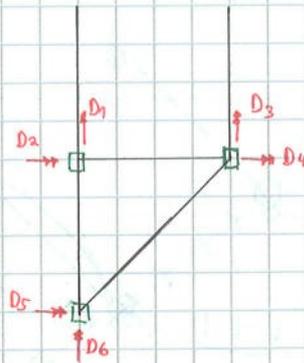
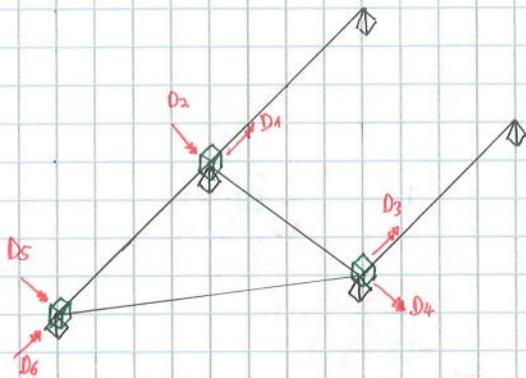
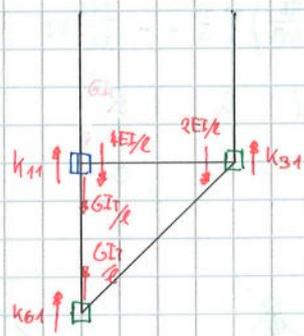
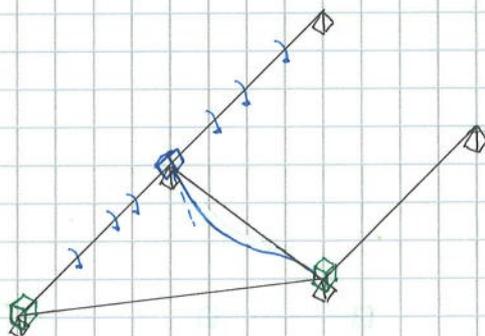


Discretisierung

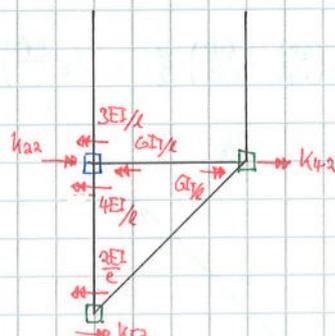
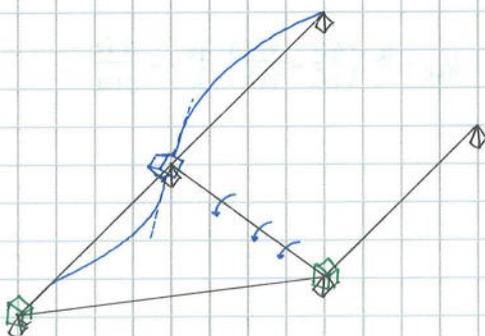


Einheitszustand  $D_1 = 1$



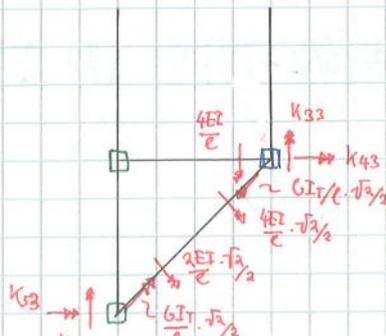
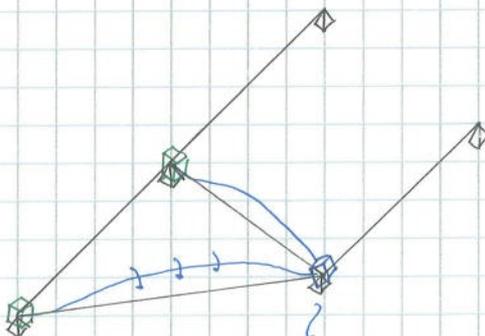
$$k_{11} = \frac{GI_T}{4} + \frac{4EI}{4} = \frac{9}{8} \cdot EI, \quad k_{31} = \frac{2EI}{4} = \frac{1}{2} EI, \quad k_{61} = -\frac{GI_T}{4} = -\frac{1}{8} \cdot EI \quad GI_T = \frac{1}{2} EI$$

Einheitszustand  $D_2 = 1$



$$k_{22} = \frac{3EI}{4} + \frac{4EI}{4} + \frac{GI_T}{4} = \frac{15}{8} EI, \quad k_{42} = -\frac{GI_T}{4} = -\frac{1}{8} \cdot EI, \quad k_{52} = \frac{2EI}{4} = \frac{1}{2} \cdot EI$$

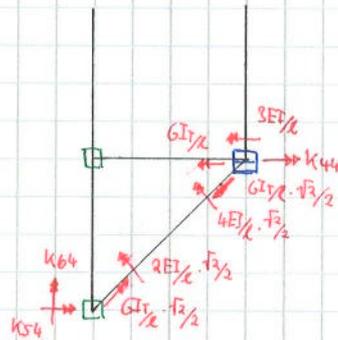
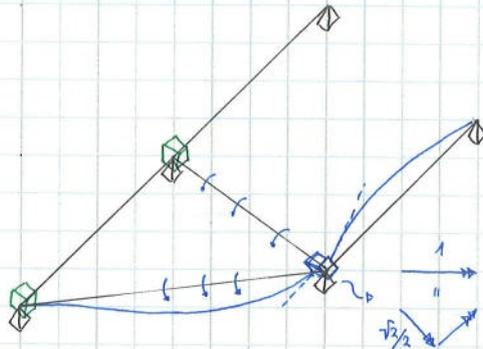
Einheitszustand  $D_3 = 1$



$$k_{33} = \frac{4EI}{4} + \frac{\sqrt{2}}{2} \cdot \left( \frac{GI_T}{\sqrt{2}} + \frac{4EI}{\sqrt{2} \cdot 4} \right) \cdot \frac{\sqrt{2}}{2} = \left( 1 + \frac{9\sqrt{2}}{32} \right) EI, \quad k_{43} = \frac{\sqrt{2}}{2} \cdot \left( \frac{4EI}{4\sqrt{2}} + \frac{GI_T}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = -\frac{7\sqrt{2}}{32} \cdot EI$$

$$k_{53} = -\frac{\sqrt{2}}{2} \cdot \left( \frac{2EI}{4\sqrt{2}} + \frac{GI_T}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = -\frac{5\sqrt{2}}{32} EI, \quad k_{63} = \frac{\sqrt{2}}{2} \cdot \left( \frac{2EI}{4\sqrt{2}} - \frac{GI_T}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{32} EI$$

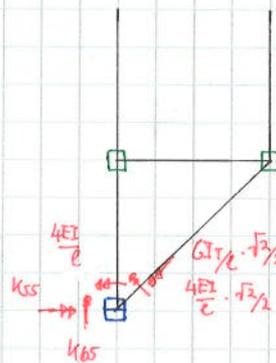
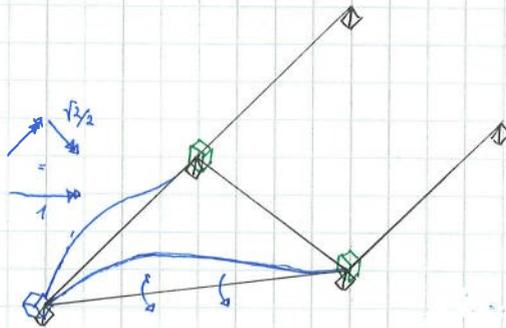
Einheitszustand  $D_4 = 1$



$$k_{44} = \frac{3EI}{4} + \frac{GI_T}{4} + \frac{\sqrt{2}}{2} \cdot \left( \frac{GI_T}{4\sqrt{2}} + \frac{4EI}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = \left( \frac{3}{8} + \frac{9\sqrt{2}}{32} \right) EI, \quad k_{54} = \frac{\sqrt{2}}{2} \cdot \left( \frac{2EI}{4\sqrt{2}} - \frac{GI_T}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{32} EI$$

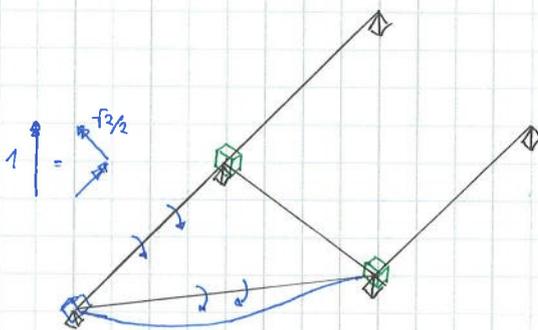
$$k_{64} = -\left( \frac{GI_T}{4\sqrt{2}} + \frac{2EI}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{5\sqrt{2}}{32} EI$$

Einheitszustand  $D_5 = 1$



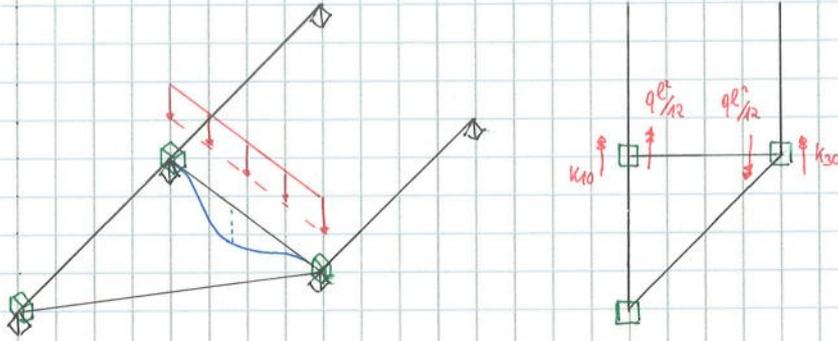
$$k_{55} = \frac{4EI}{4} + \frac{\sqrt{2}}{2} \cdot \left( \frac{4EI}{4\sqrt{2}} + \frac{GI_T}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = \left( 1 + \frac{9\sqrt{2}}{32} \right) EI, \quad k_{65} = \frac{\sqrt{2}}{2} \cdot \left( \frac{GI_T}{4\sqrt{2}} - \frac{4EI}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = -\frac{7\sqrt{2}}{32} EI$$

Einheitszustand  $D_6 = 1$



$$k_{66} = \frac{GI_T}{4} + \frac{\sqrt{2}}{2} \cdot \left( \frac{GI_T}{4\sqrt{2}} + \frac{4EI}{4\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{2} = \left( \frac{1}{8} + \frac{9\sqrt{2}}{32} \right) EI$$

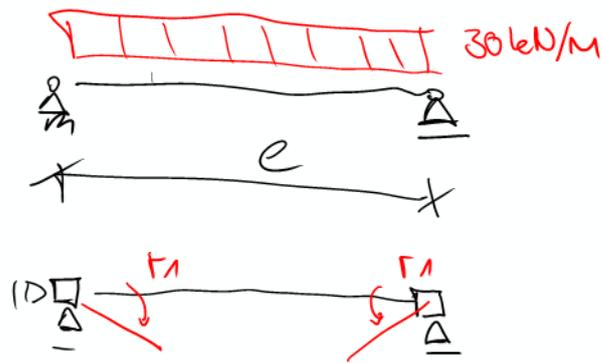
lastzustand



$$k_{10} = -\frac{q \cdot l^2}{12} = -40, \quad k_{30} = \frac{q \cdot l^2}{12} = 40$$

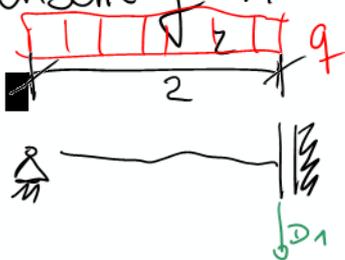
Teil b

Diskretisierung



• unter Berücksichtigung der Symmetrie  $n_G = 1$  ( $r_1$ )

• Durchsenkung in Feldmitte (Symmetrie!)

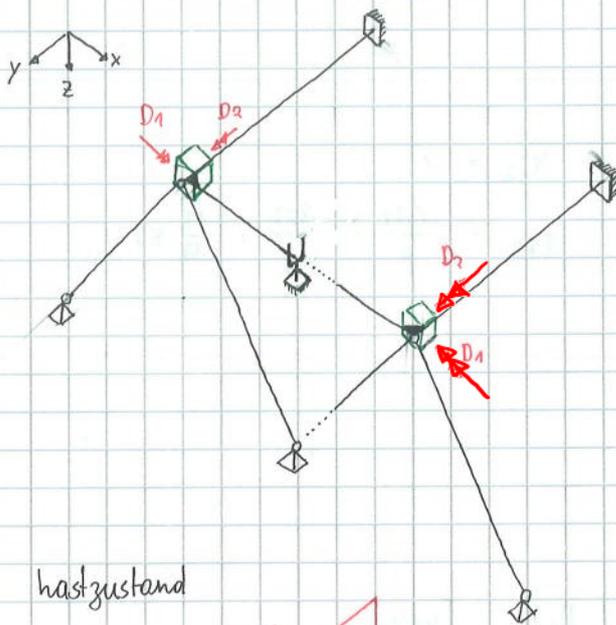


$$\rightarrow k_n = \frac{3EI}{2^3} = 300$$

$$k_{10} = -\frac{5}{8} q \cdot 2 = -37.5$$

$$\hookrightarrow D_1 = -\frac{k_{10}}{k_n} = 0.125 \text{ m}$$

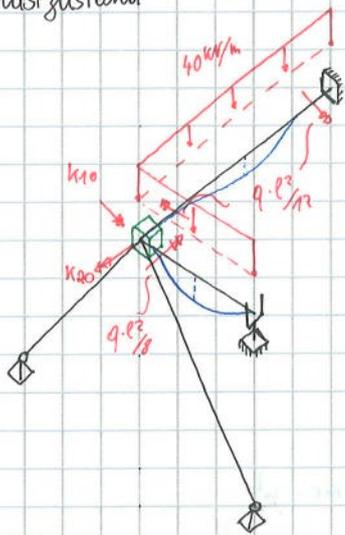
Diskretisierung



Symmetriebedingungen:

- $u_z = 0$  → Belastung antisymmetrisch
- $u_x = u_y = 0$  → keine last
- $\varphi_z = 0$  → keine last
- $\varphi_x = 0$  → Belastung antisymmetrisch
- $\varphi_y \neq 0$

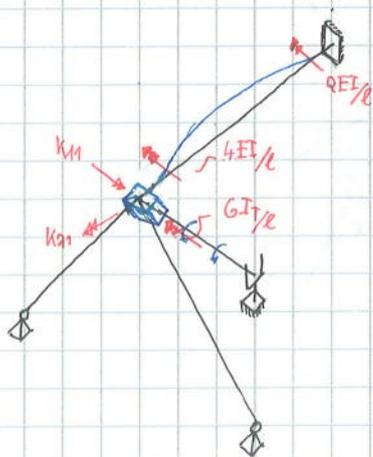
lastzustand



$$k_{10} = \frac{q \cdot l^2}{12} = \frac{40 \cdot 6^2}{12} = 120$$

$$k_{20} = \frac{q \cdot l^3}{8} = \frac{40 \cdot 3^3}{8} = 45$$

Einheitszustand  $D_1 = 1$

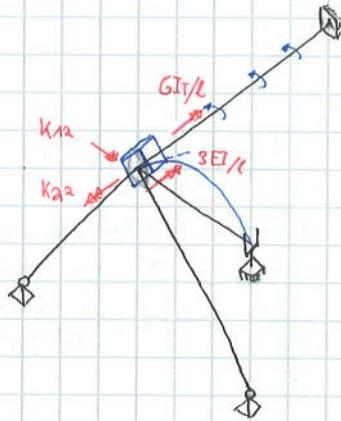


$$k_{11} = \frac{4EI}{6} + \frac{GI_T}{3} = \frac{5}{6} EI$$

$$GI_T = \frac{1}{2} EI$$

$$k_{21} = 0$$

Einheitszustand  $D_2 = 1$



$$K_{12} = 0$$

$$K_{22} = \frac{GI_T}{l} + \frac{3EI}{l} = \frac{13}{12} EI$$

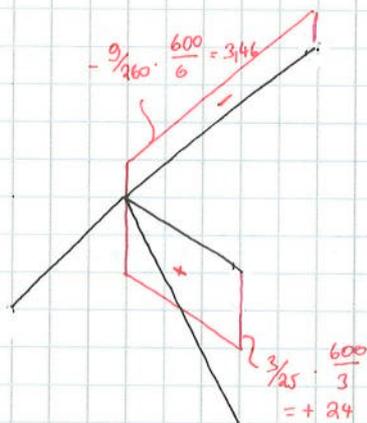
Gleichungssystem:

$$\begin{bmatrix} \frac{5}{6} & 0 \\ 0 & \frac{13}{12} \end{bmatrix} \cdot EI \cdot \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = - \begin{bmatrix} 120 \\ 45 \end{bmatrix}$$

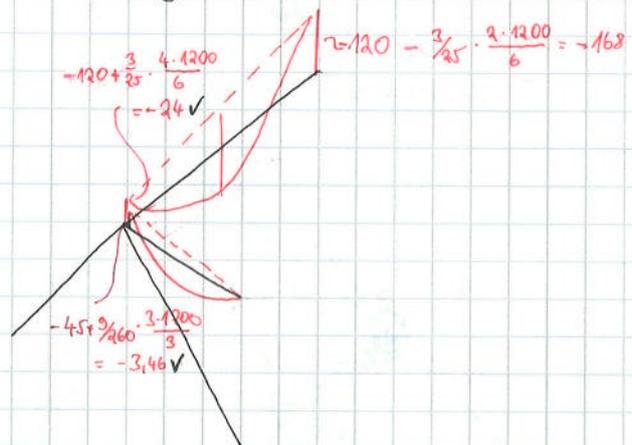
$$\rightarrow D_1 = - \frac{3}{25}$$

$$D_2 = - \frac{9}{260}$$

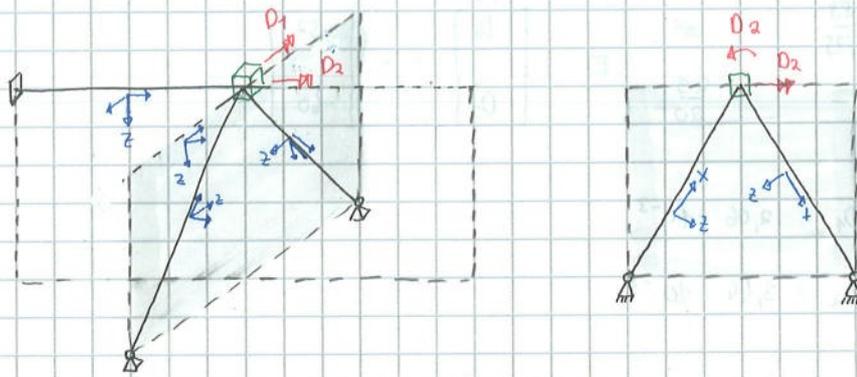
Torsionsmomente:



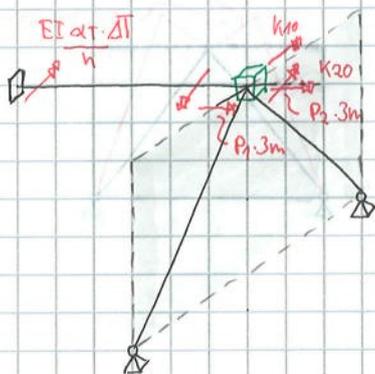
Biegemomente



Diskretisierung



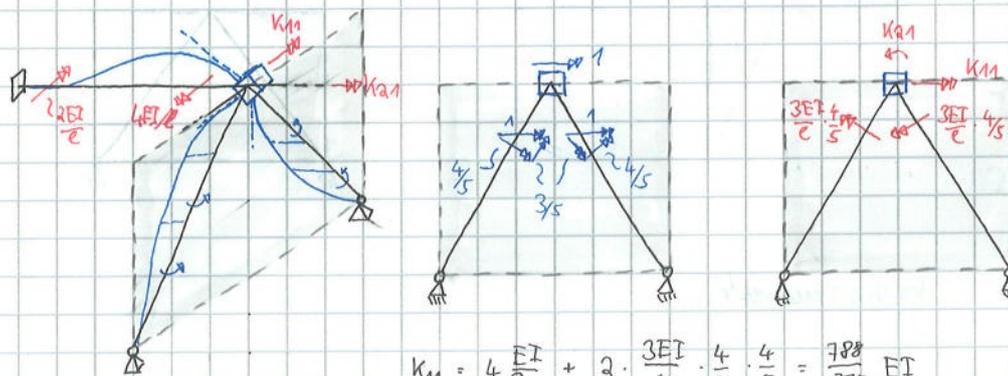
lastzustand



$$K_{10} = \frac{EI \cdot 10^{-5}}{0,30} - 20 \cdot 3 = -52$$

$$K_{20} = -20 \cdot 3 = -60$$

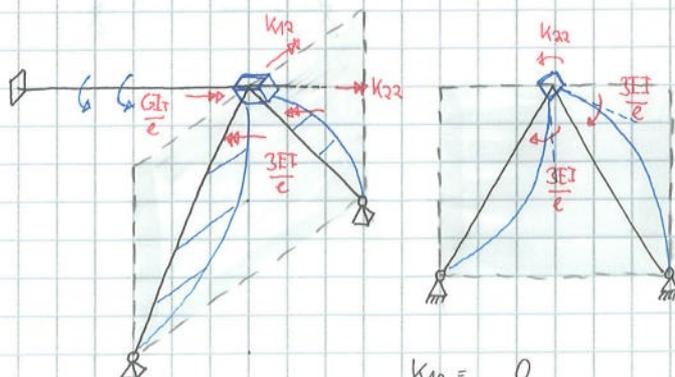
Einheitszustand  $D_1 = 1$



$$K_{11} = 4 \frac{EI}{3} + 2 \cdot \frac{3EI}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{788}{375} EI$$

$$K_{21} = 0$$

Einheitszustand  $D_2 = 1$



$$K_{12} = 0$$

$$K_{22} = \frac{GI_T}{3} + 2 \cdot \frac{3EI}{5} = \frac{29}{20} EI$$

$$GI_T = \frac{3}{4} EI$$

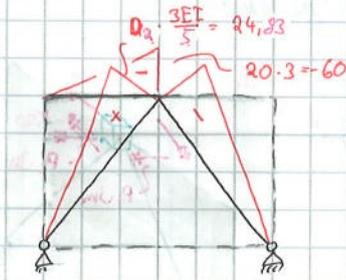
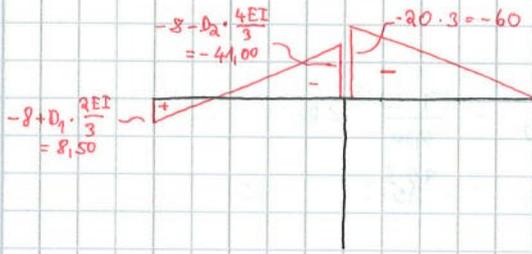
# Gleichungssystem

$$\begin{bmatrix} \frac{788}{375} & \neq \\ - & \frac{29}{20} \end{bmatrix} EI \cdot \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = - \begin{bmatrix} -52 \\ -60 \end{bmatrix}$$

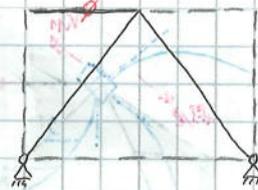
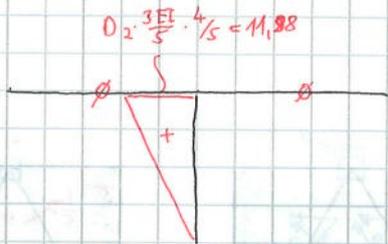
$$\rightarrow D_1 = 2,06 \cdot 10^{-3}$$

$$D_2 = 3,44 \cdot 10^{-3}$$

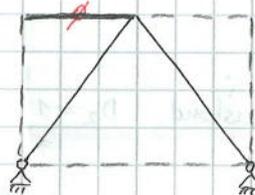
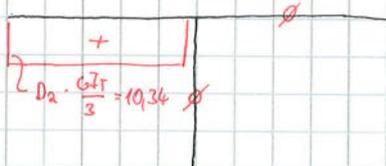
## Biegemomente $M_y$



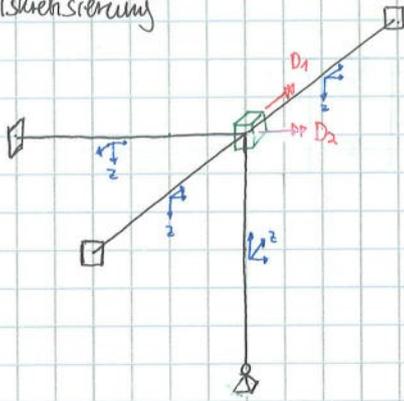
## Biegemomente $M_z$



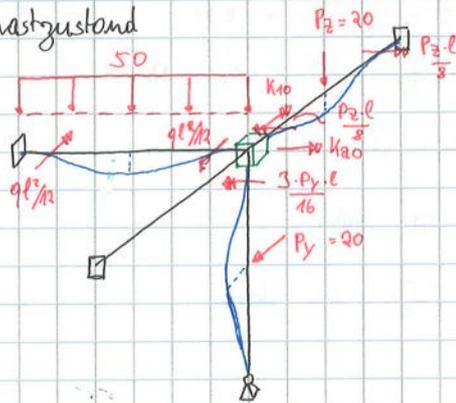
## Torsionsmomente



Diskretisierung



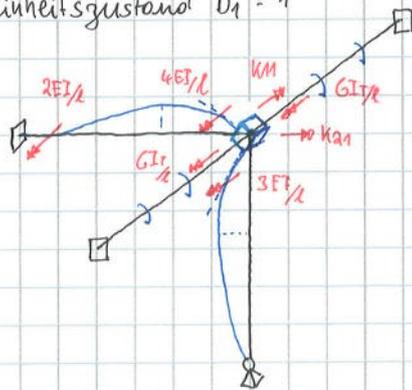
lastzustand



$$k_{10} = \frac{50 \cdot 2^2}{12} = \frac{50}{3}$$

$$k_{20} = \frac{3 \cdot 20 \cdot 2}{16} + \frac{20 \cdot 3}{8} = 15$$

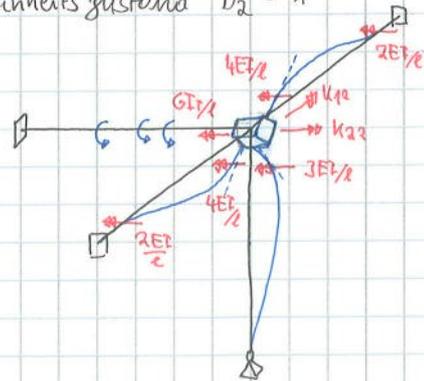
Einheitszustand  $D_1 = 1$



$$k_{11} = \frac{3EI}{2} + 2 \cdot \frac{GI}{3} + \frac{4EI}{2} = \frac{29}{6} \cdot EI$$

$$k_{21} = 0$$

Einheitszustand  $D_2 = 1$



$$k_{12} = 0$$

$$k_{22} = \frac{GI}{2} + \frac{3EI}{2} + 2 \cdot \frac{4EI}{3} = \frac{31}{6} EI$$

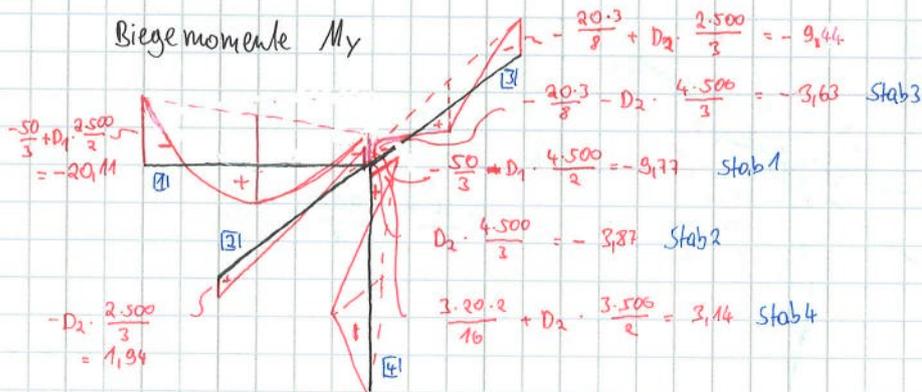
Gleichungssystem

$$\begin{bmatrix} 29/6 & \# \\ 0 & 31/6 \end{bmatrix} EI \cdot \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = - \begin{bmatrix} 50/3 \\ 15 \end{bmatrix}$$

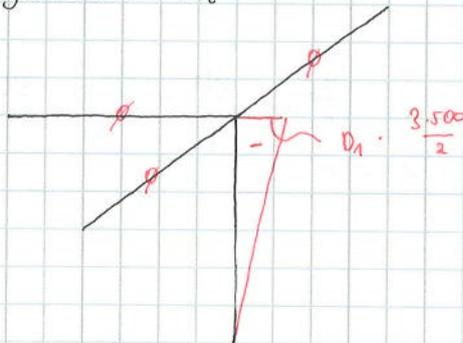
$$\rightarrow D_1 = -6,90 \cdot 10^{-3}$$

$$D_2 = -5,81 \cdot 10^{-3}$$

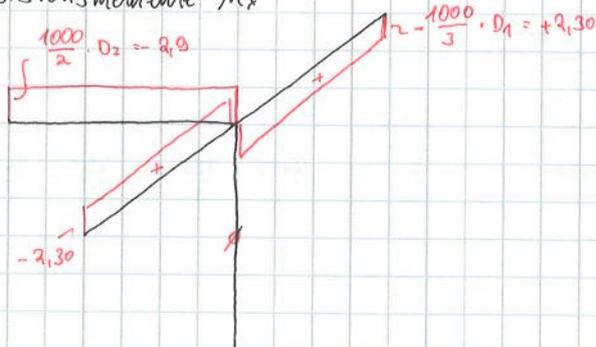
Lokale Momente:



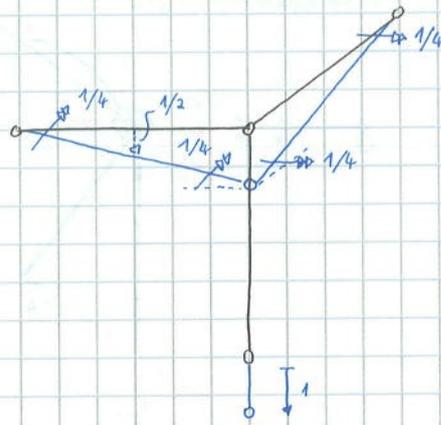
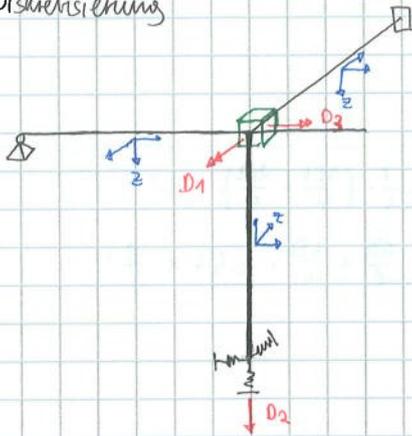
Biegemomente  $M_z$



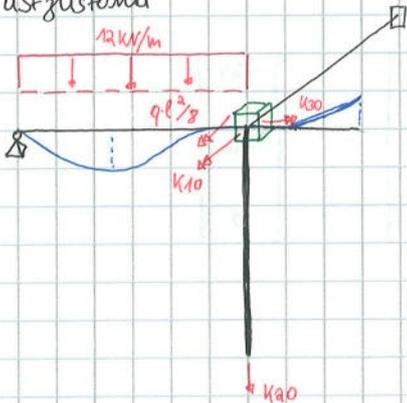
Torsionsmomente  $M_x$



Diskretisierung



Lastzustand



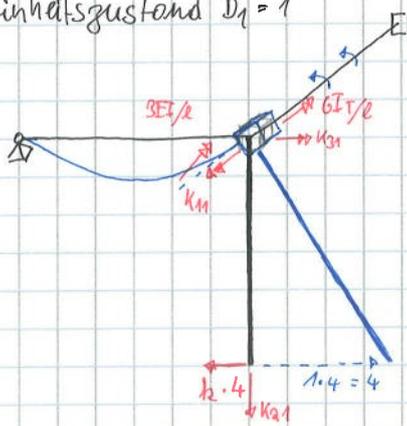
Temperaturlast: Kragarm kann sich frei verformen → keine Belastung

$$K_{10} = - \frac{12 \cdot 4^2}{8} = -24$$

$$1. K_{20} = - \frac{1}{4} \cdot \frac{12 \cdot 4^3}{8} - \frac{1}{2} \cdot 12 \cdot 4 = -30$$

$$K_{30} = 0$$

Einheitszustand  $D_1 = 1$



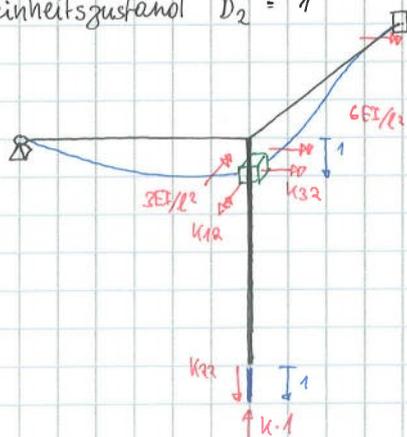
$$K_{11} = \frac{3EI}{4} + \frac{6IT}{4} + k \cdot 4 \cdot 4 = \frac{23}{12} EI$$

$$k = \frac{1}{16} EI, IT = \frac{2}{3} EI$$

$$1. K_{21} = \frac{1}{4} \cdot \frac{3EI}{4} = \frac{3}{16} EI$$

$$K_{31} = 0$$

Einheitszustand  $D_2 = 1$

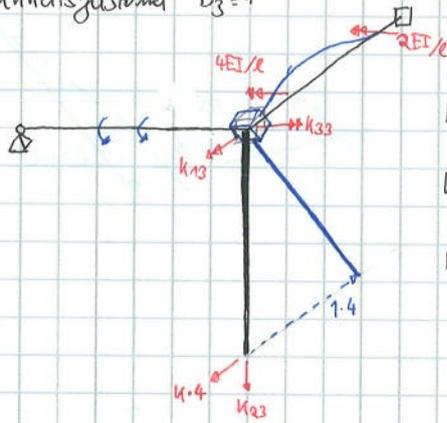


$$K_{12} = \frac{3EI}{4^2} \checkmark$$

$$K_{22} = k \cdot 1 + \frac{1}{4} \cdot \left( \frac{3EI}{4^2} + 2 \cdot \frac{6EI}{4^2} \right) = \frac{19}{64} EI$$

$$K_{32} = - \frac{6EI}{4^2} = - \frac{3}{8} EI$$

Einheitszustand  $D_3 = 1$



$$k_{13} = 0$$

$$k_{23} = -\frac{1}{4} \cdot \left( \frac{4EI}{4} + \frac{2EI}{4} \right) \quad \checkmark$$

$$k_{33} = \frac{4EI}{4} + 2 \cdot 4 \cdot 4 = 2 \cdot EI$$

Gleichungssystem

$$\begin{bmatrix} 23/12 & \# & \# \\ 3/16 & 19/64 & \# \\ 0 & -3/8 & 2 \end{bmatrix} EI \cdot \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = - \begin{bmatrix} -24 \\ -30 \\ 0 \end{bmatrix}$$

$$\rightarrow D_1 = -1,96 \cdot 10^{-5}$$

$$D_2 = 5,53 \cdot 10^{-3}$$

$$D_3 = 1,04 \cdot 10^3$$

Dasch. an Punkt 1

$$\begin{aligned} w_1 &= D_2 - l_3 \cdot D_1 \\ &+ \frac{1}{2} \cdot (-2) \cdot 2l \cdot \frac{\Delta T}{h} \cdot 2 \\ &= 10,946 \text{ mm} \end{aligned}$$

