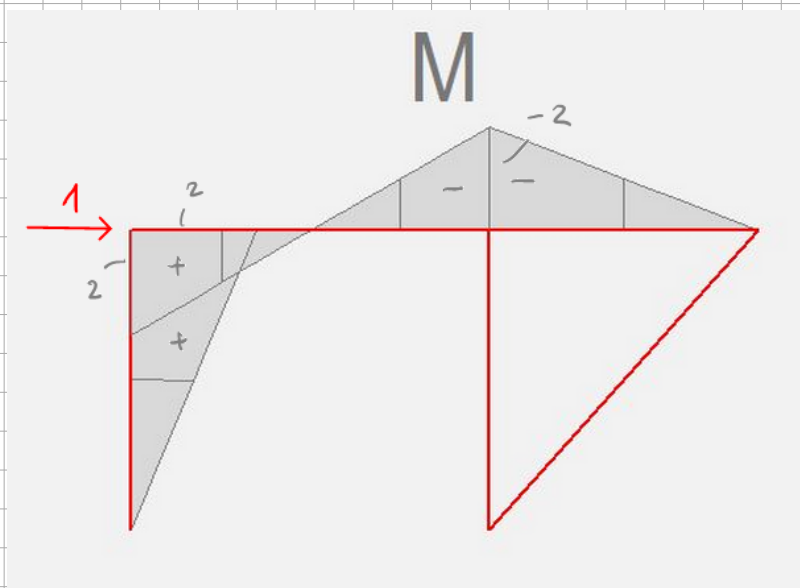
$$h(x_1) = 0$$

$$Q(x_2) = -6,25 \cdot 10^{-4} \cdot \frac{x_2}{2} + \frac{15}{800} \cdot \frac{2^2}{6} \cdot \left[\frac{x_2}{2} - \left(\frac{x_2}{2} \right)^3 \right]$$

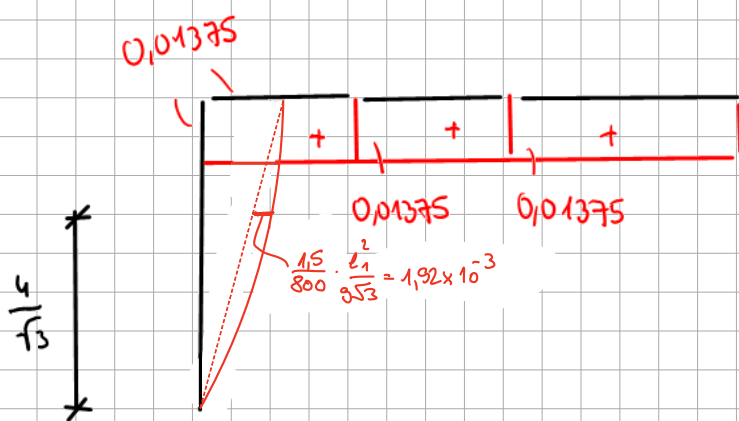
$$h(x_3) = -6,25 \cdot 10^{-4} \cdot \frac{2-x_3}{2} - \frac{15}{800} \cdot \frac{2^2}{6} \cdot \left[\frac{2-x_3}{2} - \left(\frac{2-x_3}{2} \right)^2 \right]$$

$$h(x_4) = 0,0103 \cdot \frac{x_4}{3} + \frac{1,5}{80} \cdot \frac{3^2}{6} \left[\frac{x_4}{3} - \left(\frac{x_4}{3} \right)^3 \right]$$

(d)



$$u_2 = \frac{1}{3} \cdot 2 \cdot 15 \cdot \left(\frac{4+2+2+3}{81} \right) = 0,01375 \text{ m}$$



$$h(x_1) = 0,01375 \cdot \frac{x_1}{4} + \frac{15}{800} \cdot \frac{4^2}{6} \cdot \left[\frac{x_1}{4} - \left(\frac{x_1}{4} \right)^3 \right]$$

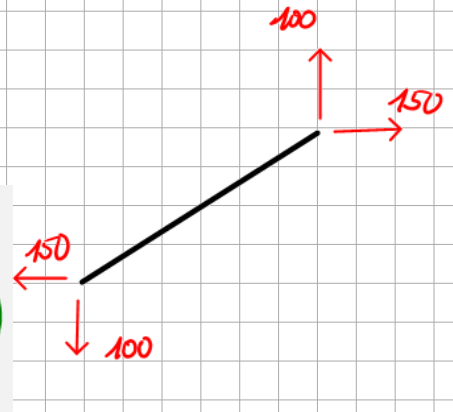
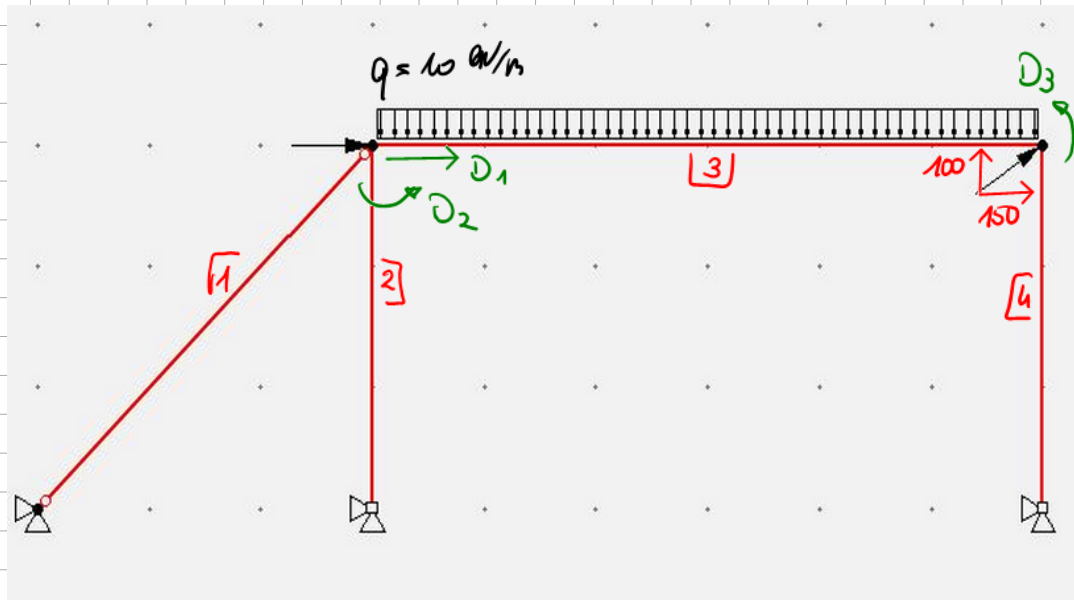
$$h(x_2) = 0,01375$$

$$h(x_3) = 0,01375$$

$$h(x_4) = 0,01375$$

Statik Musterlösungen

Probeklausur 7 - Aufgabe 1

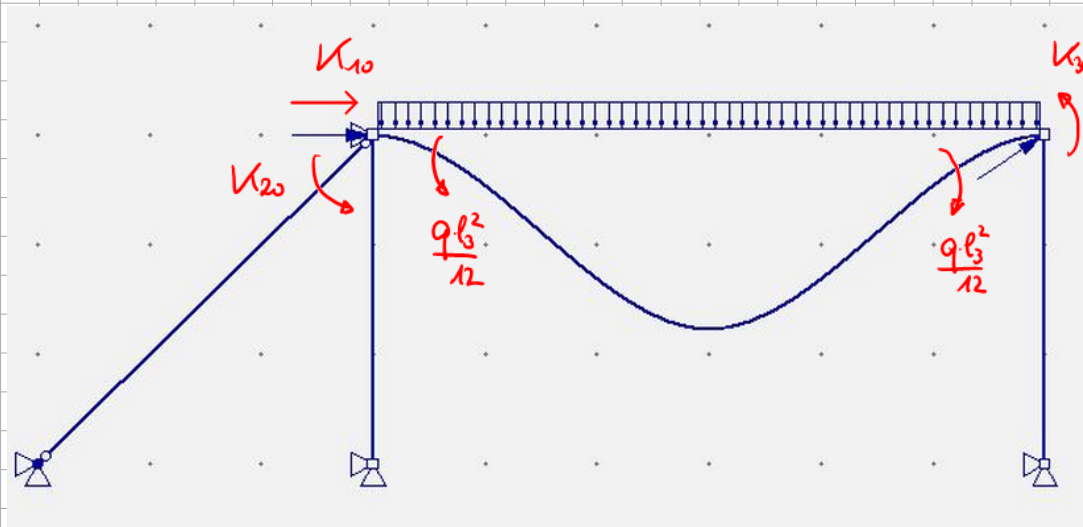


$$EI = 1000$$

$$EA \rightarrow \infty$$

$$EA_1 = 10000$$

L2

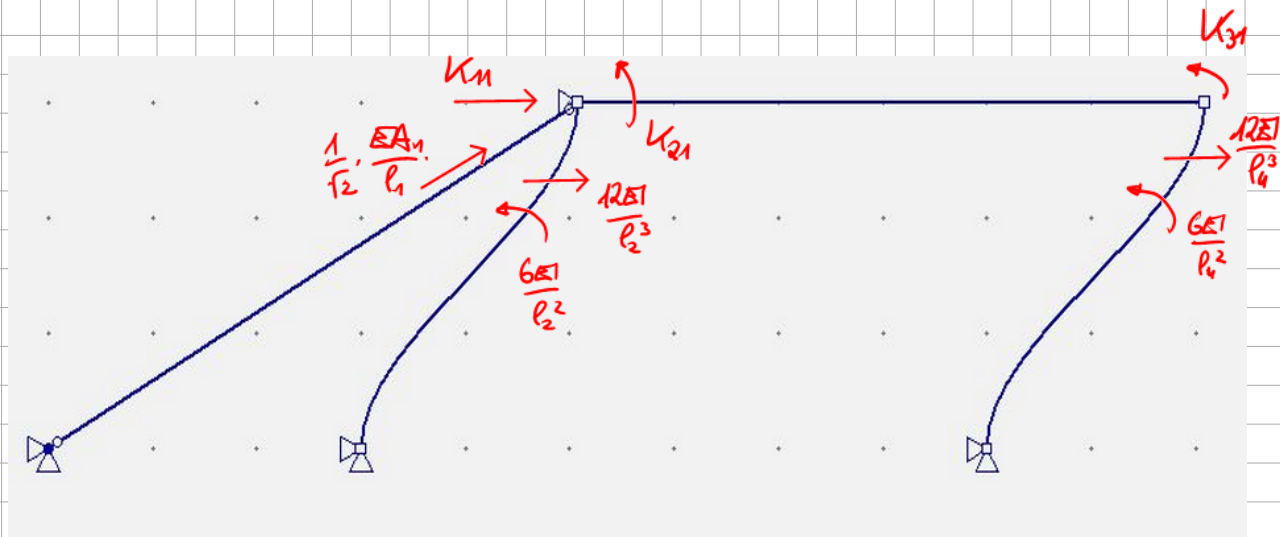


$$V_{10} = -100 - 150 = -250$$

$$V_{20} = \frac{q \cdot b^2}{12} = 30$$

$$V_{30} = -\frac{q \cdot b^2}{12} = -30$$

Ex 1

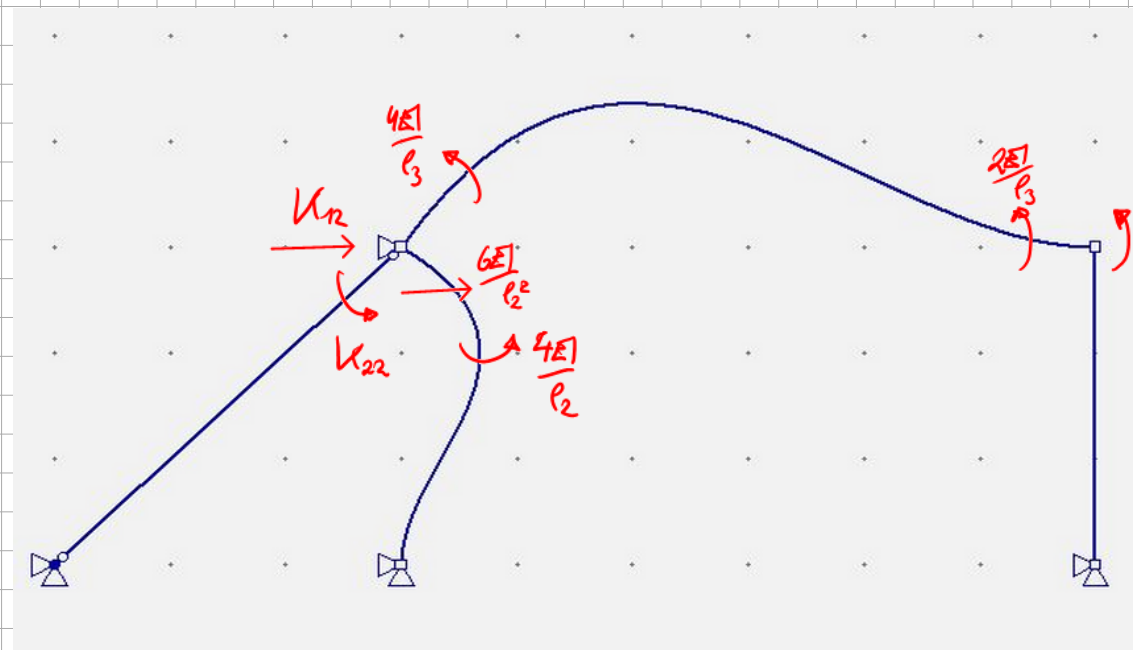


$$K_{11} = \frac{EA_1}{l_1} \cdot \frac{1}{2} + \frac{12EI}{l_2^3} + \frac{12EI}{l_4^3} = 1178,51 + 444,4 + 444,4 = 2067,399$$

$$K_{21} = \frac{6EI}{l_2^2} = 666,6$$

$$K_{31} = \frac{6EI}{l_4^2} = 666,6$$

Ex 2

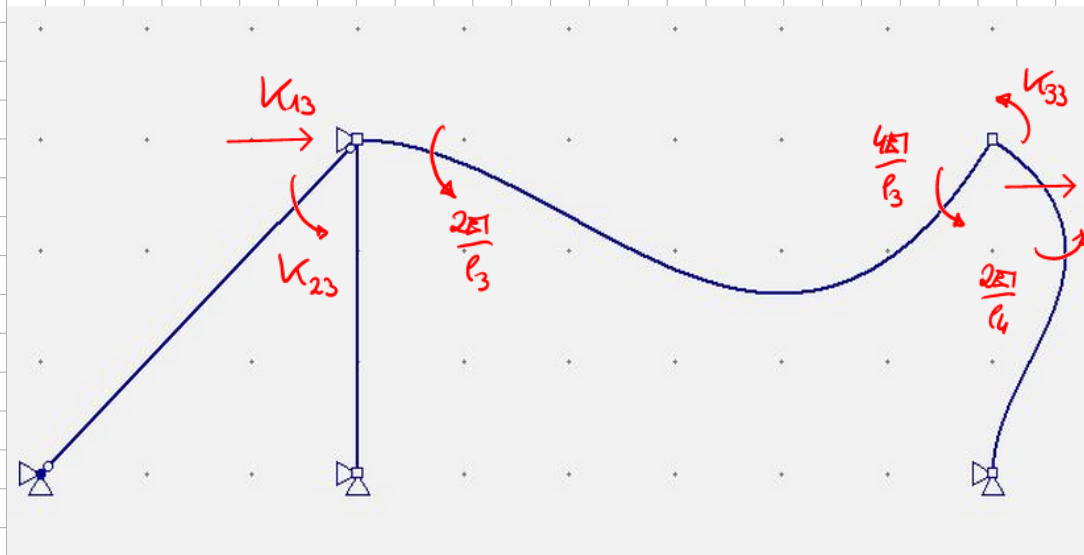


$$K_{12} = \frac{6EI}{l_2^2} = 666,6$$

$$K_{22} = \frac{4EI}{l_2} + \frac{4EI}{l_3} = 1333,3 + 666,6 = 2000$$

$$K_{32} = \frac{2EI}{l_3} = 333,3$$

E2 3



$$K_{13} = \frac{6EI}{l_4^2} = 666,6$$

$$K_{23} = \frac{2EI}{l_3} = 333,3$$

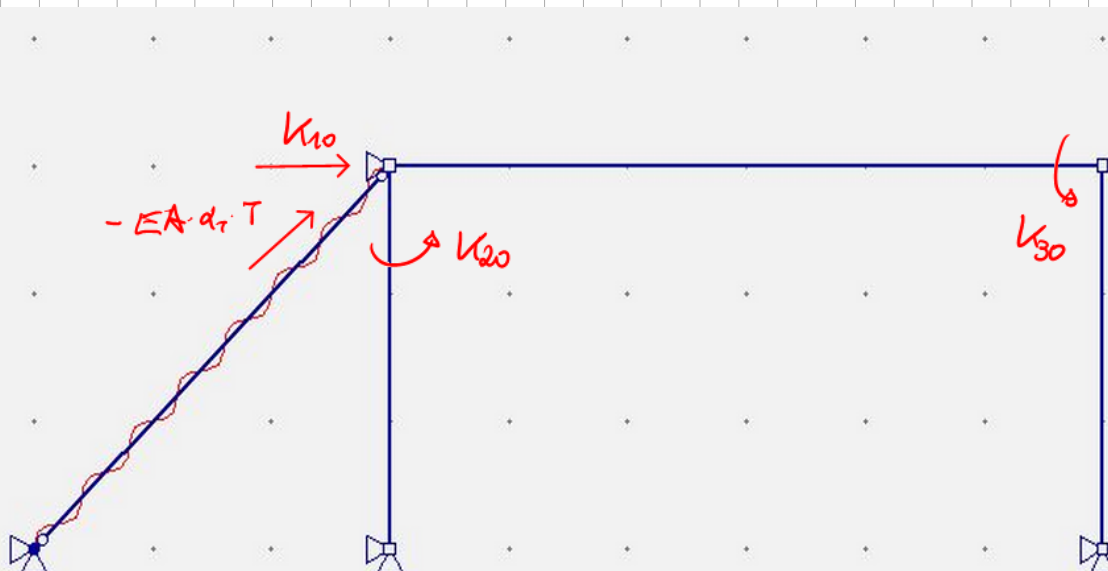
$$K_{33} = \frac{4EI}{l_3} + \frac{4EI}{l_4} = 1333,3 + 666,6 = 2000$$

$$\underline{K} = \begin{bmatrix} 2067,399 & 666,6 & 666,6 \\ 666,6 & 2000 & 333,3 \\ 666,6 & 333,3 & 2000 \end{bmatrix}$$

$$\underline{F} = -\underline{K}_0 = \begin{bmatrix} 250 \\ -30 \\ 30 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \Rightarrow \underline{u} = \begin{bmatrix} 0,1482 \text{ m} \\ -0,0604 \text{ rad} \\ -0,0244 \text{ rad} \end{bmatrix}$$

(b) neues LZ:



$$K_{10} = -\frac{1}{l_2} EA \cdot \alpha \cdot \Delta T = -10,61$$

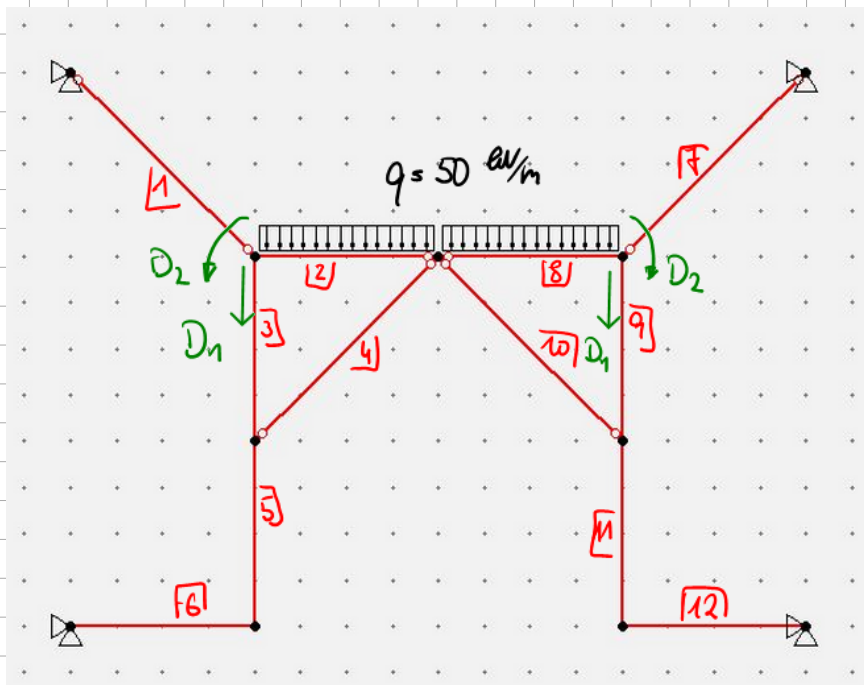
$$K_{20} = K_{30} = 0$$

K bleibt gleich

$$\underline{\underline{F}} = - \underline{\underline{K}}_0 = \begin{bmatrix} 10,61 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \rightarrow \underline{\underline{u}} = \begin{bmatrix} 0,00629 \text{ m} \\ -0,001797 \text{ rad} \\ -0,001797 \text{ rad} \end{bmatrix}$$

Probeklausur 7 - Aufgabe 2



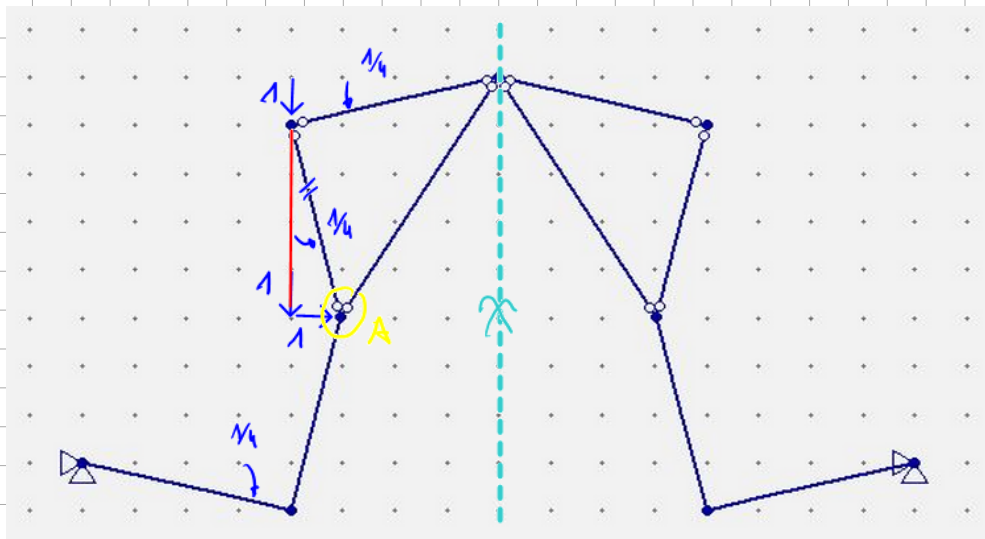
$$EA_{1,3} = 20\,000$$

$$EA_{\text{Rest}} \rightarrow \infty$$

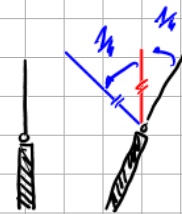
$$EI_{5,6,11,12} \rightarrow \infty$$

$$EI_{\text{Rest}} = 3500$$

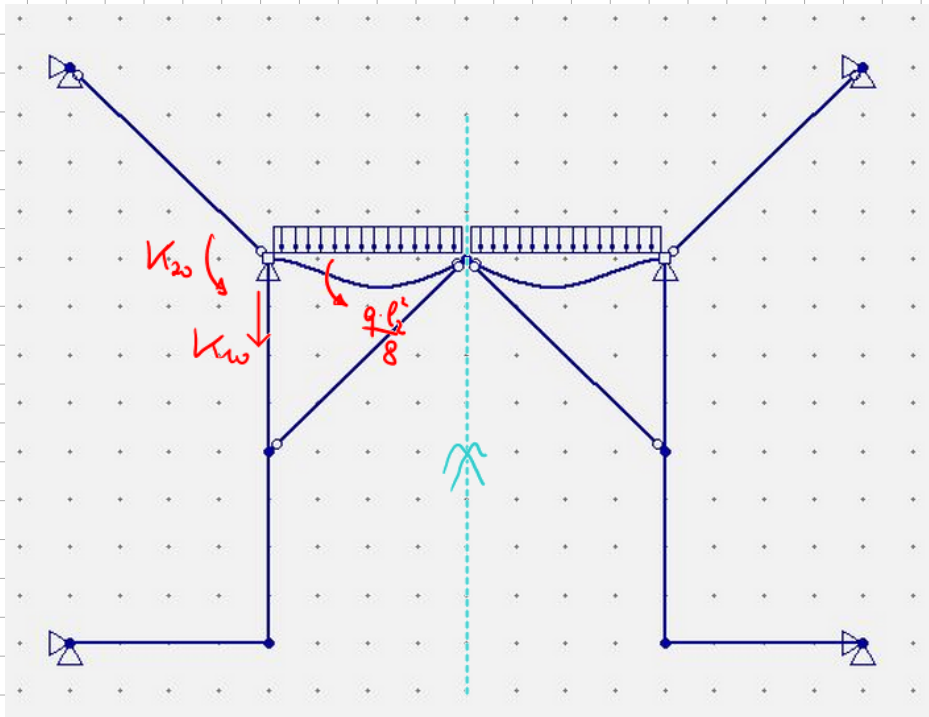
Gelenkfigur



Detail A

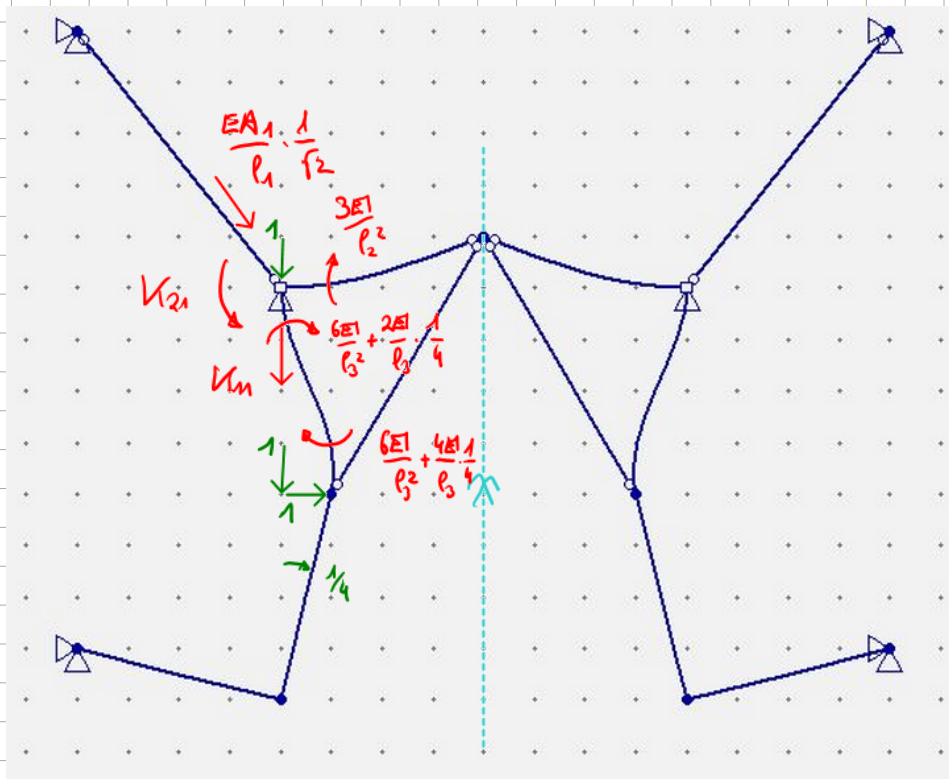


L2



$$\begin{aligned}
 V_{u_0} \bar{1} &= -q \cdot l_2 \cdot \frac{\bar{1}}{2} - q \frac{l_2^2}{8} \cdot \frac{\bar{1}}{4} = -100 - 25 = \\
 &= -125 \\
 V_{20} &= \frac{q \cdot l_2^2}{8} = 100
 \end{aligned}$$

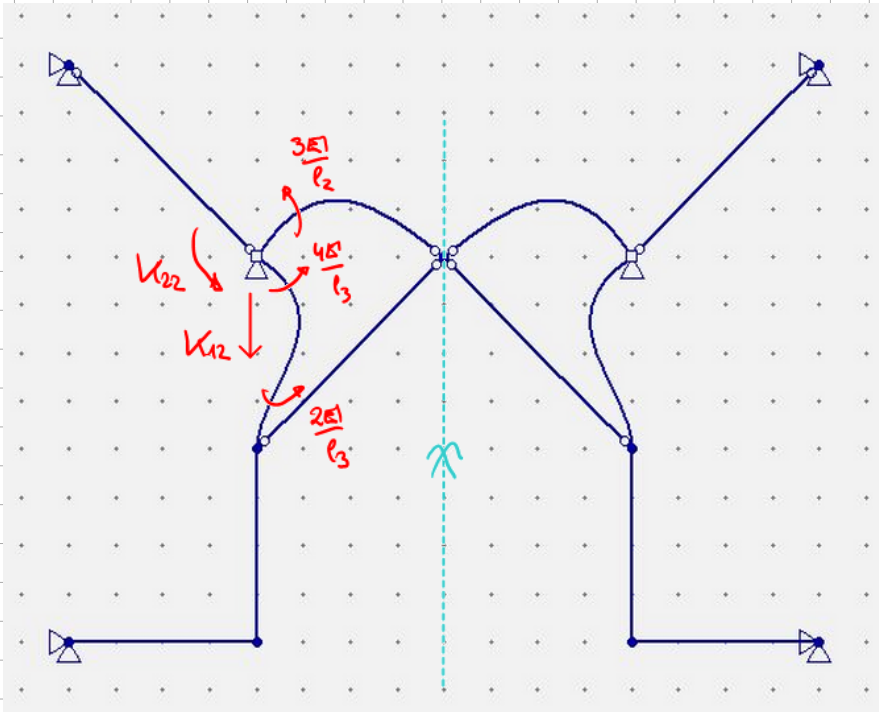
Ex 1



$$\begin{aligned}
 V_{u_1} \bar{1} &= \frac{3EI}{l_2^2} \cdot \frac{\bar{1}}{4} + \left(\frac{6EI}{l_3^2} + \frac{2EI}{l_3} \cdot \frac{1}{4} \right) \cdot \frac{\bar{1}}{4} + \\
 &+ \left(\frac{6EI}{l_3^2} + \frac{4EI}{l_3} \cdot \frac{1}{4} \right) \cdot \frac{\bar{1}}{2} + \frac{EA_1}{l_1} \cdot \frac{1}{2} \\
 &= 164,06 + 437,5 + 1093,75 + 1767,77 = \\
 &= 3463,08
 \end{aligned}$$

$$\begin{aligned}
 V_{21} &= -\frac{3EI}{l_2^2} - \left(\frac{6EI}{l_3^2} + \frac{2EI}{l_3} \cdot \frac{1}{4} \right) = \\
 &= -656,25 - 1750 = -2406,25
 \end{aligned}$$

E2 2



$$K_{22} \cdot \bar{1} = -\frac{3EI}{l_2} \cdot \frac{1}{4} - \frac{4EI}{l_3} \cdot \frac{1}{4} - \frac{2EI}{l_3} \cdot \frac{1}{2}$$

$$= -656,25 - 875 - 875$$

$$= -2406,25$$

$$K_{22} = \frac{3EI}{l_2} + \frac{4EI}{l_3} = 2625 + 3500$$

$$= 6125$$

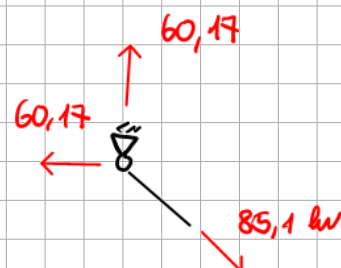
$$\underline{\underline{K}} = \begin{bmatrix} 3463,08 & -2406,25 \\ -2406,25 & 6125 \end{bmatrix} \quad \underline{\underline{F}} = -\underline{\underline{K}}_0 = \begin{bmatrix} 125 \\ -100 \end{bmatrix}$$

$$\underline{\underline{K}} \cdot \underline{\underline{u}} = \underline{\underline{F}} \Rightarrow \underline{\underline{u}} = \begin{bmatrix} 0,03404 \text{ m} \\ -2,952 \cdot 10^{-3} \text{ rad} \end{bmatrix}$$

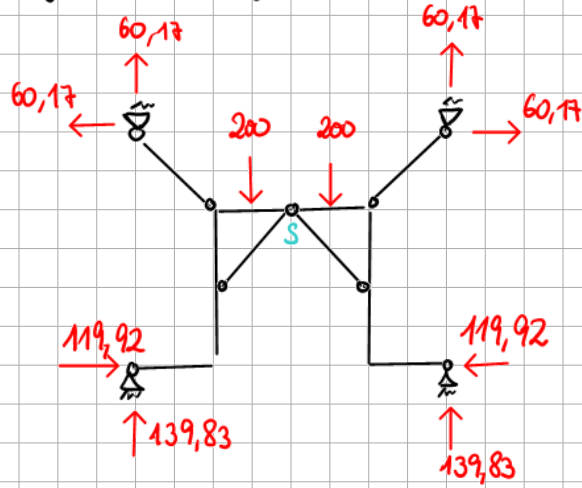
$$M_2 = -\frac{q \cdot l_2^2}{8} + \frac{3EI}{l_2^2} \cdot D_1 - \frac{3EI}{l_2} \cdot D_2 = -100 + 22,34 + 7,75 = -69,91 \text{ Nm}$$

Auflagerkräfte über Auflager links oben

$$N_1 = \frac{EA_1}{l_1} \cdot \frac{1}{l_2} \cdot D_1 = 85,1 \text{ kN}$$



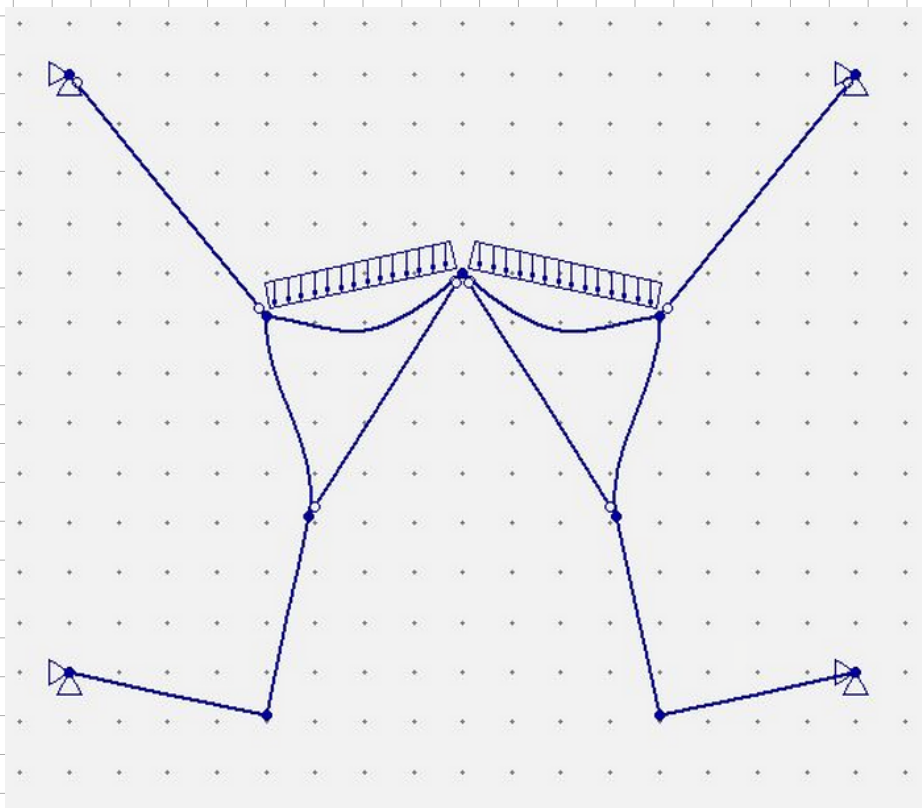
Gleichgewicht am gesamten System (unter Ausnutzung der Symmetrie)



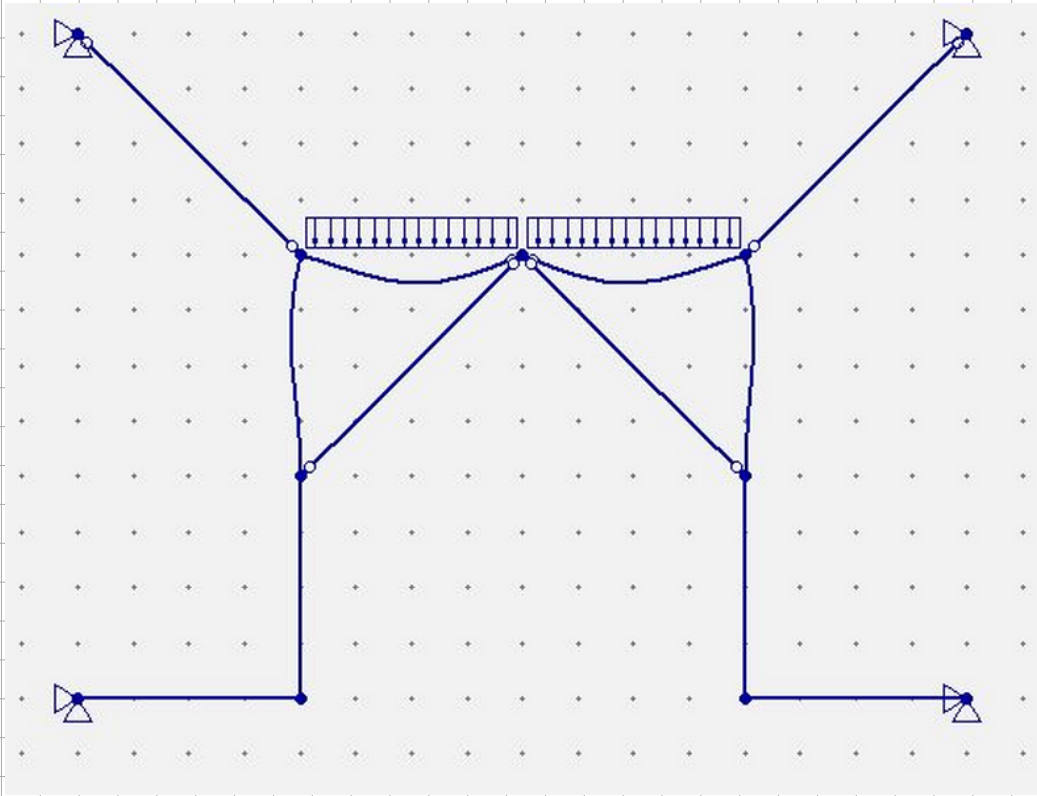
$$M_S = 0: A_H \cdot 8 = 200 \cdot 2 + 60,17 \cdot 4 - 60,17 \cdot 8 - 139,83 \cdot 8$$

$$A_H = -119,92$$

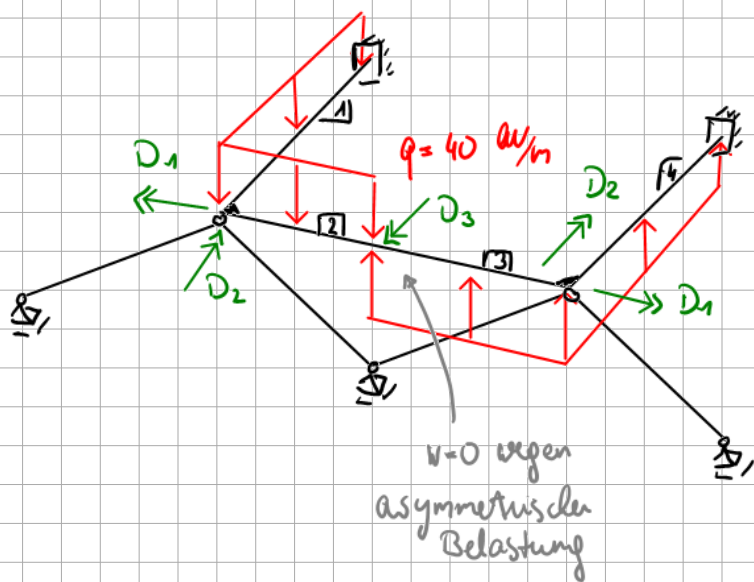
Biegelinie



Biegelinie für $EA_{1,2} \rightarrow \infty$; $\varphi_1 = 0$



Probeklausur 7 - Aufgabe 3

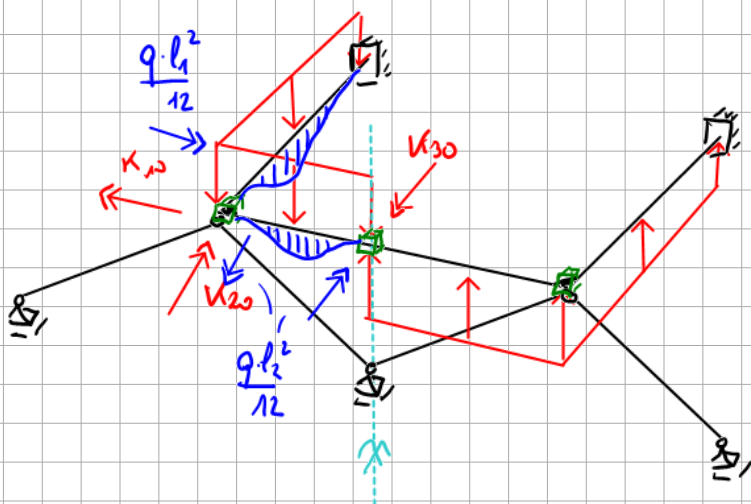


$$EA \rightarrow \infty$$

$$EI = 1200$$

$$GI_T = 600$$

LZ

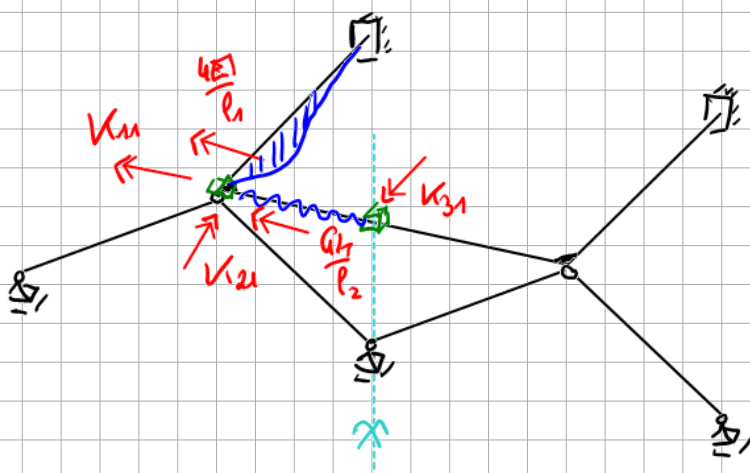


$$K_{10} = -\frac{q \cdot l_1^2}{12} = -120$$

$$K_{20} = -\frac{q \cdot l_2^2}{12} = -30$$

$$K_{30} = -\frac{q \cdot l_2^2}{12} = -30$$

EZ 1

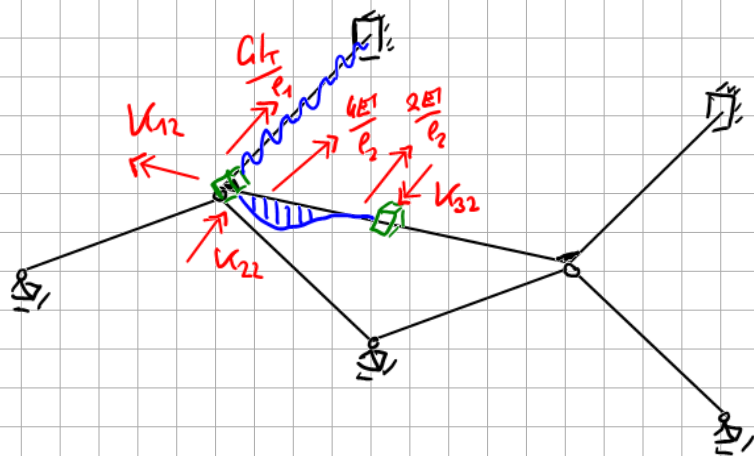


$$K_{11} = \frac{4EI}{l_1} + \frac{q \cdot l_2}{l_2} = 800 + 200 = 1000$$

$$K_{21} = 0$$

$$K_{31} = 0$$

E2 2

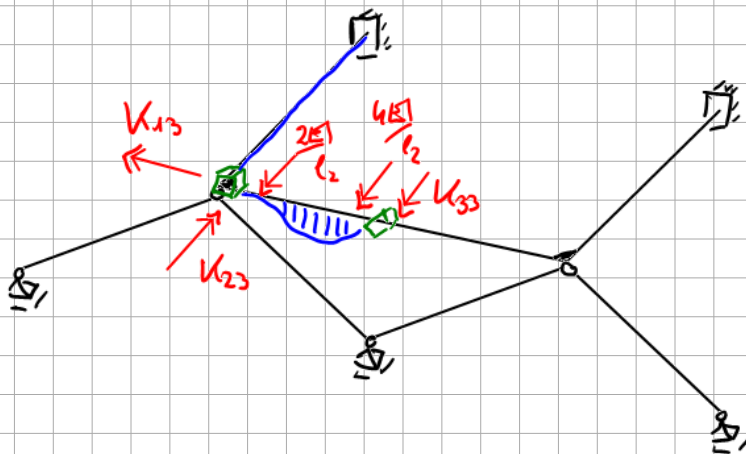


$$K_{12} = 0$$

$$K_{22} = \frac{Ql}{l_1} + \frac{4EI}{l_2} = 100 + 1600 = 1700$$

$$K_{32} = -\frac{2EI}{l_2} = -800$$

E2 3



$$K_{13} = 0$$

$$K_{23} = -\frac{2EI}{l_2} = -800$$

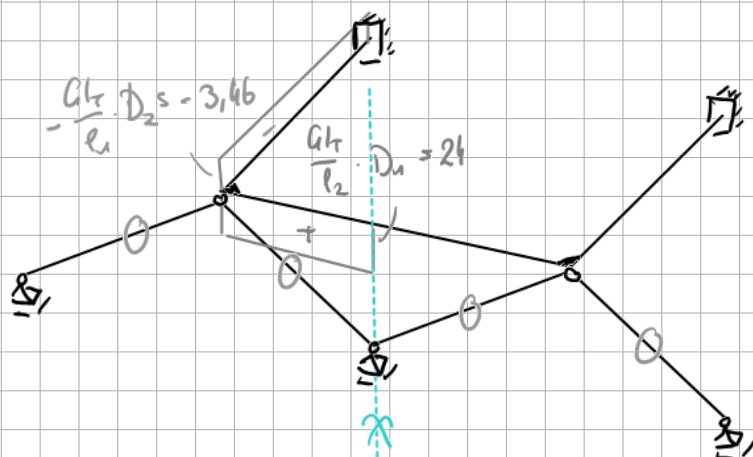
$$K_{33} = \frac{4EI}{l_2} = 1600$$

$$\underline{K} = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 1700 & -800 \\ 0 & -800 & 1600 \end{bmatrix}$$

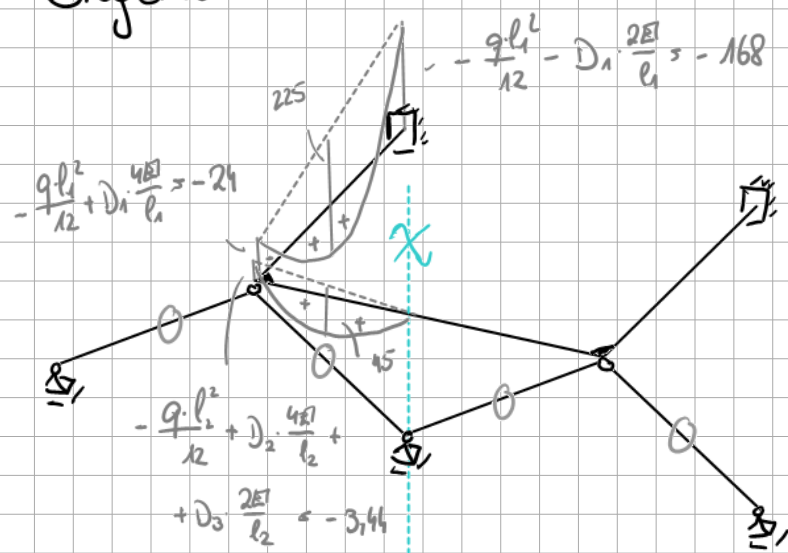
$$\underline{F} = -\underline{K}_0 = \begin{bmatrix} 120 \\ 30 \\ 30 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \Rightarrow \underline{u} = \begin{bmatrix} 0,12 \text{ rad} \\ 0,0346 \text{ rad} \\ 0,036 \text{ rad} \end{bmatrix}$$

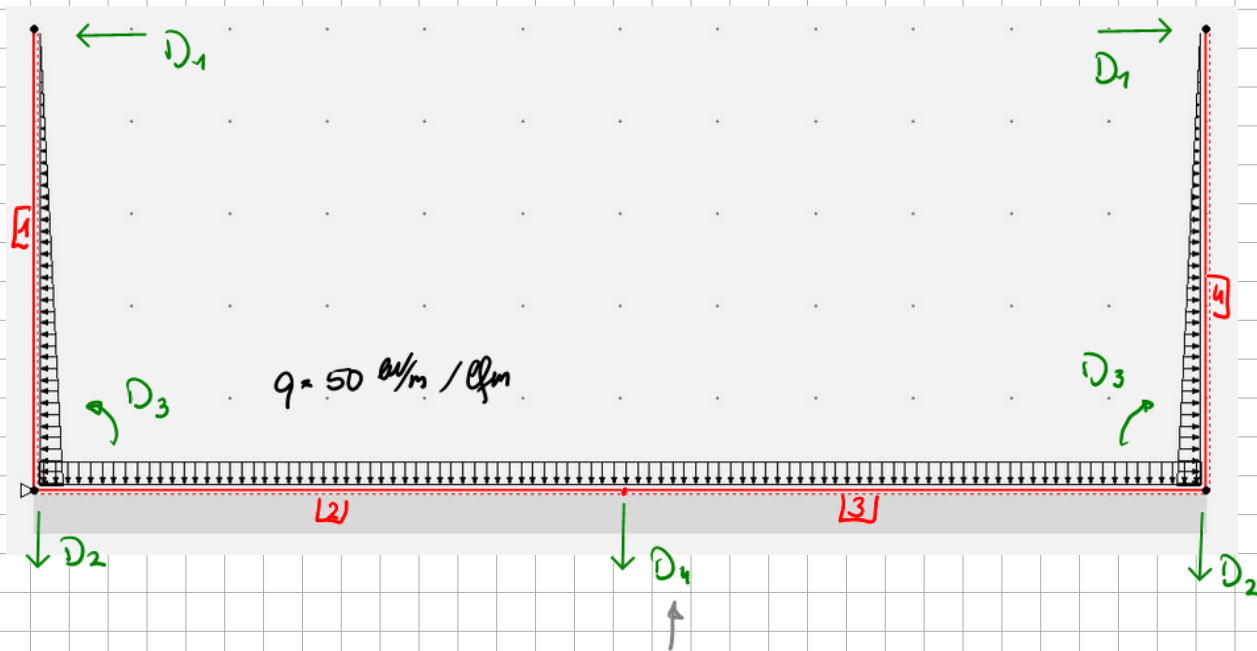
Torsionsmoment



Biegemoment



Probeklausur 7 - Aufgabe 4



$$EI = 20000$$

$$EA \rightarrow \infty$$

$$l_3 = 10000$$

$$\gamma = 10 \text{ kN/m}^2 / \text{cm}$$

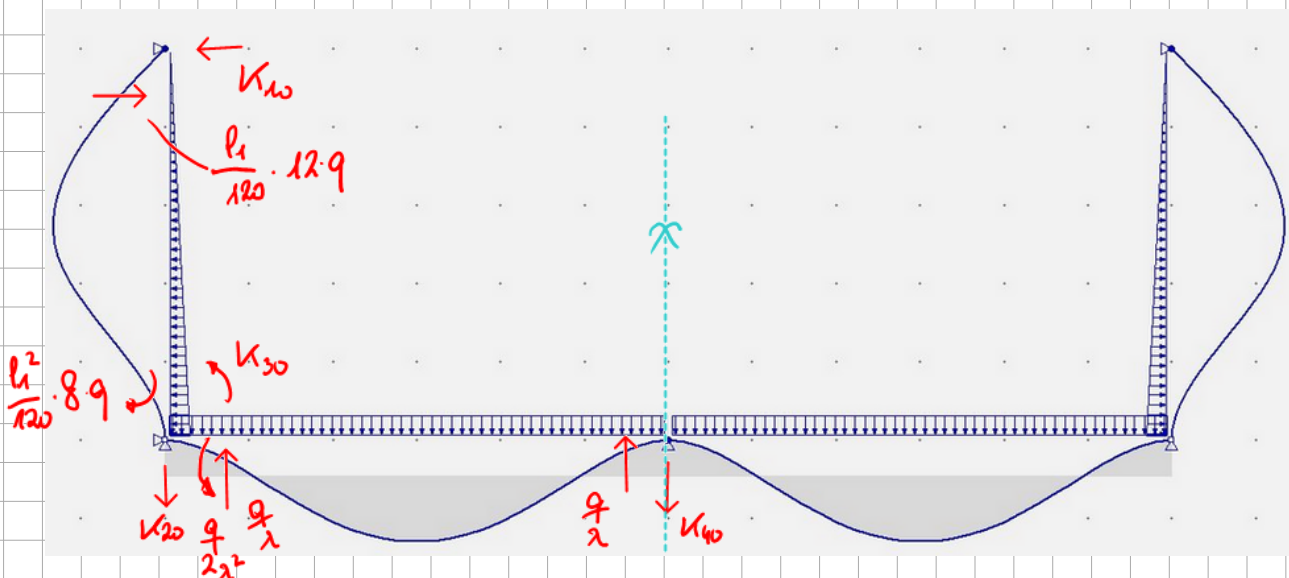
muss nicht diskretisiert werden,
es reicht aber Arbeit mit der
Tabelle für Aufgabe (b)

$$\lambda = \sqrt[4]{\frac{10000}{4 \cdot 20000}} = 0,595$$

$$\lambda \cdot l_2 = 3,56 > \pi$$

$\rightarrow \infty$ langer Balken
 $\rightarrow D_4$ ist entkoppelt!

L2



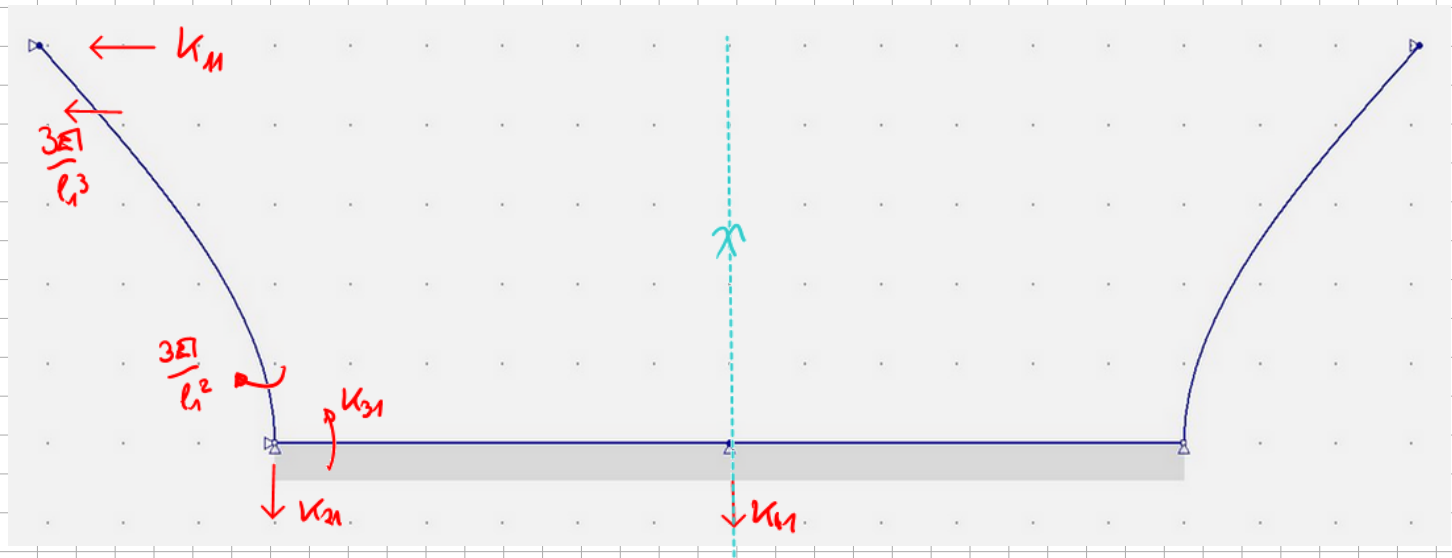
$$K_{10} = -\frac{l_1}{120} \cdot 12 \cdot q = -25$$

$$K_{20} = -\frac{q}{\lambda} = -84,09$$

$$K_{30} = \frac{q}{2\lambda^2} - \frac{l_1^2}{120} \cdot 8 \cdot q = 70,71 - 83,3 = -12,623$$

$$K_{40} = -\frac{q}{\lambda} = -84,09$$

E2 1



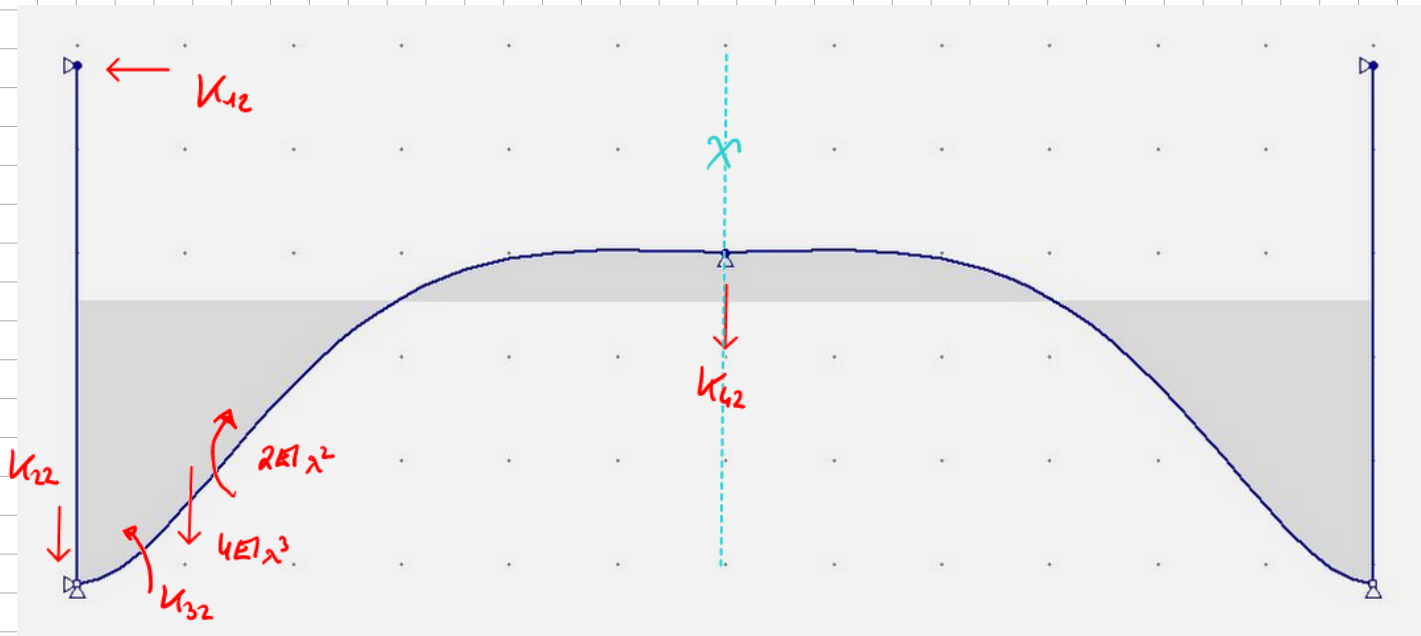
$$K_{11} = \frac{3EI}{l_1^3} = 480$$

$$K_{21} = 0$$

$$K_{31} = -\frac{3EI}{l_1^2} = -2400$$

$$K_{41} = 0$$

E2 2



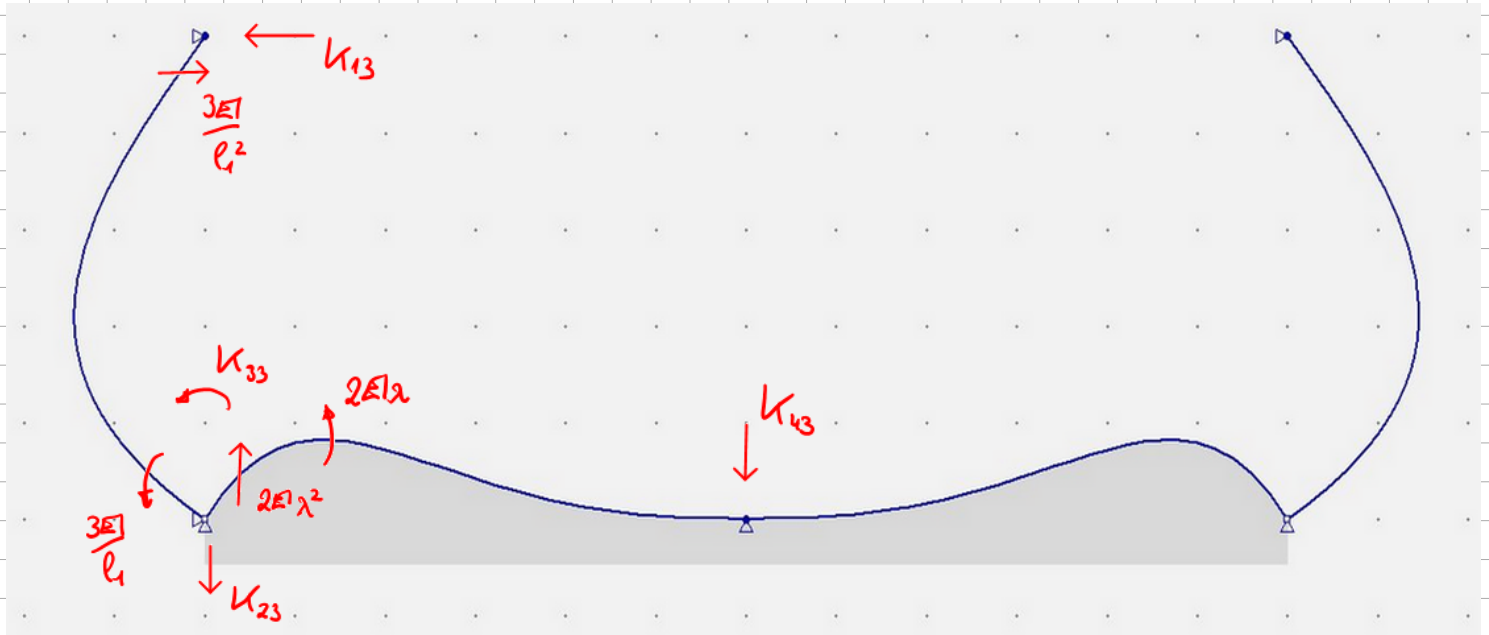
$$K_{12} = 0$$

$$K_{22} = \frac{4EI}{l_1^3} = 16\,817,93$$

$$K_{32} = -\frac{2EI}{l_1^2} = -14\,142,14$$

$$K_{42} = 0 \quad (\text{entkoppelt})$$

E2 3



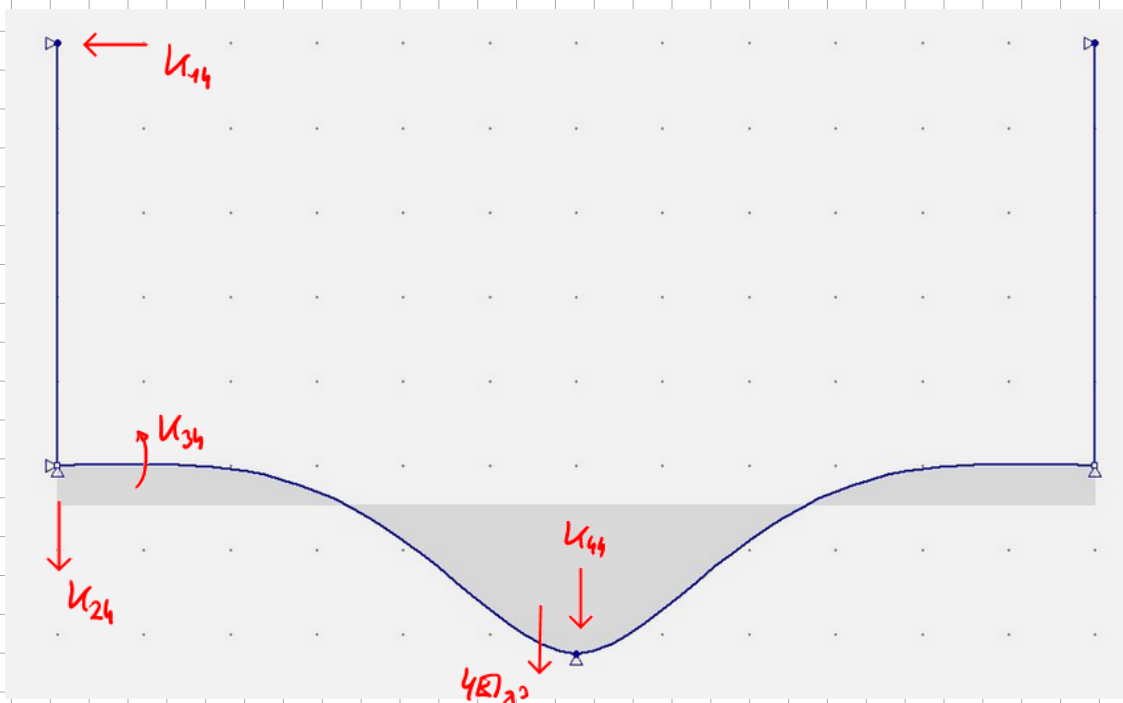
$$K_{13} = -\frac{3EI}{l_1^2} = -2400$$

$$K_{23} = -2EI\lambda^2 = -14\,142,14$$

$$K_{33} = \frac{3EI}{l_1} + 2EI\lambda = 12\,000 + 23\,784,14 = 35\,784,14$$

$$K_{43} = 0 \text{ (entkoppelt)}$$

E2 4



$$K_{14} = 0$$

$$K_{24} = 0$$

$$K_{34} = 0$$

$$K_{44} = 4EI\lambda^3 = 16\,817,93$$

$$\underline{K} = \begin{bmatrix} 480 & 0 & -2400 \\ 0 & 16817,93 & -14142,14 \\ -2400 & -14142,14 & 35784,14 \end{bmatrix} \quad \underline{F} = -\underline{K}_0 = \begin{bmatrix} 25 \\ 84,09 \\ 12,623 \end{bmatrix}$$

$$\underline{K} \cdot \underline{u} = \underline{F} \Rightarrow \underline{u} = \begin{bmatrix} 0,1396 \text{ m} \\ 0,01983 \text{ m} \\ 0,01752 \text{ rad} \end{bmatrix}$$

$$(b) \quad D_4 = -\frac{K_{40}}{K_{44}} = \frac{84,09}{16817,93} \approx 5 \cdot 10^{-3} \text{ m}$$

(c) D_1 muss 0,2 m betragen, damit der Deckel gerade noch nicht kippelfällt

$$(12,4 \text{ m} = l_2 + l_3 + 2 \cdot D_1)$$

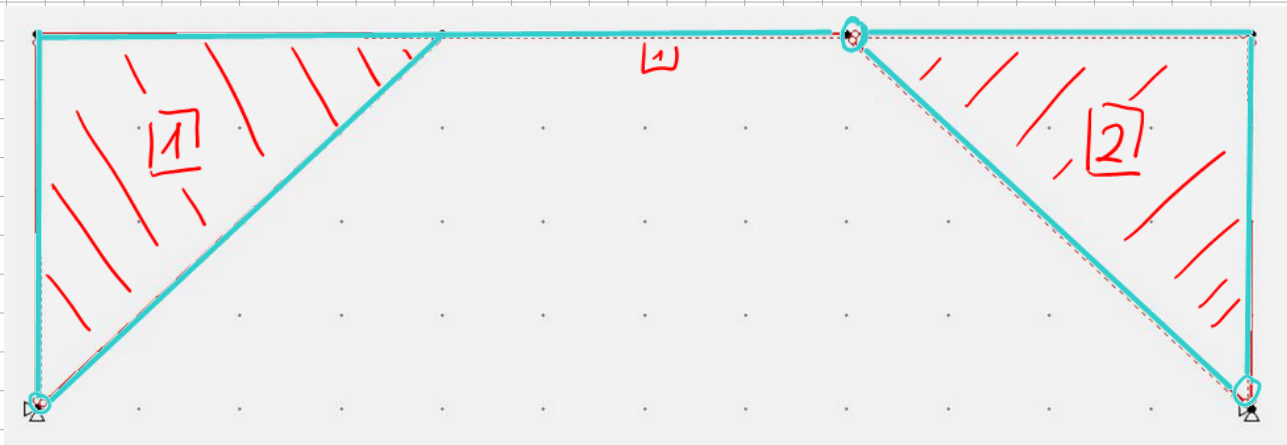
Da die Berechnung des VV linear ist, kann die Vichte skaliert

werden. $\frac{D_{1,\text{neu}}}{D_{1,\text{alt}}} = \frac{0,2}{0,1396} = 1,432 \approx \frac{\gamma_{\text{neu}}}{\gamma_{\text{alt}}}$

$$\rightarrow \gamma_{\text{neu}} = 1,432 \cdot \gamma_{\text{alt}} = 14,32 \text{ a/m}^2 / \text{efm}$$

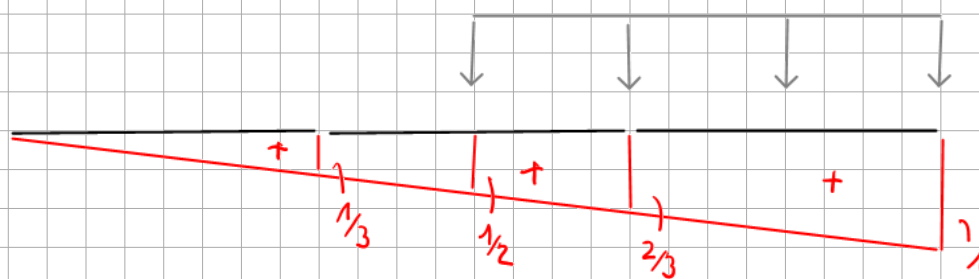
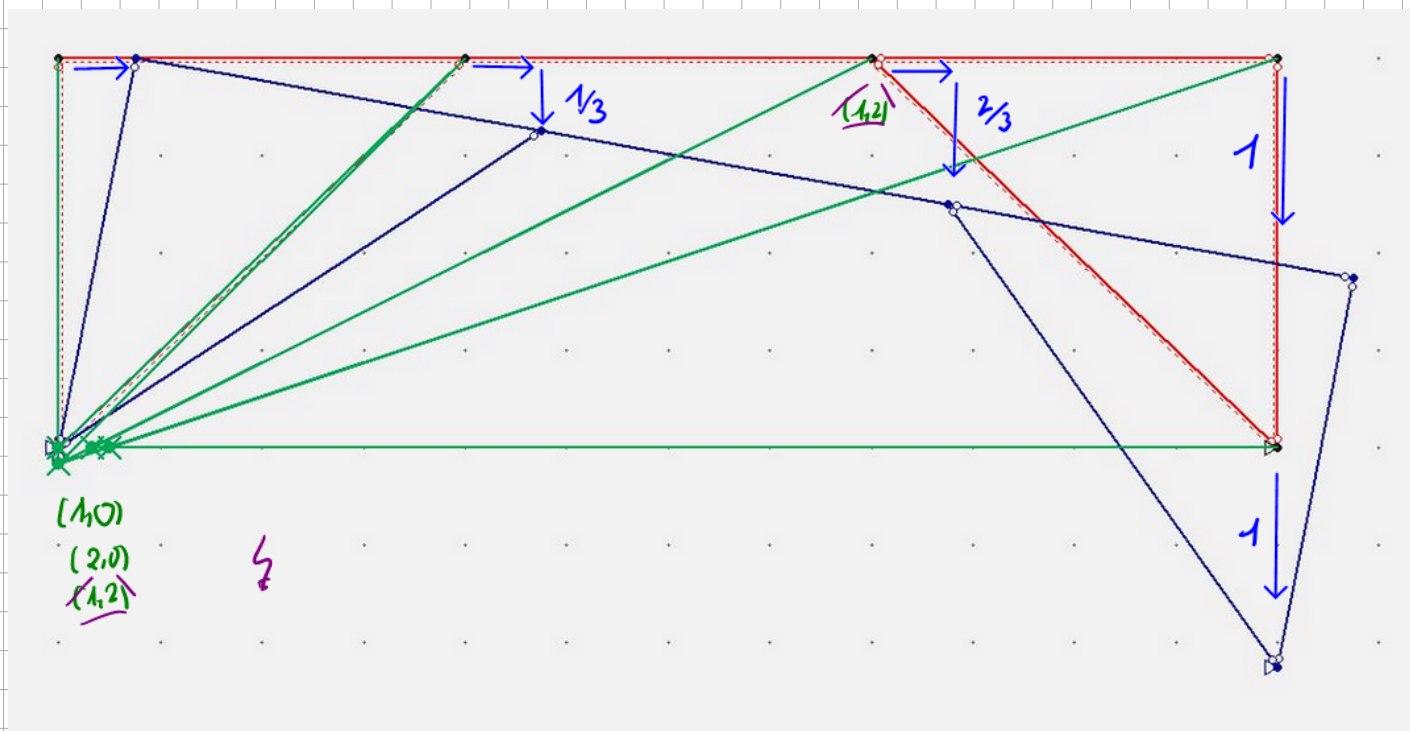
Probeklausur 7 - Aufgabe 5

(a)



3-Gelenk-Tragwerk \rightarrow statisch bestimmt

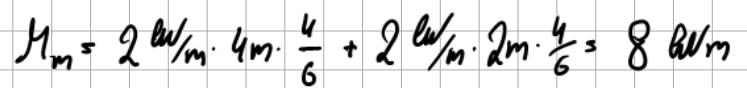
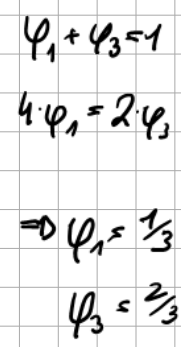
(b)



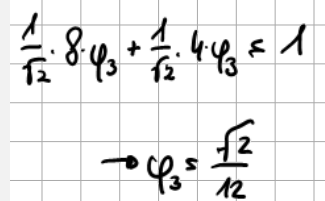
ungünstigste Laststellung

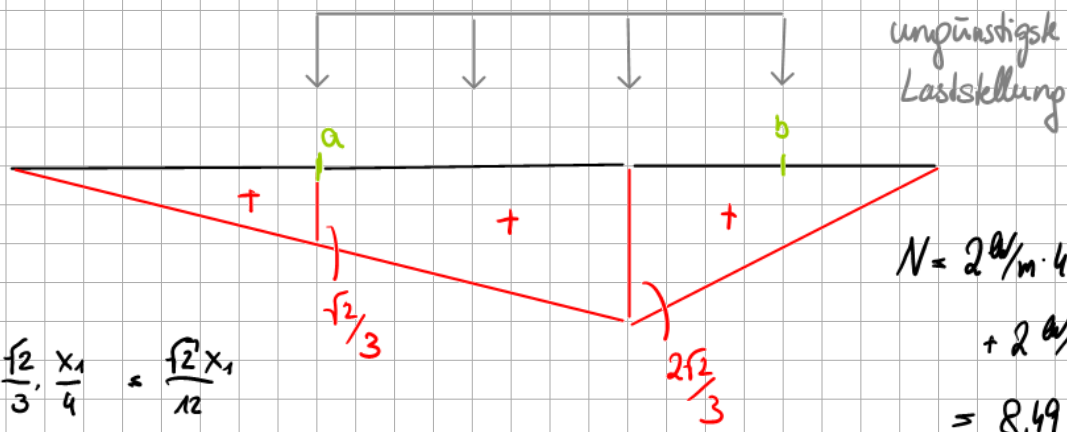
$$B_v = 2 \text{ kN/m} \cdot 6 \text{ m} \cdot \frac{1}{2} \left(\frac{1}{2} + 1 \right) = 9 \text{ kN}$$

(c)



(d)





$$\eta(x_1) = \frac{\sqrt{2}}{3} \cdot \frac{x_1}{4} = \frac{\sqrt{2}}{12} x_1$$

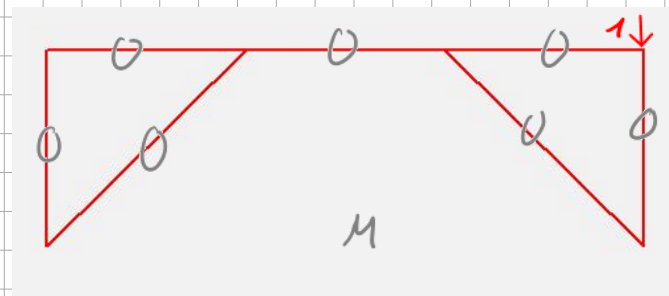
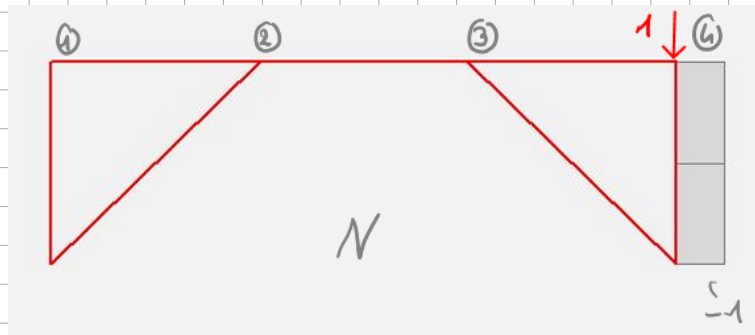
$$\eta(x_2) = \frac{2\sqrt{2}}{3} - \frac{2\sqrt{2}}{3} \cdot \frac{x_2}{4} = \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6} x_2$$

$$\text{I } \eta(a) = \frac{a\sqrt{2}}{12} = \frac{2\sqrt{2}}{3} - \frac{b\sqrt{2}}{6} = \eta(b)$$

$$\Rightarrow a=4 \\ b=2$$

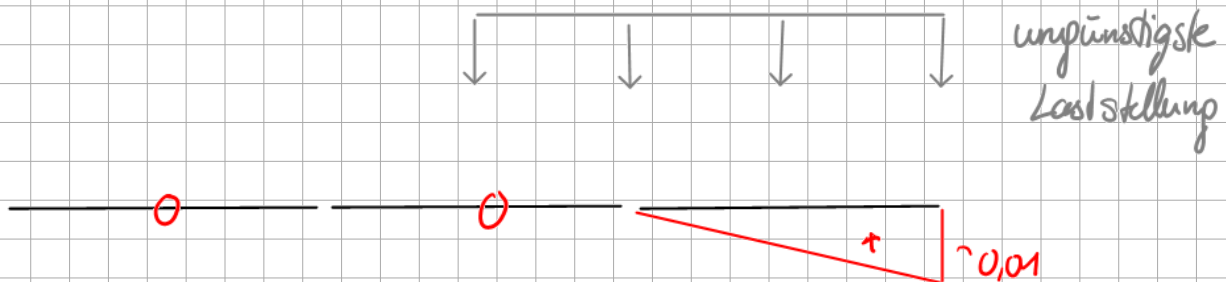
$$\text{II } b+8-a=6 \rightarrow b=a-2$$

(e)



$$W_h = 1^2 \cdot \frac{4}{8k} = 0,01 \text{ m}$$

aus N- und M-Verlauf folgt: $w_1 = w_2 = w_3 = 0$



$$W = 2 \text{ kN/m} \cdot 4 \text{ m} \cdot \frac{1}{2} \cdot 0,01 + 2 \text{ kN/m} \cdot 2 \text{ m} \cdot 0 = 0,04 \text{ m}$$