MODEL OF INTERACTION OF FRESH AND SALT WATER IN TIDAL ESTUARIES UNDER FORMATION OF INTERNAL WAVES

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Abstract

Estuarine vertical mixing is defined as a function of river water discharge and tidal range. Conditions of formation of mixing layers and internal waves in an estuary are discussed. 1D Navier-Stokes equation is used to describe water motion in a plane turbulent mixing layer. The period of self-oscillations of the velocity field in a mixing layer calculated by the model is compared with that estimated from measurement results in partially mixed St. Lawrence estuary.

Introduction

Tidal estuary is a semi-enclosed coastal water body where waters of different density interact. In these coastal systems, the interaction of strong and stratified flows with irregular topographies produces some of the most interesting naturally occurring physical processes found in the ocean.

The type of estuarine vertical circulation and stratification depends on many factors, the main of which is the ratio of river water flow to tidal prism, defined as the volume of water brought into the estuary by the flood tide. All estuaries could be roughly subdivided into three types: well-mixed (or vertically mixed), salt-wedge (highly stratified) and partially mixed (slightly stratified) estuaries (Dyer, 1973). If the tidal volume is much larger than the river flow, turbulent mixing dominates, and estuary is a well-mixed one, in which the salinity of water is practically homogenous through the depth of the estuary. Strong tidal currents eliminate the vertical layering of fresh water floating above denser seawater, and salinity is typically determined by the daily tidal stage. Salt-wedge estuaries occur when a rapidly flowing river discharges into the deep ocean bay, where tidal currents are comparatively weak. A sharp boundary is created between the water masses, with fresh water floating on top and a wedge of saltwater on the bottom. Some mixing does occur at the boundary between the two water masses, but generally it is slight. In slightly stratified or partially mixed estuaries, saltwater and freshwater mix at all depths; however, the lower layers of water typically remain saltier than the upper layers. Salinity is largest at the mouth of the estuary and decreases as one moves upstream. Inside the water column there is a zone with high gradient of salinity, where the mixing layer is generated. This zone is a possible source of instability and internal waves. Interaction of freshwater and saltwater in partially mixed estuary produces complex multilayer currents in estuaries of this type.

Classification of types of estuarine mixing

The balance of forces that establishes a steady state in estuarine circulation involves advection of freshwater from a river and introduction of sea water through turbulent mixing, produced by tidal currents. The ratio of freshwater input to sea water mixed in by the tides determines the estuary type. One way of quantifying this is by comparing the volume W of freshwater that enters from the river during one tidal period, with the volume W_t of water brought into the estuary by the tide and removed over each tidal cycle. W is sometimes called the river volume, while W_t is known as the tidal prism. It is important to note that it is only the Simmons criterion $\alpha = W/W_t$ (Simmons, 1955), that determines the estuary type, not the absolute values of W or W_t. In other words, estuaries can be of widely different size and still belong to the same type. This fact will be confirmed by consideration of mixing processes in estuaries of the rivers St. Lawrence and Yenisei.

Water dynamics in partially mixed estuary

If the tidal volume W_t is of order or larger than the river volume and the ratio W/W_t equals to 0.1 – 1, the estuary can be classified as a slightly stratified estuary, also known as the partially mixed estuary with entrainment (Tomczak & Godfrey, 2003). The fresh water movement in the upper layer against the stronger tidal current produces intense current shear at the interface. This creates instabilities in the form of internal waves which become unstable and break. When the tops of the breaking waves separate from the interface they inject salt water into the upper layer. The result is a net upward transport of mass and salt known as entrainment. As a general definition, entrainment is the transport of mass drawn from a less turbulent medium into a more turbulent medium. Entrainment is a one-way process; there is no transport of mass from the more turbulent medium to the less turbulent medium.

The characteristics of the partially mixed estuary are: the salinity of water in the upper layer increases towards the sea, partly due to salt water entrainment from the lower layer. Salinity in the lower layer remains constant at the oceanic value. Vertical salinity profiles show oceanic salinity below the interface and a gradual salinity increase from station to station above the interface as the ocean is approached. Thus salinity of the surface layer increases and in order to discharge river flow the seaward surface flow is considerably enhanced. This causes an increase of the compensating landward bottom current of salt water. As a consequence a system of two layers of water of different densities, which are moving in the opposite directions, is formed (Figure 1). A particularly interesting aspect of the slightly stratified estuary is the intensification of the circulation produced by the entrainment process.

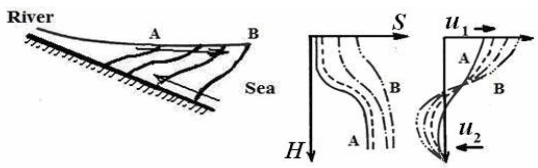

Figure 1. Change of salinity S and velocity profiles u along the partially mixed estuary; velocity of the surface (u_1) and bottom (u_2) currents.

An amount W of fresh water enters the estuary at its head during a tidal cycle. This water moves towards the sea in the upper layer. Entrainment from below adds salt water to the layer, increasing the volume transport of the upper layer. This process continues, and the volume transported in the upper layer becomes several times larger than the fresh water volume supplied by the river, say to kW, where k is the magnification factor for the estuary. In the lower layer, a corresponding volume $(k–1)W$ has to be supplied compensate for the loss from entrainment into the upper layer. It is seen that the introduction of the volume W into an oceanic environment produces a vigorous circulation in which the additional volume $(k–1)W$ is drawn from the sea into the estuary and returned after mixing with the fresh water. This illustrates the thermohaline character of the estuarine circulation quite well; without the "priming" of the system by fresh water the estuary would be nothing but an appendix to the coastal ocean with identical water mass and very little mean movement.

Because water is constantly added to the upper layer along the length of the estuary, the interface does not come up to the surface but remains submerged even at the mouth, where its location is hard to determine from the salinity profile. It is, however, well defined through the depth of zero mean flow and can thus be determined through current measurements over a tidal period, if required. Observations show that magnification factors of 30 and more are easily obtained in natural estuaries (Tomczak & Godfrey, 2003).

Slightly stratified estuaries usually require a large lower layer volume in comparison to the volume of the upper layer to provide enough oceanic water for the entrainment process. This condition is met particularly well in true fjords, where the fresh water is derived from the melting of glaciers and small rivers inflows, and produces a thin upper layer that is only a few meters thick and spreads over a lower layer which may be hundreds of meters deep.

The considerable middle part of the partially mixed estuary is occupied by two-layer current with the horizontal salinity gradient in the surface and bottom layers linearly increasing down the estuary. The undiluted freshwater in the surface layer occurs only near the head of the estuary. The vertical salinity profile changes weakly along this part of estuary (Figure 1).

Parameter of stratification *n* for St. Lawrence and Yenisei estuaries

The type of mixing and water stratification in an estuary can be estimated with the help of well-known criterion of stratification n

$$n=\Delta S/S_m, \quad (1)$$

where $\Delta S=S_b – S_s$, and $S_m=0.5(S_b+S_s)$

The type of mixing of fresh and salt waters and stratification of the water column in an estuary can be defined with the help of Table 1 (Mikhailov, 1997):

Table 1. The type of mixing and stratification of water in an estuary as a function of parameter n.

n	Mixing	Stratification
0 – 0.1	Complete	Weak or none
0.1 – 1.0	Partially mixed	Moderate
>1.0	Salt wedge	Strong

The estimate of n for the St. Lawrence estuary varies within the range 0.1–1.26 (Table 2), thus suggesting the complex structure of water in this area. In particular, parameter n calculated for the reach between ile d'Orlean and ile d'Coudres islands (Figure 2) (Dolgopolova & Isupova, 2010) shows that ~23 km downstream of ile d'Orlean Island, the flow is strongly stratified with the presence of a wedge of saline water.

The results of measuring turbulent velocity pulsations and salinity near ile d'Coudres (Bourgault et al, 2001) show that the interaction between fresh and saline waters in this part of the estuary with the formation of internal waves occurs regularly about 1 h before high water. Thus, by parameter n, the upper part of the St. Lawrence estuary can be referred to as partially mixed, moderately stratified estuary with transition into a strongly stratified estuary.

The depth of St. Lawrence estuary appeared to be quite enough (Figure 3) for development of internal waves and entrainment process that will be shown below.

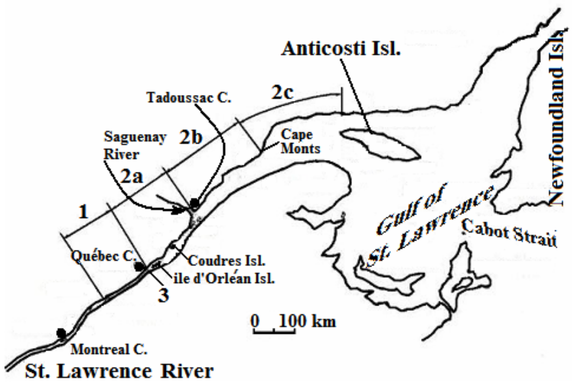

Figure 2. Scheme of St. Lawrence River mouth: 1 – river mouth reach; 2a, 2b and 2c – upper, middle and lower estuary correspondingly; 3 – upper boundary of the estuary. The division of the estuary into several reaches is made in accordance with (Dolgopolova & Isupova, 2010).

Simmons parameter n calculated with the help of salinity profiles for the Yenisei estuary (typical examples are presented in Figure 4, (Harms et al, 2002)) lies in the range 0.3÷0.4 (Table 2), which in accordance with Table 1 shows partially mixed moderately stratified current in this estuary.

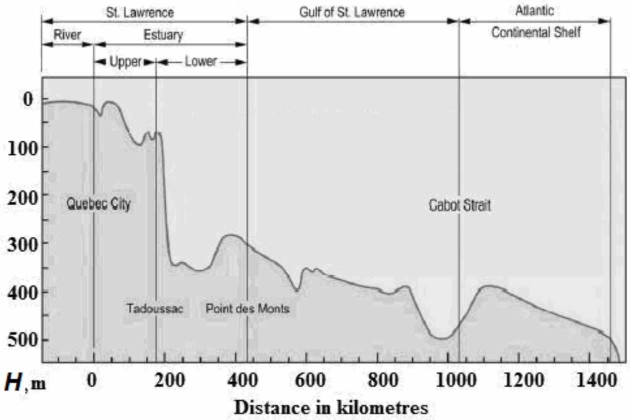

Figure 3. Longitudinal bottom profile of St. Lawrence estuary.

Simmons criterion $\alpha = W/W_t$ for St. Lawrence and Yenisei estuaries

These two estuaries were chosen for comparison as they both have an abrupt sill on their bottoms (Figures 3, 4). Let us estimate the type of estuarine circulation by α for St. Lawrence and Yenisei estuaries. Cumulative mean annual discharge Q of the river St. Lawrence and its tributary Saguenay into the estuary equals to 13383 m^3/s. Then fresh water flow during 6 hours equals to W = 2.9·10^8 m^3 (see Table 2).

Estimate of tidal volume W_t could be made by two ways (Dolgopolova & Isupova, 2010). The assessment by the first usual method can be obtained as a product of the estuary area F and the tidal range h at the sea cross section of the estuary. This estimate of the volume of tidal prism is very approximate, because it strongly depends on the choice of the boundary between the estuary and the gulf.

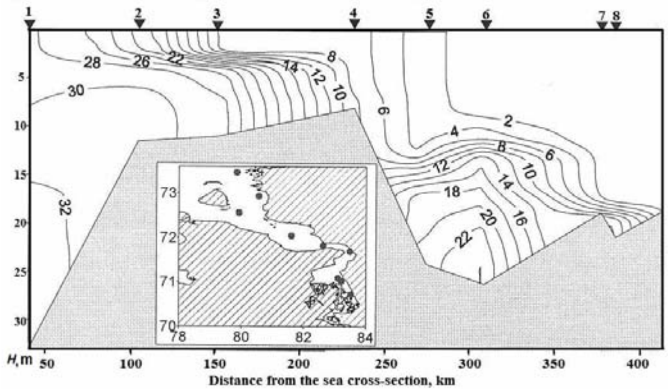

Figure 4. Longitudinal bottom profile and vertical salinity distributions (‰) in Yenisei estuary (Harms et al, 2002).

Table 2. Characteristics of the estuaries of St. Lawrence and Yenisei rivers (F, B, h, – is the area of an estuary, width and tidal range at the mouth cross-section correspondingly)

Parameter	St. Lawrence	Yenisei
h, m	0.12 – 0.68	0.4
Q, m^3/s	13383	19800
W, m^3	2.9·10^8	4.3·10^8
F, m^2	1.1 10^4	2.0·10^4
W_t km^3	2.2·10^{10}	8.0·10^9
W/W_t	0.01	0.05
B, H, u m, m, m/s	50 /300 /3.0	120 /20 /1.2
W_t', m^3	3.9·10^8–2.2·10^9	1.6·10^9
W/W_t'	0.14 – 0.76	0.35
Ri	0.25–0.5	–
Ri$_L$	4.3	2.5–3.6
n	0.1 – 1.26	0.3 – 0.4

The second method can be used in case if the magnitude of reverse velocity of tidal flow u is available. The estimate of the tidal volume W_t' in this case is obtained as a product of tidal discharge and cross-sectional area. The results of calculations of W/W_t and W/W_t' for the estuaries of St. Lawrence and Yenisei presented in the Table 2 reveal partially mixed currents with slight stratification in both estuaries, that is confirmed by measurements of salinity profiles (Bourgault et al, 2001, Harms et al, 2002).

Shear instability

The development of shear instabilities (or Kelvin-Helmholtz instabilities) is thought to be the one of the main mechanisms, by which mixing is enhanced within the pycnocline of partially mixed estuaries. The resulting vertical fluxes of mass determine the mean baroclinic pressure field that drives the residual salt circulation. It is possible to observe such instabilities in a natural system where a persistent shear is maintained and where the main external forcing is provided by the periodic action of tides, as in a stratified tidal estuary.

Typically in oceanography it is accepted that a stratified current is unstable for Ri< 0.25, that is velocity shear is considered to be sufficient to overcome the tendency of the current to remain stratified. Gradient Richardson number is defined as $Ri = N^2 S^{-2}$, were vertical shear squared S^2 and buoyancy frequency squared N^2 are $S^2 = \overline{(\partial u/\partial y)^2} + \overline{(\partial v/\partial y)^2}$, $N^2 = -(g/\rho)\partial\rho/\partial y$, y is vertical direction positive upward. Using the experimental results obtained in St. Lawrence estuary during the flood – ebb cycle (Bourgault et al, 2001), the Richardson number is found to vary in the range $0.25 \le \mathrm{Ri} \le 0.5$ for the period hw – 1 – lw – 2 and Ri > 1 for the ebb flow period.

Several acoustic images of pycnocline made in St. Lawrence estuary during the flood – ebb cycle which are presented in the paper (Bourgault et al, 2001) show Kelvin-Helmholtz instabilities and mixing activity from the moment before high water (hw – 1) till low water – 2 (lw – 2), whereas during the ebb flow (lw–2 – lw+2) the measurements show quiet conditions. The extension of range of Richardson number for which the stratified flow in the estuary appeared to be unstable could be explained by interaction of this flow with irregular topography of the St. Lawrence estuary (Figure 3).

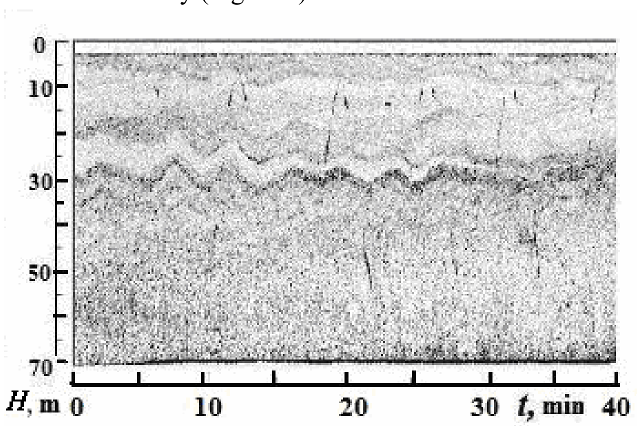

Figure 5. Image of the pycnocline presented in the paper (Bourgault et al, 2001) at two hours before high tide.

The interaction of the stratified fluid with abrupt topography perturbs the isopycnals and may be a source of density currents and high-frequency internal gravity waves. These topographically generated perturbations tend to destabilize the water column by increasing the vertical shear and thus constitute a source of energy for development of shear instabilities. Figure 6 shows a large wave train during the transition from ebb to flood tide between lw+2 and lw+3, when observed gradient Richardson number becomes low again. The vertical zone of nonlinearity reaches nearly 50 m. The wave train presented in Figure 6 looks as if it was induced by a tidally forced flow over a sill or a bank. The existence of large internal waves in this region could be explained by a sudden depression of isopycnals that forms over the bank edge during flood tide.

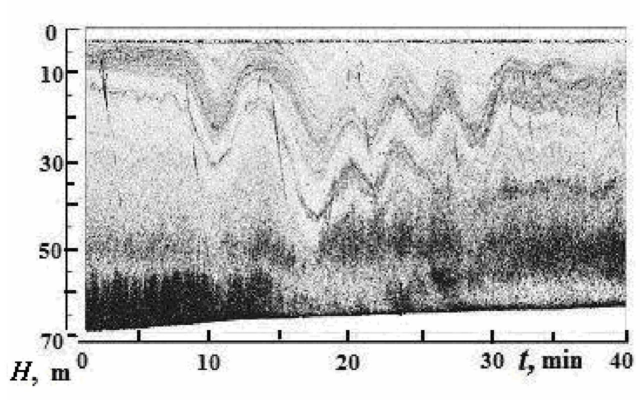

Figure 6. Image of the pycnocline presented in the paper (Bourgault et al, 2001) at two hours after low water.

The depression then propagates against the flow, in the seaward direction, where it gets trapped at the bank edge, the amplitude increases, and the wavelength decreases.

The thickness of the mixing layer varies in the range b = 10 m ÷ 22 m, then the wave length estimated by $\lambda \approx 2\,\pi\,b$ gives λ = 63 ÷ 138 m. At the transition from ebb to flood flow the thickness of the mixing layer becomes extremely large and λ = 314 m.

There is no enough data to calculate gradient Richardson number for Yenisei estuary. So following (Dyer, 1973, Mikhailov, 1997) one can estimate stability of the current in the Yenisei estuary by Richardson number for a layer Ri_L

$$\mathrm{Ri}_L = \frac{gH}{V^2} \cdot \frac{\Delta\rho}{\rho_m} \qquad (2)$$

where H is the flow depth; V is flow velocity in the upper water layer; Дс=c_2–c_1, c_m=(c_1 + c_2)/2, i.e., the difference between the densities and the mean water density in the bottom and surface layers.

The value of Richardson number (2), calculated with the use of profiles of conditional density (Bourgault et al, 2001)) for the segment of the upper estuary of the Saint Lawrence River between the ile d'Coudres Island and the Saguenay River mouth (Figure 2) was found to be Ri_L = 4.3 (Dolgopolova & Isupova, 2010), and falls into the range $2 < \mathrm{Ri}_L < 20$, corresponding to a partially mixed estuary (Dyer, 1973).

Calculation of Ri_L by (2) for the sites 2 and 3 of the Yenisei estuary (Figure 4) yields the range 2.5–3.6, which confirms that Yenisei estuary is also a partially mixed one. There is no evidence on existence of internal waves in Yenisei estuary. It is reasonable to suppose that in Yenisei bay no internal waves are generated because of insufficient depths of the estuary.

Mixing layers in an estuary

Different mixing layers occur widely in nature. They emerge between bottom and under ice currents of different velocities in ice covered flows (Dolgopolova, 2008), at the boundary of density flow at the mouths of rivers inflowing lakes, reservoirs and seas (Samolyubov, 1999), at the tidal river mouths where salt and fresh water currents move in the opposite directions (Dyer, 1973) etc. The existence of large-scale eddies in mixing layers is confirmed by many experimental studies (Dolgopolova, 2003, Tomczak & Godfrey, 2003).

Model of mixing layer

The model of mixing layer fluctuations will be applied to the mixing layer formed in an estuary between fresh water upper current moving seaward and brackish water layer of larger density moving upstream near the bottom. The motion in the mixing layers is known to have self-oscillation character (Speranskaya, 1982), and the velocity profile in such mixing layer is formed under the competing influence of the viscous and turbulent forces. Let us consider a plane turbulent mixing layer with a thickness b, velocities u_1 and u_2 of the oppositely directed currents and assess the period of its oscillations T (Figure 7).

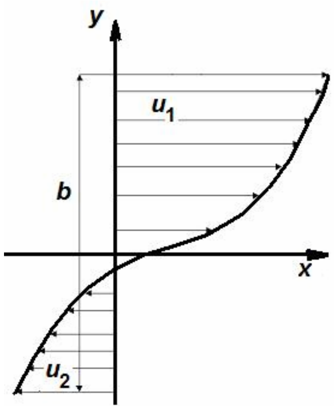

Figure 7. Scheme of two layers moving in the opposite directions.

The destruction of the mixing layer is assumed to be instantaneous and result in a jump-like return to the initial conditions. The 1D Navier-Stokes equation in dimensionless variables is used as an input equation of the model, and only the stage of mixing layer formation is considered:

$$\frac{\partial u^+}{\partial t}+\frac{\partial u^+ w^+}{\partial y^+}=-\frac{\partial p^+}{\partial x^+}+\nu_m^+\frac{\partial^2 u^+}{\partial y^{+2}} \qquad (3)$$

where $u^+ = u/u_1$, $w^+ = w/u_1$, u, w are the longitudinal and vertical components of flow velocity; u_* is the shear velocity; $x^+ = x/b$, $y^+ = y/b$, x, y are stream wise and vertical coordinates, $t^+ = tu_1/b$, $(p/\rho)^+ = p/(\rho u_1^2)$, $\nu_m^+ = \nu_m/bu_1$ ν_m is the kinematic viscosity; ρ is liquid density; p is pressure; t is time.

Boundary conditions for equation (3) are the following

$$\begin{aligned} &y^+ = 0, \quad u^+ = 1/2(1+u_1/u_2) \\ &y^+ = 0.5, \qquad u^+ = 1 \\ &y^+ = -0.5, \qquad u^+ = u_1/u_2, \qquad |u^+| < \infty \end{aligned} \qquad (4)$$

The destruction of the mixing layer is assumed to be a Poisson random process, and Poisson averaging is introduced for the Navier-Stokes equation, where T is the time of development of the mixing layer:

$$< f(y) > = \frac{1}{T}\int_0^\infty e^{-t/T} f(y,t)dt \qquad (5)$$

Following Prandtl the turbulent viscosity is specified in the model in the form

$$\nu_t = l^2 d\bar{u}/dy, \; l = \alpha b, \qquad (6)$$

where l is the mixing length, α is a dimensionless empirical constant, and $d\bar{u}/dy = (u_1 - u_2)/b$.

Using the probabilistic averaging (5) of the equation (3) and boundary conditions (4) we obtain the solution of (1) depending on the expression $\sqrt{\nu_\Sigma^+ T^+}$, where $\nu_\Sigma^+ = \nu_m^+ + \nu_t^+$ is the total viscosity (Dolgopolova, 2003). Using the Prandtl expression for the vertical velocity distribution in the mixing layer we obtain the value of $\sqrt{\nu_\Sigma^+ T^+}$ = 0.32 – 0.33, which does not depend on the ratio of u_1/u_2. The expression for turbulent viscosity (6) can transformed as

$$\nu_t = \alpha^2 b(u_1 - u_2). \qquad (7)$$

Experimental magnitude of α in (7), which was found in studies of velocity field in the mixing layer formed at a detached flow (Speranskaya, 1982), equals to $\alpha = 4.25\cdot10^{-2}$. In turbulent mixing layers, in which $\nu_t >> \nu_m$, one may assume $\nu_\Sigma \cong \nu_t$. Using obtained magnitudes of $\sqrt{\nu_\Sigma^+ T^+}$ = 0.32 – 0.33 and $\alpha = 4.25\cdot10^{-2}$, we have the estimate of self-oscillation period of the velocity field in a plane turbulent mixing layer

$$T^+ = \frac{61}{1-(u_2/u_1)}. \qquad (8)$$

In order to verify the model, expression (8) was used to calculate the period of fluctuations of the mixing layer in the partially mixed St. Lawrence estuary. Necessary parameters of the flow were borrowed from (Bourgault et al, 2001). The calculation gives T = 4.2 min which is in good correspondence with the period observed and presented in Figure 5. The model calculations of the period of self-oscillations in conditions of transition from ebb to flood yields T=10.2 min, which slightly exceeds the value obtained from acoustic image in Figure 6 (T=7.2 min). This can be explained by the fact that the model does not take into account intensification of instability of the flow resulting from interaction of tidal waves with abrupt change of the bottom profile in the St. Lawrence estuary.

Conclusions

A section of partially-mixed estuary is usually occupied by two-layer current. Between these two layers with different densities and moving in the opposite directions mixing layer is developed. Vertical shear associated with the two-layer current in partially-mixed estuary is sufficient to destabilize the pycnocline, leading to Kelvin-Helmholtz instabilities. The range of Richardson number characterizing these instabilities is extended to Ri ~ 0.5 due to interaction of abrupt bottom topography with tidal waves. The period of oscillations of mixing layer calculated by the suggested model is in good agreement with the measurements carried out under quasi-stable conditions during high water stage.

Analysis of different criteria for estimation of water mixing type in the St. Lawrence and Yenisei estuaries shows that Simmons criterion б describes vertical mixing of water column over the entire estuary; parameter of stratification n yields the most accurate determination of water mixing type at the site of salinity measurement; and Ri_L number reflects the degree of stability of the flow. The estimates of these parameters for the Saint Lawrence estuary shows that the upper estuary belongs to partially mixed type, though all types of mixing given in Table 1 are typical for different parts of the estuary. In the middle and lower estuary, the mixing of river and sea water follows the type of a “salt wedge” and the estuary is strongly stratified. The transition from ebb to flood tide is the period when the largest internal wave train is produced.

Acknowledgment

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References

Bourgault, D., Saucier, F.J. & Lin, C.A. (2001, 106) Shear instability in the St. Lawrence Estuary, Canada: a comparison of fine-scale observations and estuarine circulation model results. *J. Geophysics Research*, pp. 9393–9409.

Dolgopolova, E.N. (2003, 30). On the interaction between flow and erodible bed. *Water Resources*. No. 3, pp. 268–274.

Dolgopolova, E.N. (2008, 35). Vertical Transfer Coefficient in Natural Streams. *Water Resources*. No. 4, pp. 408–416.

Dolgopolova, E.N. & Isupova, M.V. (2010, 37). Classification of Estuaries by Hydrodynamics Processes. *Water Resources*. No. 3, pp. 268–284.

Dyer, K.R. (1973). *Estuaries: A Physical Introduction*. London: John Wiley & Sons.

Harms, I.H., Hübner U., Backhaus, J.O., Kulakov, M., Stanovoy, V., Stepanets, O.V., Kodina, L.A. & Schlitzer, R. (2002). Salt intrusions in Siberian river estuaries: observations and model experiments in Ob and Yenisei. *EGS XXVII General Assembly*, Nice. Abstract #4537. pp. 27–46.

Mikhailov, V.N. (1997). *Hydrological Processes in River Mouths*. Moscow: GEOS (in Russian) .

Samolyubov, B.I. (1999). *Near-bottom stratified currents*. Moscow: Scientific World (in Russian).

Simmons, H.B. (1955, 81). Some effects of upland discharge on estuarine hydraulics. Proc. Amer. Soc. Civil. Engrs. Sep. Paper 792. pp. 1–20

Speranskaya, A.A. (1982). Boundary layers in geophysical hydrodynamics. Doctoral Dissertation, Moscow State University, (in Russian).

Tomczak, M., & Godfrey, J. S. (2003). *Regional Oceanography: an Introduction.* Delhi: Daya Publishing House.