USING RELIABILITY METHODS FOR QUANTIFYING UNCERTAINTIES IN A 2D-MORPHODYNAMIC NUMERICAL MODEL OF RIVER RHINE

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Abstract

Reliability analysis was used as an effective method to calculate the uncertainties in the results of a two-dimensional morphodynamic numerical model of river Rhine. For a 60 km long stretch from Iffezheim to Speyer a historical hydrograph of 10 years was simulated to calibrate the model, including artificial bed load supply and dredging activities. The simulation package of Telemac (www.opentelemac.org) was used with the modules Telemac2D, Sisyphe and DredgeSim. Such long term simulations incorporate a large scope of natural and numerical uncertainties. Three different reliability methods were used to calculate the sensitivity of the numerical results according to variances in 9 input parameters.

The reliability analysis presents confidence intervals of the results based on the calculated sensitivities with a given probability for every point in time and space. For the Iffezheim-Speyer model the most sensitive parameters were found and their effect on the bottom evolution was quantified. From this the simulation results can be divided into zones with different levels of uncertainty in time and space.

Introduction

Morphodynamic modeling incorporates a lot of uncertainties due to unknown initial and boundary conditions, the natural variability or the imprecision of model parameters and the deficient description of the complex physical processes. Large scale and long term modeling is often needed to answer morphodynamic tasks. Therefore the demand for calibration and validation and also the uncertainty of prediction increases. Evaluation and interpretation of numerical results becomes very important. Here reliability analysis can be helpful as it quantifies the uncertainties in time and space and according to its source.

Beside the sources of uncertainties named above only the influence of uncertain input parameters to the results are considered here. The aim of this article is to show the advantage of using even a quite simple reliability method. It is well known that the bottom evolution cannot be predicted very precisely with a morphodynamic model. This means that the calculated value, hopefully the most probable, can vary inside a certain range. Mathematically expressed the calculated value is the mean value and the certain range is equivalent to the confidence interval.

Description of used reliability methods

Three methods were applied for analyzing the reliability of the model of river Rhine in time and space. The first order Scatter Analysis (SA) was used for a simple sensitivity analysis. A wide range of model parameters can be checked with this method. The relatively broad results were used for comparing the influence of the different parameters and their chosen range. The two other methods are based on the Monte Carlo principle and therefore need much more computation time. These methods can be used even for strong nonlinearities.

Scatter Analysis

The Scatter Analysis belongs to the first order methods. So it is only adequate for linear or slightly non-linear problems. From the root mean square (rms) the deviations are assumed. The rms can be calculated from the first derivation multiplied by the standard deviation For the confidence limits only the first order terms are taken into account. The confidence interval of the evolution for a 68 % probability is two times the rms and for a 95 % probability 4 times the rms. For the reliability analysis shown here the 95 % confidence limits and the related tolerance range were used. For a detailed description please refer to (Kopmann, Schmidt, 2010).

The rms of the state variable evolution E, which describes the bed level changes e.g. in a river, influenced by the friction coefficient ks, can be calculated as in equation (1).

$$rms(E) = \frac{1}{2}\left[E(ks_0 + \sigma) - E(ks_0 - \sigma)\right] \quad (1)$$

$E(ks_0+/-\sigma)$ are results from simulation runs with $ks_0+/-\sigma$. The calculations of the deviations or the tolerance limits for n uncertain parameters need only n*2 + 1 simulation runs. (Nikitina, 2008).

The distortion for the evolution E

$$\delta E = \frac{1}{2} E''(ks_0)\sigma^2 << rms \qquad (2)$$

can be calculated with the second derivative of E (E'') concerning an uncertain parameter (in this case ks) and the standard deviation of this parameter. In case of a linear function of E, the second derivative would be zero. The distortion can be used as an indicator for linearity. It should be much smaller than the rms, otherwise the function is not slightly non-linear and the method is not adequate for this special problem. However, the distortion can only be used as an indicator for slight non-linearity in case of symmetric distributions.

Monte Carlo CL

The MC-CL method is a specialized Monte Carlo method which focuses on the confidence limits. It is not limited to linear problems and determines the confidence limits approximatively while using as few as possible simulations. In case of strong non-linearities the confidence limits cannot be deduced from the root mean square (rms) any more. Moreover it is not possible to calculate the rms from the deviations. A connection between the confidence limits and the root mean square only exists in case of non-distorted gaussian distribution as in linear functions. For strong non-linear functions the root mean square and the confidence limits are not equivalent, not proportional and furthermore there is no functional connection between them. A more detailed description of this method can be found in Kopmann & Schmidt (2010), Nikitina & Clees, (2009).

Metamodel

All Monte Carlo methods require a large sample number for precise determination of the confidence limits and need even more samples for the probability density function. In order to reduce the number of required samples and / or increase the precision, a computationally efficient interpolation (metamodel) can be used. Such a model can be constructed using a moderate number of simulations. Afterwards a huge number of model results can be created by the metamodel. With these results the confidence limits and the probability density functions (PDF) can be found with a higher precision. The metamodel is using radial basis functions. For details refer to Buhmann (2003) and Nikitina et al (2010). The used simulations for constructing the metamodel should be chosen in such a way, that the whole parameter space is covered as even as possible. As for the MC-CL method the DoE generator is used to create the parameter set. In order to guarantee an optimal construction of the metamodel a uniform distribution of each parameter must be assumed.

Application of River Rhine model

For a 60 km long stretch of river Rhine from Iffezheim to Speyer a two-dimensional morphodynamic numerical model was applied. A historical hydrograph of 10 years was simulated to calibrate the model including artificial bed load supply and dredging activities. The simulation package of Telemac (www.opentelemac.org, (Villaret et al 2010)) was used with the modules Telemac2D calculating the depth averaged hydrodynamics, Sisyphe calculating the bed load transport and DredgeSim calculating the dredging and disposal activities. Such long term simulations incorporate a large scope of natural and numerical uncertainties.

From the experiences gained during the calibration 9 parameters were declared as uncertain:

The active layer thickness, the pre-factor of the Meyer-Peter Mueller formula, the parameter of the slope effect of Koch & Flokstra (1981), the parameter for the secondary current approach of Engelund (1974), the sieve line including the mean grain size of the transported material and of the artificial bed load supply and the Nikuradse roughness coefficient of three different zones (river channel, bank area, groynes). The chosen mean values from the calibration and the approximated minimum and maximum values are shown in table 1. The corresponding formulas for all parameters can be found in Villaret (2011).

Table 1: Mean, minimum and maximum values of the chosen uncertain parameters

Uncertain parameter	**Min value**	**Mean value**	**Max value**
Active layer thickness [m]	0.0833	0.1	1.0
Pre-factor of Meyer-Peter Mueller formula [-]	4	6	8
Parameter for slope effect [-]	0.8667	1.3	1.7333
Parameter for secondary current [-]	0.7	1	1.3
Mean grain size of bed load material [m]	-10%	0.0205 (0.013-0.024)	+10%
Mean grain size of supply material [m]	-10%	0.019	+10%
Nikuradse friction coefficient at river channel [m]	0.016	0.02	0.024
Nikuradse friction coefficient at bank areas [m]	0.0233	0.03	0.0367
Nikuradse friction coefficient for groynes [m]	0.2333	0.3	0.3667

Results of the Scatter Analysis

Through the SA method timely and spatially distributed deviations were calculated over two years. For the most sensitive parameters also calculations over 10 years were done in order to see the evolution of the deviations over time. Altogether 19 two-year simulations and three 10-year simulations were done. The time requirement for a two-year simulation on a parallel compute server at BAW is about 30.5 h using 64 parallel processors (64 cores). A ten-year simulation takes about 9 days.

First the validity of this method is checked. As explained above the method gives only quantitative reliable values if the problem is slightly linear. The distortion can be used as an indicator. In Figure 1 the deviation and distortion of the bottom evolution according to the mean grain size for a representative node in the river channel is shown. The assumption of linearity seems to be valid just for the first months. Afterwards the distortion is not only much smaller than the deviation but for some periods even bigger. From this follows, that the SA method can merely give some trends or estimations. Quantitative reliable results can be produced by non-linear methods like MC-CL. Nevertheless qualitatively analysis can be done with the SA method.

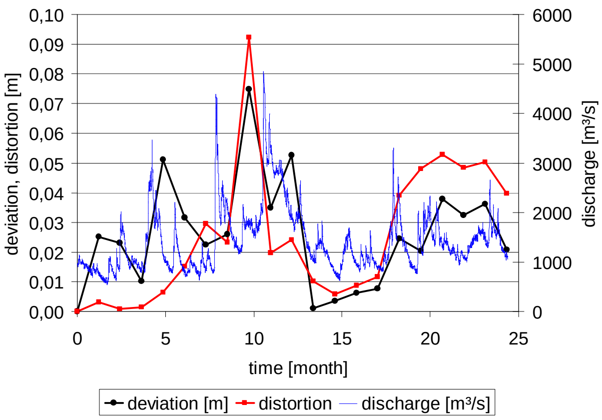

Figure 1: 68% deviation and distortion of the bottom evolution according to the mean grain size at a representative point in the river channel

To identify the most sensitive parameters the deviations of the bottom evolution according to all nine uncertain parameters after one year are compared in Figure 2 along the middle of the fairway. For a better comparison the highly scattered values were displayed using a smoothing function. The three most sensitive parameters are the active layer thickness (ALT), the friction coefficient of the river channel (KS RIVER CHANNEL) and the parameter for the slope effect (BETA). The two next sensitive parameters are the mean grain size of the bed load material (DM) and the parameter for secondary currents (ALPHA). Not at all points the order of sensitivity is the same. For a better differentiation a representative river stretch is chosen to show the spatial distribution of the deviation. Systematically the deviation is higher in the shear zone between groyne field and river channel. This can be explained by the very coarse resolution of the grid. The complex sediment and flow processes in groyne fields cannot be reproduced by the coarse 2D model. In the bends the river channel has less deviation than the near bank parts. From this follows, that the mean bottom level can be predicted quite well whereas the slope due to secondary currents effect is more uncertain. Figure 3 exemplarily shows a river stretch with higher deviation in groyne fields and in bends.

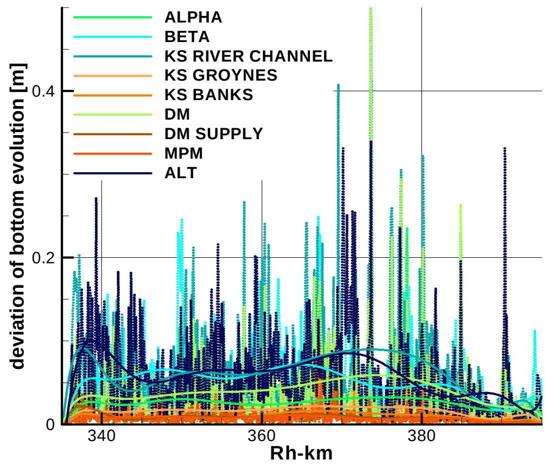

Figure 2: Comparison of bottom evolution deviations according to all 9 uncertain parameters after 1 year along the river channel midway (lines: using a smoothing function, dotted: original data)

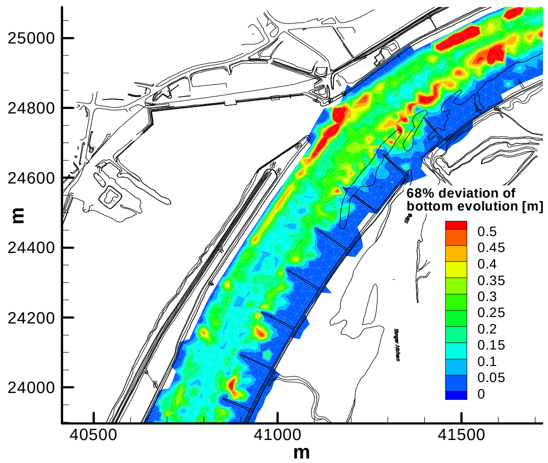

Figure 3: Bottom evolution deviation according to all nine parameters for a representative river stretch after two years

The comparison of the deviation after one and two years in Figure 4 shows a strong increase at around Rh-km 380. This can be ascribed to dredging and disposal activities. The first dredging and disposal activities took place after one year. The dredging was steered by a given dredging horizon, in a way that these parts were artificially forced to a certain level. As expected the deviations at these parts were quite small. The dredged material was disposed in the disposal areas. As the amount of dredged material varied due to different parameters, the deviations in these disposal areas were very high. The river parts with stabilization were completely unaffected by the variation of the changed parameters and the bed load supply area was only influenced by the grain size distribution of the supply material.

From this first analysis it can be reasoned that in models with automatic dredging and disposal the bottom evolution of the disposal areas are more uncertain than of the dredging areas.

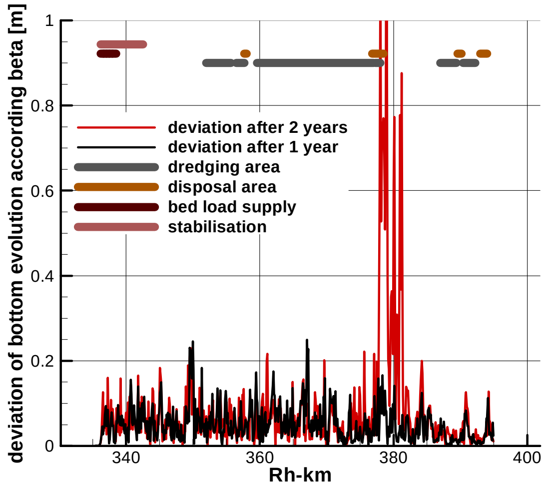

Figure 4: Comparison of deviations of bottom evolution according the parameter for slope effect after 1 and 2 years

The analysis over time along the river fairway or at representative locations was quite scattered. That's why an averaged value for all model nodes with a deviation greater than zero was built. The high uncertainties due to the disposal areas have a big influence on this averaged value. Due to this these river parts were neglected for some analysis. After 2 years the mean value for the bottom deviation was calculated to be 5 cm (28 %) lower as if the whole area would be considered.

The behaviour of the 68% deviation in time is shown in Figure 5 and Figure 6. Generally the averaged value over the whole model area increases over the time (Figure 5 and green line Figure 6). Only in some rare occasions it decreases (e.g. flood event 18 months in Figure 5). The qualitatively behaviour of the deviation for all parameters is the same. But for the most sensitive ones the increase over time is stronger. From Figure 6 can be derived, that the increase of uncertainty is higher during smaller discharges (e.g. low water conditions during the 3rd year). It seems that declines mostly occur during high water conditions. Contrarily to the assumption that the uncertainty is proportional to the amount of sediment transport (at least in this example) high water conditions lead to a state of the system which is more independent of the parameters. This has to be verified further. Unfortunately the averaged 68% deviation didn't reach a maximum level even over such a long period, but follows a trend. On the other hand the local deviation at some point in the river channel as well as the averaged value over the fairway excluding the disposal areas has indeed a maximum level and no trend.

As expected, the overall uncertainty increases with time and long term simulation should be analysed very carefully. Nevertheless for some parts the local deviations reach a maximum and level around a mean value (e.g. in the river channel).

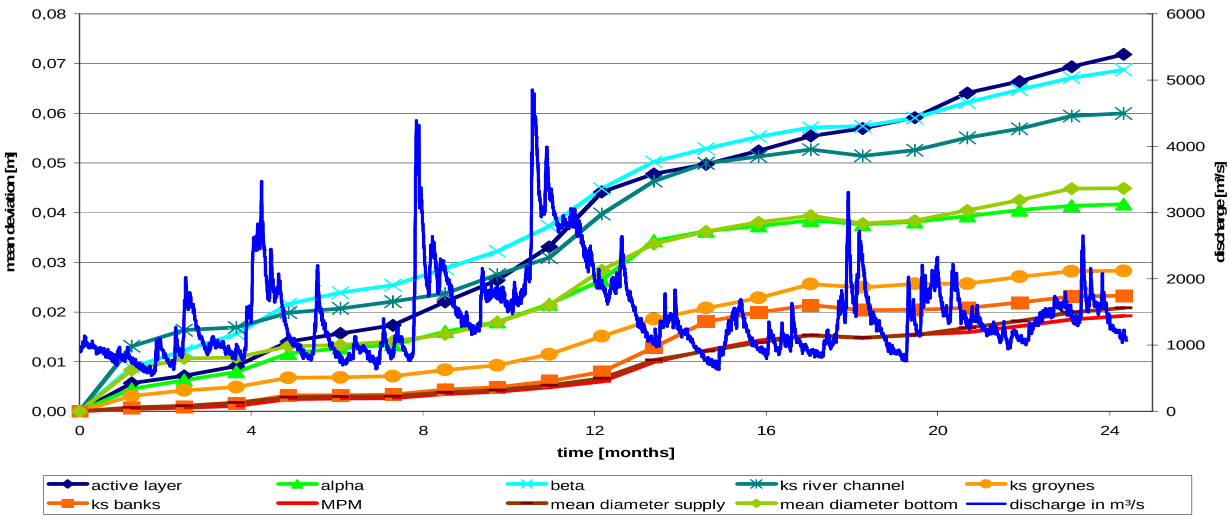

Figure 5: Mean deviation of the bottom evolution over time calculated with the Scatter Analysis for the 9 uncertain parameters

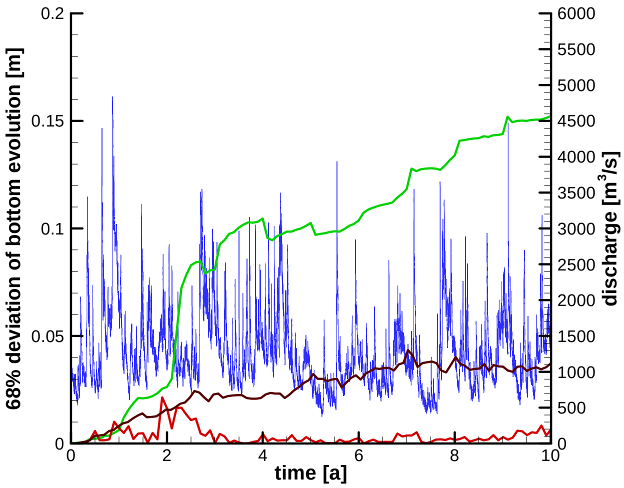

Figure 6: 68% deviation of the bottom evolution according to the river channel friction coefficient calculated with the Scatter Analysis for 10 years (green: mean value for the whole model area, red: representative point in the river channel, brown: averaged value for the fairway without disposal areas)

Results of the MC-CL method

A quantitative interpretation can be done using the MC-CL method. With the SA only small differences in the sensitivity of the different parameters were found, that is why all 9 uncertain parameters were taken into account again. With 150 simulation runs a small estimation error of 0.002 could be reached. In order to reduce the computing time only 17 months were simulated. On a parallel compute server at BAW one simulation needed round about 21 h using 64 parallel processors (64 cores). The program for the statistical analysis `CLcomp` from SCAI (Nikitina et al, 2010) needs 20 min. The MC-CL method needed altogether 64 cores for approx. 130 days. This is nearly 8 times more than the SA for the same modeling time period.

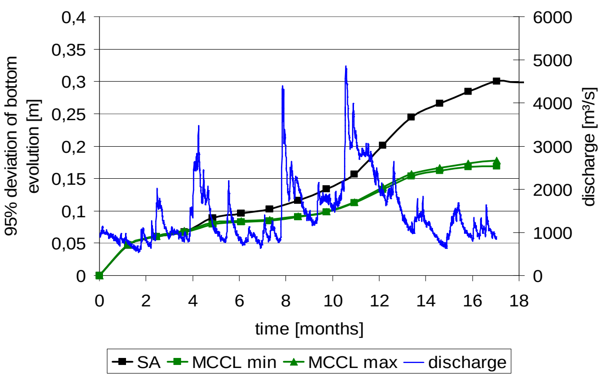

Figure 7: Comparison of the 95 % deviation of bottom evolution calculated with the Scatter Analysis and MC-CL

In Figure 7 the results of the 95 % deviation (2 σ) for the bottom evolution from the SA and MC-CL are compared to the mean values over the model area. It can be seen that the Scatter Analysis gives very good results over the first 5 months. This matches the prognosticated validity of the SA by comparing distortion to deviation as shown above (Figure 1). After the first 5 months the SA overestimates the deviation. Nevertheless the qualitative results are satisfying. For the MC-CL method the two 95% deviations are calculated to represent the minimum and maximum confidence limits. In our case both values are almost the same. This means that the probability distribution of the bottom evolution is nearly symmetric, at least for the mean value over the model area.

The MC-CL results can be analyzed quantitatively. From this method the confidence interval can be derived for a 95% probability. For the application the averaged 95% confidence interval of the bottom evolution for the whole model area increases up to 35 cm after 17 months (Figure 7). Along the fairway the 95% confidence interval varies between 0.2 and 0.5 m unaffected by the time. Only the disposal areas and the parts upstream (Rh-km 375 – 382) differ, here the values rise up to 6 m (Figure 8Figure 8).

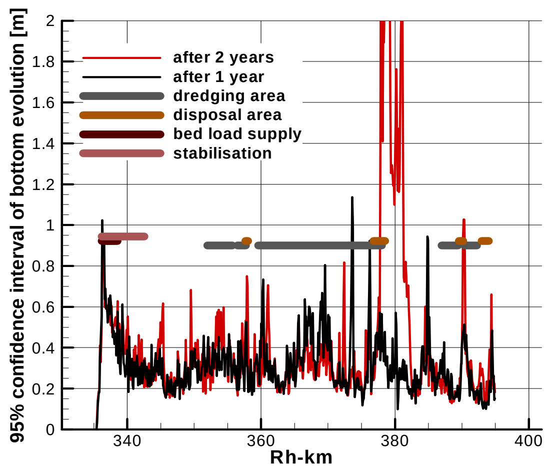

Figure 8: 95 % confidence interval of bottom evolution calculated with the MC-CL after 17 months (red) and after 1 year before the dredging activities (black)

Results of the metamodeling

For the metamodeling another 150 simulations were needed. The calculations of the MC-CL method could not be used, as for this method the chosen probability distribution functions (PDF) of all parameters need to be equally distributed. This guarantees a good description of the whole parameter field. For the MC-CL method mostly Gaussian distributions of the parameters were assumed. Exemplarily three representative locations were chosen to present a PDF of the calculated bottom evolution. One point was located in the river channel, another inside a groyne field and the last one was in the disposal area. As explained above the disposal area has the most uncertain and therefore wide and flat PDF. The most distorted one is at the groyne field, which might originate from the complex flow situation and therefore non-linear behavior. At the river channel the uncertainty is the lowest and almost Gaussian distributed. Here a nearly linear behavior can be stated.

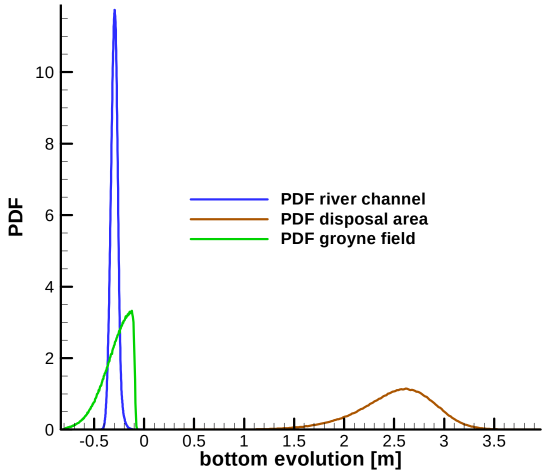

Figure 9: Probability density function of bottom evolution for representative points in the river channel (blue), inside a groyne field (green) and in the disposal area (brown)

Conclusion

With help of reliability methods the influence of chosen distributions of input parameters can be analyzed. Not only can the influence of each parameter be estimated, but also areas of higher and lower uncertainty. These uncertainties can be quantified with confidence intervals and additionally a probability density function for the calculated results can be specified.

For the shown application the three most sensitive parameters are the active layer thickness, the friction coefficient of the river channel and the parameter for the slope effect. This is not surprising as all three of them belong to the so called “soft parameters”, which are widely used for calibration.

The disposal areas have the highest uncertainties, due to accumulation of varying dredging amounts at a relatively small location. Additionally the lack of not considering the complex processes in groyne fields and bends shows significantly higher confidence intervals in these areas. While the mean level of the bottom is not so strongly affected, the approximation of slopes in bends is more uncertain. The three dimensional effects of secondary currents were only estimated with a formulation in a 2D approach.

The averaged confidence interval for the whole model area usually increases in time, even for a long period over 10 years. The theory that high waters induce a decrease in the local deviations, which can lead to a decrease of the averaged confidence interval, has to be proved furthermore. Locally the confidence intervals vary a lot in time and can even go down to zero again. An averaged confidence interval for the river channel reaches a maximum at some point and is not accompanied by a trend.

The Scatter Analysis, even though it is a method for only slightly non-linear systems, gives qualitatively good results. The better suited non-linear method of Monte Carlo CL needs at least 8 times more computing time. With this method quantitative analyses can be done. For the application the mean 95% confidence interval of the bottom evolution for the whole model area increases up to 35 cm after 17 months. The local confidence intervals in the river channel are mostly about 30 cm. This matches the authors experiences, who assess such model results not to be better than +/- 10 - 15 cm. The advantage of reliability methods is the quantification and spatial and timely differentiation of the uncertainty.

The metamodel can be applied after a Monte Carlo CL method in order to show a probability density function (PDF) at a certain point. In the shown example the Gaussian shaped PDF in the river channel reflects the nearly linear model behavior. An analysis of the PDFs at different locations can give further insights in the model results.

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References

Buhmann, N.D. (2003). *Radial basis functions: theory and implementations*. Cambridge University Press.

Kopmann, R., Schmidt, A. (2010). Comparison of different reliability analysis methods for a 2D morphodynamic numerical model of River Danube. *River Flow 2010 – International Conference on Fluvial Hydraulics, Braunschweig, 8.-10.9.2010* pp.1615-1620.

Engelund F. (1974). *Flow and Bed Topography in Channel Bends, Journal of the Hydraulics Division, Vol. 100, HY11, 1974, , pp. 1631-1648.*

Koch F.G., Flokstra C. (1981). *Bed level computations for curved alluvial channels, XIX Congress of the International Association for Hydraulic Research, New Delhi India, pp. 357-364.*

Maerker, C., Malcherek, A., Riemann, J. Brudy-Zippelius, T. (2011). Modelling and analyzing dredging and disposal activities using Telemac, Sisyphe and DredgeSim. *Telemac Users Club 2011, Chatou, 19.-21.10.2011* pp.92-98.

Nikitina, L., Nikitin, I., Clees, T. (2008). Studie Zuverlässigkeitsanalyse morphodynamischer Modelle. Abschlussbericht zum Arbeitspaket 1, *Fraunhofer Institut Algorithmen und Wissenschaftliches Rechnen*.

Nikitina, L., Nikitin, I., Clees, T. (2009). Studie Zuverlässigkeitsanalyse morphodynamischer Modelle. Abschlussbericht zum Arbeitspaket 2, *Fraunhofer Institut Algorithmen und Wissenschaftliches Rechnen.*

Nikitina, L., Nikitin, I. Stefes-lai, D., Clees, T. (2010). Studie Zuverlässigkeitsanalyse morphodynamischer Modelle. Abschlussbericht der Zuverlässigkeitsanalyse für stark nichtlineare Funktionen y(x) mittels einer speziellen, durch RBF-Metamodellierung beschleunigten Monte-Carlo-basierten Methode zur CL-Berechnung. *Fraunhofer Institut Algorithmen und Wissenschaftliches Rechnen.*

Villaret, C., Hervouet, J.-M., Kopmann, R., and Merkel, U. (2011). Morphodynamic modelling using the Telemac finite-element system. *Computer & Geosciences.*