# EVALUATING SEDIMENT TRANSPORT EQUATIONS AND PARAMETER SENSITIVITY USING THE SRH-2D MODEL

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## Abstract

Channel planform, pattern, morphology, erosion, deposition are to a large extent determined by the amount of sediment transported by the flow. Engineering applications such as channel stability, reservoir sedimentation and bridge pier scouring rely on knowledge of the transport processes for design, management and maintenance. In order to analyze these processes, the use of different numerical models is often indispensable.

Sedimentation and River Hydraulics Two–Dimensional model, SRH-2D is a depth-averaged numerical model using the finite volume method on a hybrid meshes. Sediment transport in this model can be computed using four different equations which have been developed for different bed conditions.

Here the applicability of the implemented four sediment transport equations for a gravel bed river reach on the River Spöl in Switzerland is evaluated and the sensitivity of the best predicting equation to the different parameters is established.

The model results show that of the four sediment transport equations tested, the equation developed for sandy river beds by Engelund and Hansen (1972) results in the largest discrepancy with field measurements. The Parker (1990) and Wilcock & Crowe (2003) equations show comparably similar results. The Meyer-Peter and Müller (1948) equation modified by Wong and Parker (2006) (MPM) gives the most satisfactory results compared to the field measurements.

The sensitivity of the MPM equation when estimating of erosion and deposition was tested using different active layer thickness values and available adaptation length equations. Additionally, the effect of Manning's roughness coefficient on estimations of the bed shear stress is also carried out. The calculated values of the erosion depths are dependent on the chosen active layer thickness, where it was observed that only marginal change occurs for only the lower range of thickness values. The selection of adaptation length equations has a large impact on the spatial distribution of erosion and deposition areas. Further, it is proven that calibration of Manning's roughness value has a high correlation to bed shear stress estimates. It is well known that bed shear stress, velocity, sediment particle size and channel cross section are critical parameters governing sediment transport.

## Introduction

Rivers are dynamic systems governed by hydraulic and sediment transport process. Over time, the river responds to changing conditions in its environment by modifying its cross sectional shape, thus increasing or decreasing its local sediment carrying capacity, observed as patterns of erosion and deposition.

To analyze these dynamic processes, several approaches have been developed. These approaches can be theoretical, experimental or numerical, where applications of both linear and nonlinear models are available.

Physical modeling and computational simulation are the two major tools used in river engineering analysis. Both come with their respective advantages and disadvantages. Physical models can provide directly visible results, but are often cost prohibitive and time consuming. On the other hand, computational simulation gives direct, real scale predictions without scale distortion and in most cases provides a cost effective alternative. However, as stated by Wu (2007) the reliability of computational simulation relies on factors describing the relevant physical processes such as the proper choice of governing equations, boundary conditions and the empirical formulas used.

At the heart of this study, the SRH-2D model was chosen as it has been implemented for modeling two-dimensional hydraulic, sediment, temperature, and vegetation for river systems. The model is under development at the U.S. Bureau of Reclamation and has been tested under a variety of different river engineering tasks and predicted the processes well (e.g. Lai and Randle, 2007; Lai and Greimann, 2008, 2010; Lai, 2010, 2011; Lai et al., 2011a). SRH-2D offers four different alternative sediment transport equations to analyze sediment transport.

Most sediment transport formulas entail relationships including Einstein's and Shields' parameters. According to Bates et al. (2005) this formulation basically has two major sub-sets; one set having a threshold for incipient motion (the likes of Meyer-Peter and Müller, 1948) and the other without a threshold (the likes of Engelund & Hansen, 1972). In addition, both formulations involve different assumptions, experimental methods and other controlling factors. Due to these variations and the complexity of natural river dynamics, the predictions from each equation can be expected to be highly variable. Hence, most sediment transport estimations in a river reach are done through a best fit analysis of several years of measurement data against the estimation of individual equations.

Further, Yang (1996) states that, because of the tremendous uncertainties involved in estimating sediment discharge at different flow and sediment conditions under different hydrologic, geologic, and climatic constraints, it is extremely difficult to recommend one formula for engineers and geologists for practical applications under various circumstances. However this is not an excuse not to consider the available application guidelines and recommendations before applying a transport equation to a known bed and hydraulic condition. In this paper special consideration is given to determine a suitable equation which can be applied to a gravel bed river.

## SRH-2D Model

As noted in Lai & Greimann (2010) SRH-2D solves the 2D depth averaged shallow water equation through a finite volume approach ensuring mass conservation both locally and globally.

The sediment transport (equation (1)), the consecutive change in bed elevation (equation (2)) and their respective discretization (equations (3) & (4)) implemented in the model are as follows:

$$\frac{\partial hC}{\partial t}+\frac{\partial UhC}{\partial x}+\frac{\partial VhC}{\partial y}=\frac{1}{L_b}(q_t^*-\sqrt{U^2+V^2}\,hC) \qquad (1)$$

$$(1-p_b)\left(\frac{\partial z_b}{\partial t}\right)=-\frac{1}{L_b}(q_t^*-\sqrt{U^2+V^2}hC) \qquad (2)$$

$$\frac{(hC)^{int}-(hC)^k}{\Delta t}+\frac{\partial U(hC)^{int}}{\partial x}+\frac{\partial V(hC)^{int}}{\partial y}=0 \qquad (3)$$

$$\frac{(hC)^{k+1}+(hC)^{int}}{\Delta t}=\frac{1}{L_b}[q_t^*-\sqrt{U^2+V^2}(hC)^{k+1}] \qquad (4)$$

Where q* is the equilibrium sediment transport rate that can be computed using either of the following equations: (1) Engelund and Hansen (1972), (2) Parker (1990), (3) Wilcock and Crowe (2003) and (4) Meyer-Peter & Müller (1948) modified by Wong and Parker (2006), h is the water depth, x and y are the horizontal cartesian coordinates, t is the time, C is the depth-averaged volumetric sediment concentration, U and V are the depth-averaged velocity components in the x and y directions respectively, $z_b$ is the bed elevation, and $p_b$ is the bed material porosity. $L_b$ is the non-equilibrium adaptation length; and in this paper for the evaluation of the four equations the average adaptation length of Philips and Sutherland (1989) was used:

$$L_b=4000(\theta-\theta_c)d\,;\;\;\theta=\frac{\tau_b}{(s-1)\rho g d} \qquad (5)$$

In which d is the sediment particle diameter, $\tau_b$ the bed shear stress, s = $(\rho_s/\rho)$, $\rho$ the density of water, $\rho_s$ the sediment density, g the gravitational acceleration, and $\theta$ is the critical Shields parameter.

## Sediment Transport Equations in SRH - 2D

**Engelund and Hansen (1972)**

Engelund and Hansen (1972) (E&H) proposed an equation based on the stream power approach. The rate of energy used in transporting materials should be related to the rate of materials being transported. This equation is recommended for sandy rivers:

$$f'\emptyset=0.1\theta^{5/2} \qquad (6)$$

$f'=2gSD/V^2$, $\emptyset=q_t/\sqrt{(s-1)gd^3}$ and $\theta=\tau/(\gamma_s-\gamma)d$

Where g is the gravitational acceleration, S is the energy slope, V is the average flow velocity, $q_t$ is the total sediment discharge by volume per unit width, s is the specific gravity of sediment, $\gamma_s$ and $\gamma$ are specific weights of sediment and water respectively, d is the median particle diameter, D is the mean water depth, and $\tau$ is the shear stress along the bed.

**Parker (1990)**

Parker's approach is developed based on the "equal mobility" hypothesis to describe observed behavioral features of bed load transport in gravel bed streams. According to the hypothesis, bed armor regulates entrainment of particles by the stream, resulting in various sizes being approximately equal in mobility, with particle sizes transported at rates proportional to their presence in the bed material (Klingeman, 2002). The Parker (1990) bed load transport equation is given as:

$$\frac{q_{bi}g(s-1)}{p_i(\frac{\tau_b}{\rho})^{1.5}}=11.93f(\emptyset_i) \qquad (7)$$

Where $\emptyset_i$ is a measure of the shear stress relative to the reference shear stress:

$$\emptyset_i=\theta_i/(\varepsilon_i\theta_c) \qquad (8)$$

Where $\theta_c$ is reference Shields parameter; $\theta_i$ is the Shields parameter of the sediment size class i, and $\xi_i$ is the exposure factor, $q_{bi}$ is the bed sediment discharge for sediment size class i, $\tau_b$ is the bed shear stress.

**Wilcock and Crowe (2003)**

As noted in Chaundhry (2008) some bed load models (Parker, 1982, 1990; Powell et al. 2001) exclude sand-size particles from the formulation and consider the transport of sand as throughput. However, developed with the same "equal mobility" hypothesis as Parker 1990, the Wilcock and Crowe 2003 (W&C) model considers both sand and gravel transport in the formulation i.e. mixed bed transport. The equation is stated as follows:

$$\frac{q_{bi} g(s-1)}{p_i(\tau_b/\rho)^{1.5}} = 14 f(\emptyset_i) \tag{9}$$

The function *f is* computed by:

$$\begin{aligned} f(\emptyset) &= \left(1 - \frac{0.894}{\sqrt{\emptyset}}\right)^{4.5} \qquad && \emptyset \geq 1.35 \\ f(\emptyset) &= (0.000143)^{7.5} && \emptyset \leq 1.35 \end{aligned} \tag{10}$$

Where $\emptyset_i$ is given the same as equation (8).

**Meyer-Peter and Müller (1948) modified by Parker and Wong (2006)**

This is the most widely used gravel bed sediment transport equation. It was first developed by Meyer-Peter and Müller (1948) based on shear stress and later modified by Parker and Wong (2006) through an improvement on the grain shear stress approximation:

$$\frac{q_t}{\sqrt{\rho g^2 d^3}} = 4.93 \left[ \left( \frac{d_{50}^{1/6}}{20n} \right)^{1.5} \theta - 0.047 \right]^{1.6} \tag{11}$$

Where n is the Manning coefficient for total roughness and $d_{50}$ the median bed sediment diameter.

## Study Area and Numerical Modeling Set Up

The study reach is located in Switzerland on the River Spöl extends some 500 m in length and is located 2 km downstream of the Livignio dam on the Swiss-Italian border. Figure 1 shows the flow depth in the reach at steady state flow of 1.44 $m^3/s$ discharge. Due to the sediment retention and large dam size, only clear-water flow conditions exist in the investigation reach.

In order to prepare the sediment transport, both flow calibration and mesh independence studies have been carried out in the flow module. The boundary conditions for the hydraulic modeling are: a flood flow hydrograph as inflow (Figure 2 (a)) and at the downstream boundary, the corresponding rating curve (Figure 2 (b)). After completing a mesh independence study, a final mesh consisting of mixed (fine and coarse) resolution is adopted for sediment transport modeling.

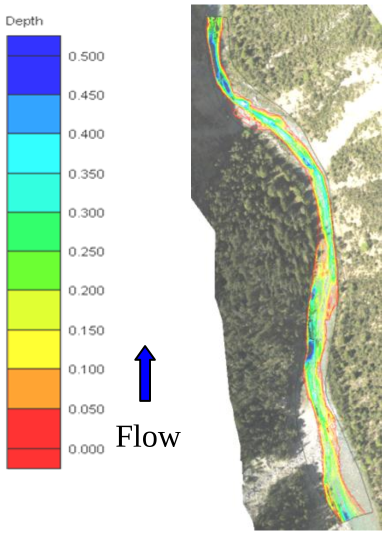
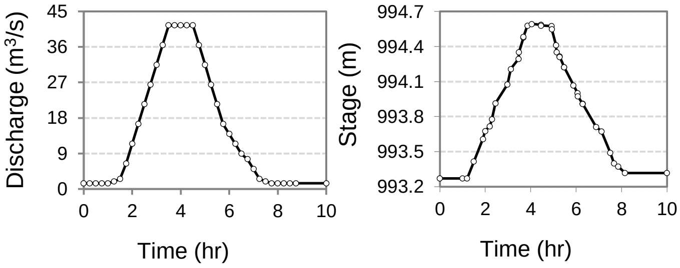



Figure 1: Study reach and water depth at a steady state flow of 1.44$m^3/s$




(a) Inflow hydrograph (b) Rating curve

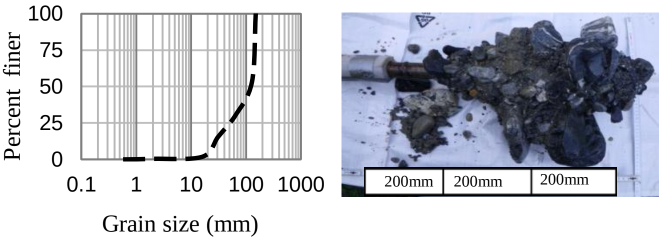
Figure 2**:** Upstream and downstream boundary conditions

The adopted mesh consists of 13,287 elements. Regions of contraction and curvature are assigned with finer elements in order to capture the complex hydraulics. After having the hydraulic model calibrated in the flow module for the adopted mesh, the rest of the sediment transport parameters i.e. active layer thickness and adaptation length are kept constant to evaluate the variations of the selected transport equations' predictive capacity.

**Sediment boundary conditions and substrate map**

As both the actual substrate map and the photo-sieving analysis show, Spöl is a gravel bed river. The river bed in the model was represented by 21 bed classes each having seven sediment size classes. A typical sample of the Spöl bed material and its gradation is shown in Figure 3. The percentage finer and the freeze core samples from Spöl bed show coarser bed material composition.

In SRH-2D there is an option to assume that the inflow sediment load is at equilibrium, and is computed at the inlet based on the bed material composition and specified sediment transport equation. This assumption can be used in the absence of measured inflow sediment data. The inflow sediment in this paper is assumed to be at equilibrium.




(a) Sediment gradation (b) Freeze core sample

Figure 3: Example of bed grain size in the River Spöl

## Results and Discussion

Here the expectation is to observe similarities, variations and performances of each transport equation implemented in SRH-2D given their specific derivation concept and application recommendations as a background. Depending on the various inputs used to derive the transport equations i.e. laboratory and experimental data sets, as previous studies (Girma & Horlacher, 2004) and (Yang, 1996) proved, the prediction in natural rivers might significantly over or underestimate the actual transport rate.

### Results from Engelund and Hansen (1972)

The E&H approach as shown in Figure 4 and Figure 5 overestimates both erosion and deposition extents and depths. As stated in Julien (2010), the rate of sediment transport using the equation does not go to zero or calculates transport for large grain sizes. This is caused since it does not consider the concept of incipient motion. Hence, this will limit its application for rivers with low flow velocities and large grain sizes, making it preferable for sandy bed rivers. Considering the flow and the bed condition in Spöl with the above background of derivation, the overestimation observed in the results was thus expected.

### Results from Parker (1990) & Wilcock and Crowe (2003)

Compared to the measured data, both of these approaches similarly underestimate the extent of erosion as well as deposition. Equal mobility stresses that bed armoring controls entrainment of particles by the streams, requiring higher flood level to transport the bed surface. Investigating the results only from the two equations, the Parker (1990) approach failed to capture the small patches just as Wilcock and Crowe (2003). This failure can be attributed to Parker's consideration of sandy/fine particles as a throughput.

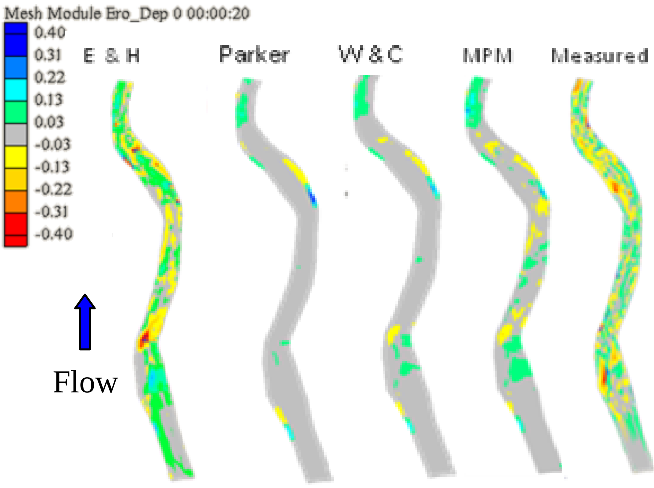



Figure 4: Predicted and measured erosion and deposition depths

### Results from Meyer-Peter and Müller modified by Parker and Wong (2006)

Compared to the rest, the MPM approach provides the most satisfactory result. As Yang (1996) suggested, the MPM equation should be used for river beds ranging from coarse sand to coarse gravel. In the case of the Spöl this recommendation is proven to be right as the results in Figure 4 and Figure 5 indicated.

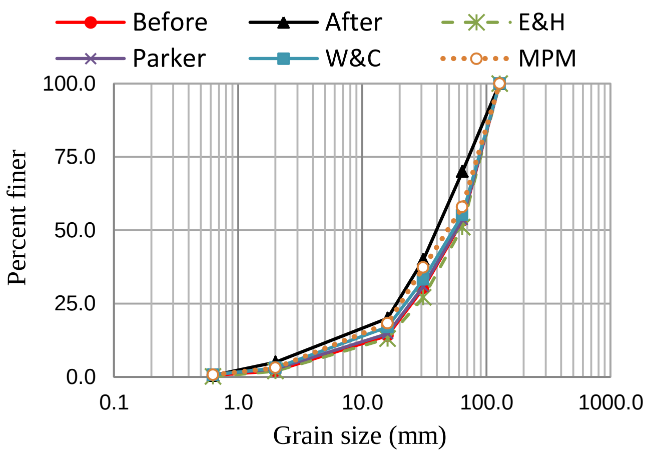



Figure 5: Sediment gradation curve prediction and measurement

The measured and predicted sediment gradation curves after the test flood at a point where deposition is observed are shown in Figure 5. The Figure confirms the above explanations. The only equation which adequately follows the measured aggradation trend is the MPM equation. Here once more, both the Parker and the Wilcock and Crowe equations show similarities and predict little change to the bed composition. Unlike the rest the Engelund and Hansen approach predicted the locations of erosion and deposition contrary to the observed locations.

### Parameter sensitivity of the MPM modified equation

Following the satisfactory estimates of the MPM equation, a parameter sensitivity analysis is conducted to examine the sensitivity of the changes in active layer thickness and selection of adaptation length equation.

#### Active layer thickness ($\delta_a$)

The effect of active layer thickness is investigated by setting the thickness as different multiples of the 90% finer grain size diameter ($d_{90}$). The computation results for $1*d_{90}$, $2*d_{90}$ and $3*d_{90}$ are shown in Figure 6. As indicated in Armanini (2010) active layer thickness can be taken as a function of $d_{90}$ except for dune beds.

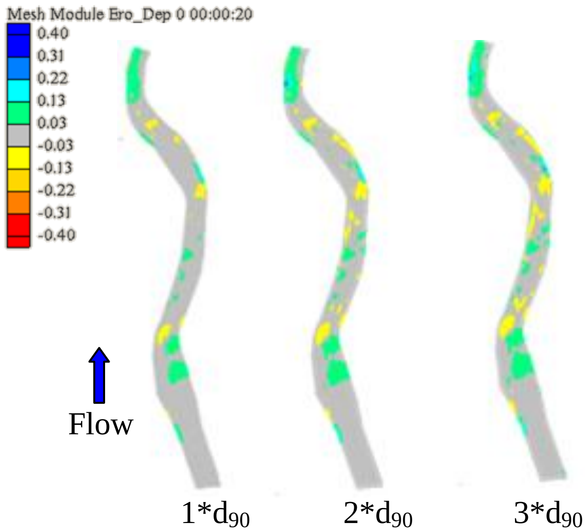



Figure 6: Sensitivity to active layer thickness

$$\frac{\partial \delta_a p_{ak}}{\partial t} = \left(\frac{\partial z_b}{\partial t}\right) + p_{ak}^{*}\left(\frac{\partial \delta_a}{\partial t} - \frac{\partial z_b}{\partial t}\right) \qquad (12)$$

Where $\delta_a$ is the active layer thickness, $z_b$ is bed level, $p_{ak}$ is the active layer volumetric fraction and $p_{ak}^{*}$ is the sub-surface fraction of sediment size class k.

It is evident from Figure 6 and equation 12 that the active layer thickness is sensitive at low thickness values due to limits in the supply of fine materials to the transport capacity. However, any further increment in the parameter to a higher thickness values (here above $2d_{90}$) results only in a marginal change to erosion and deposition depths. This finding is in agreement with the armoring study by Reed et al. (1998). It is thus found that that active layer thickness has effect on erosion/deposition depths.

#### Adaptation length ($L_b$)

In SRH-2D there are three equations to compute the adaptation length in equation 4. In Figure 7 the result from Phillips and Sutherland (1989), Van Rijn sand dune formula and Van Rijn (1987) formula are shown respectively.

Figure 7 clearly shows that the three equations deliver three distinct results, mainly varying in the spatial distributions and extent of erosion and deposition. The first approach, Philips and Sutherland (1989), gives a conservative result and is in a good agreement with the measurement. It can be seen that the selection of the appropriate adaptation length equation is necessary to address the areal extent of erosion and deposition patterns.

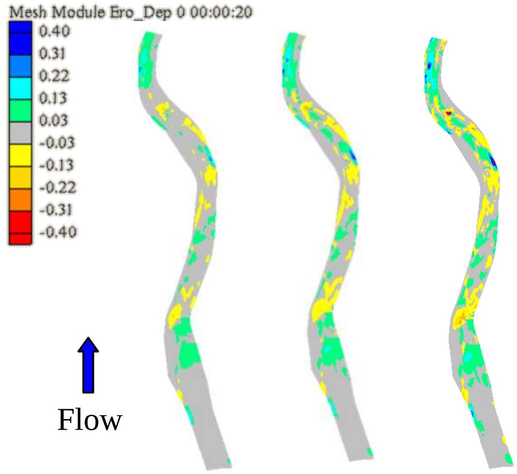



Figure 7: Sensitivity of result to adaptation length formulas; Philips & Sutherland (1989), Van Rijn Sand dune and Van Rijn (1987) respectively

#### Manning's roughness

Finally, following the fact that the MPM equation is based on shear stress estimates and the bed shear stress is a function of roughness to a second degree (equation 13); a test was carried out to figure out the sensitivity of bed shear stress on the Manning's roughness.

$$(\tau_x, \tau_y) = \rho C_f (U, V)\sqrt{U^2 + V^2} \;\; ; \;\; C_f = \frac{gn^2}{h^{1/3}} \qquad (13)$$

Where $\tau_x$ and $\tau_y$ are bed shear stresses in the horizontal x and y direction.

In most hydraulic flow calibrations one of the main parameters used to calibrate flow is the Manning's roughness coefficient (n). For instance, in vegetated rivers or in coarse meshes, the Manning's roughness is calibrated to achieve a "good" water surface level calibration. Hence, equation 13 and Table 1 prove consecutively the bed shear stress prediction will be quite wrong, negatively impacting the accuracy of the sediment transport model.

To illustrate this point, Table 1 presents two different points having the same velocity and water depth but variable roughness. Due to the only variation in roughness, the bed shear stress estimates differ by a factor of three.

Table 1: Effect of different Manning's roughness on the shear stress estimation

| Point | U | V | h | n | $C_f$ | $\tau$ |
|---|---|---|---|---|---|---|
| 1 | 0.03 | 0.84 | 0.38 | **0.06** | 0.05 | **34.44** |
| 2 | 0.01 | 0.94 | 0.32 | **0.09** | 0.12 | **102.88** |

The lesson here is, first calibrate the flow in the flow module using n with confirmation of the roughness coefficient to the nominal recommended value corresponding to bed composition. Then the consequent estimation of bed shear stress and transport will be appropriate.

## Conclusion and Recommendation

Many investigators including Yang (1996) have compared bed load transport formulas. The conclusions are usually different because different data have been used. However, as mentioned in Wu (2007) almost all existing formulas have better predictions for flume data than for field data. The reasons are that bed load transport is more complex and the measurement instruments are less efficient in natural rivers.

The sediment transport results obtained from the different equations are relatively consistent (follow similar trends) and are comparable to the measured data. The Engelund and Hansen (1972) approach is recommended for sandy bed rives. Since River Spöl is a gravel bed river the application of this equation is not recommended and was reflected by poor model performance. On the other hand, different models developed on the same basis and recommended to similar beds show comparable results as the Parker (1990) and Wilcock & Crowe (2003) approaches proved. The best performing model was found to be MPM developed for gravel bed rivers.

Overall, before commencing any sediment transport computation it is a must to have appropriate data regarding sediment inflow, substrate map and bed roughness. Afterwards the selection of transport equations should be carried out depending on the bed type and nature in relation to the recommended application range of the equations.

Calibration of transport prediction in SRH-2D can be done using three parameters: the active layer thickness, adaptation length and Manning's roughness. The choice of active layer thickness impacts the depth of erosion and deposition, however increasing the thickness beyond a threshold value has marginal impact on the predicted erosion/deposition depths. The selection of a suitable adaptation length equation significantly influences the pattern of erosion and deposition areal extents. Finally, Manning's roughness has a critical control over the estimation of bed shear stress, thus calibration of the hydraulic variables alone should first be carried out.

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