2-D NUMERICAL MODEL FOR THE BEDFORMS EVOLUTION

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Abstract

In this paper a 2-D model for the bedforms dynamics is developed and employed in the numerical computations. This model consists of two main components namely: (1) cellular automaton model for flow velocity field; and (2) model for sediment particles transport. Each of them is described in details. The first part of this study focuses on the cellular automaton model that is used for simulation of the flow velocity field. This method retains the vertical mean velocity distributions on the shift and lee sides of the bedform. On the other hand, the second part of the paper is devoted to the model for sediment particle transport. In this approach the main physical processes that govern the sediment particles behavior (i.e. entrainment, deposition, transport and inter-particles interaction) are taken into account. The deposition of particles is considered as a stochastic process. Moreover, the simulation of the particle movement is based on the balance of fundamental forces exerting on the spherical particle in the fluid, (i.e. drag, lift and gravity). The numerical algorithm, employed in the computations, is discussed in details. The preliminary results show that this model may be a useful tool for analysis of the evolution of small bedforms as sand-waves and the flow conditions required for their formation.

Introduction

River bedforms and its motion have attracted an important level of attention since the 1960s. This topic has been usually considered experimentally, including laboratory and field measurements (i.e.: Nelson et al. 1993, Coleman and Melville 1996, Nikora et al. 1997, Best and Kostaschuk 2002, Sukhodolov et al. 2006, Kostaschuk et al. 2009, Aberle et al. 2010, Shugar et al. 2010) to mention a few. On the other hand, in spite of the huge progress in the numerical modeling, especially, of flow velocity field that has been achieved recently, the numerical investigations associated with the simulations of dynamics of dunes and other river bedforms are still limited and only a few researchers have been involved in the analyses of these problems from the numerical point of view (i.e.: McLean and Smith 1986, Yoon and Patel 1996, Yue et al. 2006, Stoesser et al. 2008, Grigoriadis et al. 2009, Ancey 2010).

The focus of this paper is on the development of the numerical model of river bedforms dynamics with the special emphasis on the generation of the flow velocity field over small dunes. Such aim will be realized through the use of the model that is not too much time consuming and therefore is relatively simple in contrast to the usually used: Large Eddy Simulation (LES) or Reynolds-averaged Navier-Stokes (RANS) models. On the other hand, the presented model is able to reproduce the realistic representation of the bedforms shapes and especially the realistic flow velocity field over the erodible bed that is the most important advantage of this approach. These goals will be obtained by employing in the numerical computations the 2-D model of bedforms evolution that consists of two main components namely (1) Lattice Gas Cellular Automaton Model (LGCAM) for flow velocity field over the erodible bed in which the main rules are similar to those given by Frisch et al. (1986); and (2) the model for sediment particles transport.

Model for flow velocity field over bedforms

The flow velocity is usually simulated by the one of the following approaches: Large Eddy Simulation (LES) or Reynolds-averaged Navier-Stokes (RANS) equations. For example, Yue et al. (2006) and Stoesser et al. (2008) have shown recently that LES may be successfully used to simulate the coherent structures over two-dimensional fixed dunes. Their numerical results have shown a very good agreement with the observed data and have simulated the velocity with respect to all important flow features as: vortex or roller at the crest of the dune or real representation of the reattachment region. However, for the investigation of the bedforms evolution and sediment transport it would be ideally to explore and describe the hydrodynamic by use of the Direct Numerical Simulation (DNS). From one point of view this method allows to avoid the averaging in time and space but on the other hand, it is still impossible to use it in modeling of high Reynolds number rough-bed flows (Coleman and Nikora, 2011). Nevertheless, the main weakness point of all of the above presented approaches is that they had been used only for the description of the flow velocity field over the fixed bedforms and not been applied to the modeling of velocity over the erodible bed. Moreover, they are very time-consuming and the results due to the large number of data are difficult to use during their interpretations.

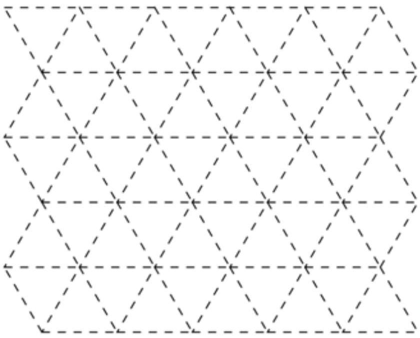

Figure 1: Triangular lattice used in the flow velocity model

One of the possible solutions in such situation is to employ in the simulations a Lattice Gas Cellular Automaton Model (LGCAM). Generally, the cellular automata are the models consisting of regular grid of cells that can be represented by one of the finite number of states and they follow mostly the simple discrete rules therefore are called automata. The main advantages of this approach that we believe makes it sufficient to this study are as follows: (1) velocity in each cell may be calculated parallelly that substantially reduces the computational time; and (2) boundary conditions are very easy to implement and thus the model may reproduce the realistic flow velocity field over the erodible bed. Frisch et al. (1986) constructed lattice-gas automata (FHP) to use them for simulation of the classical non-linear fields and to analyze the turbulence phenomena. They mathematically proved that their models asymptotically tend to the incompressible 2-D and 3-D Navier-Stokes equations. Recently, Narteau et al. (2009) and Zhang et al. (2010) presented a new 3-D cellular automaton model for bedforms' dynamics in which the flow velocity field was simulated with use of the lattice gas cellular automaton model and sediment transport was calculated by other automaton. They applied it to analyze the morphodynamics of bedforms and to study the sediment transport and finally claimed that this model could be used for the wide range of the flow conditions to reproduce specific dunes' features in different geophysical environments (Zhang et al. 2010).

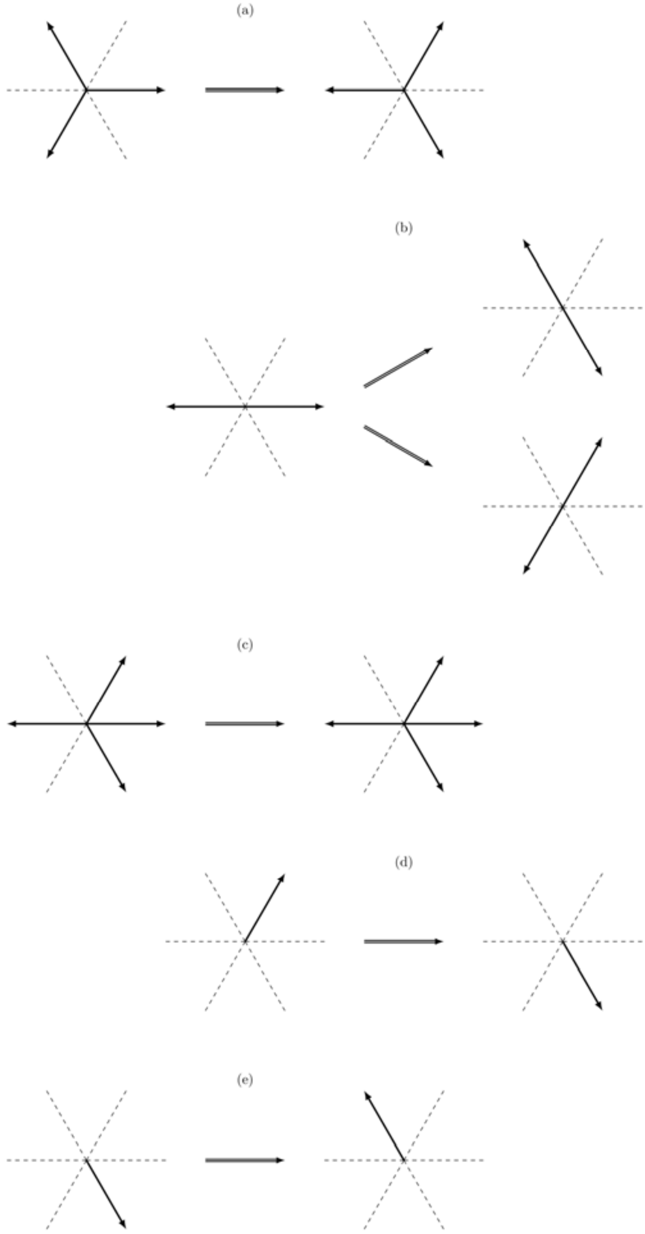

Figure 2: Rules of the model evolution

In our simulation we applied the Lattice Gas Cellular Automaton Model (FHP1) in which the main rules are similar to those given by Frisch et al. (1986):

1. FHP1 is a 2D model over a triangular lattice. It means that each node of this lattice has a hexagonal neighborhood (Fig. 1);
2. Nodes are connected by links. In each of the links is one cell with one virtual particle that means that in each vertex may be six particles. Therefore, the state of the node can be described by six bits;
3. All particles have the same mass and are indistinguishable. That means that we do not

follow the particle during the evolution of the model;

4. Each cell is coupled to the lattice vectors divided by the time step Δt which is always set equal to 1;
5. Boundary conditions at the input and output are set as periodic.
6. Evolution of the model is proceeded by the collision between particles in each lattice vertex independently and the five following cases of this process are taken into the model account (Fig. 2):
   i. If in the node there are three particles then evolution proceeds according to the diagram shown in Fig. 2a;
   ii. If there are two particles the evolution proceeds as presented in Fig. 2b;
   iii. If the situation is not described by the one of above cases (i or ii), for example, in the node there are four particles as is shown in the Fig. 2c the situation does not change in the next iteration;
   iv. The rule describing the particle collision with the free surface is presented in Fig. 2d and

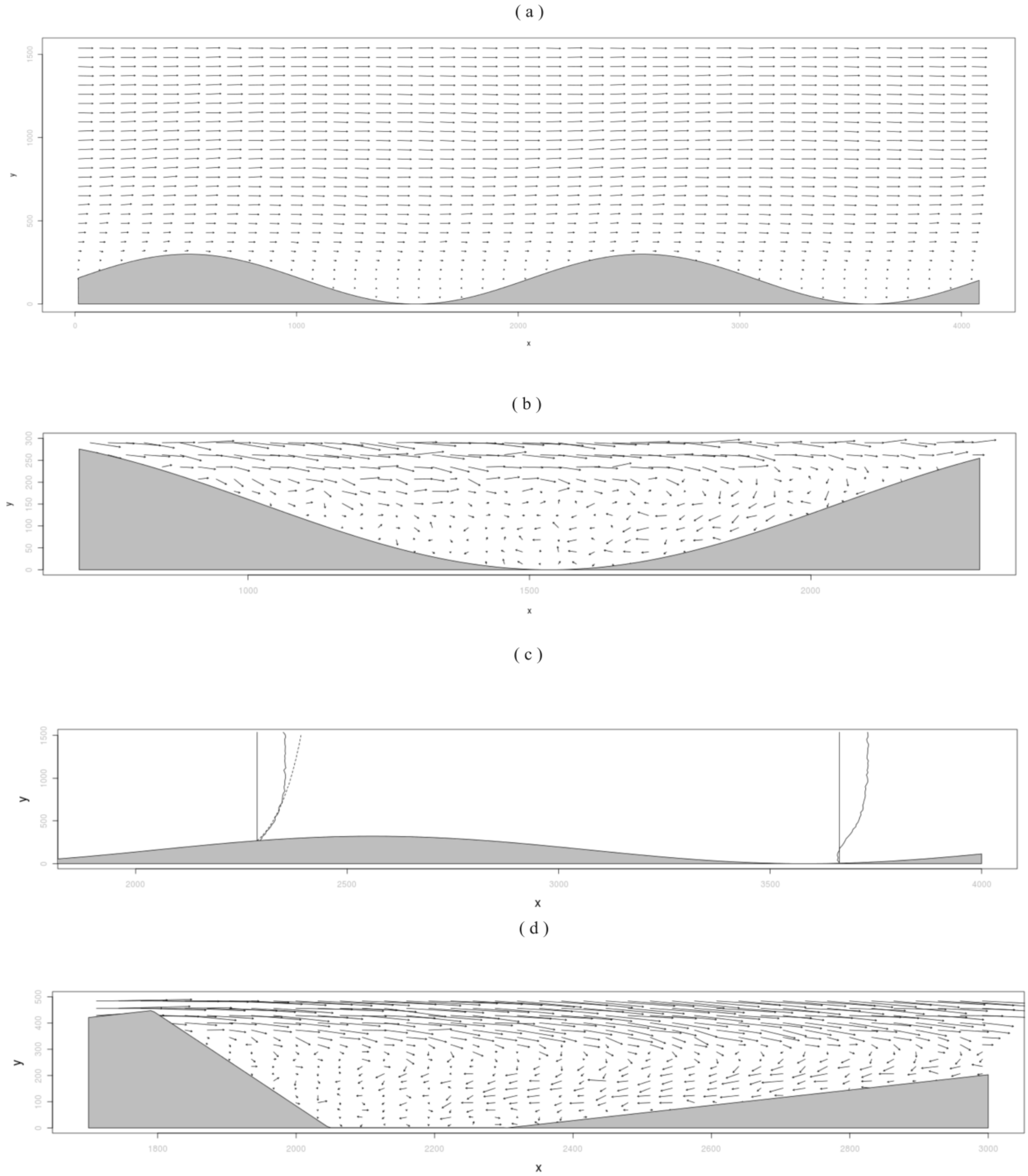

Figure 3: Results of the numerical simulation of the flow velocity field over the symmetric and the anti-symmetric dune

represents the so-called slip boundary conditions (Wolf-Gladrow, 2005);

v. The collision with the channel bottom described by the so-called no-slip boundary conditions and this situation is shown in the Fig. 2e (Wolf-Gladrow, 2005).

7. Results of the simulations are presented as averaged within the area of 32 x 32 nodes.

Figure 3 summarizes the simulation results of the velocity field over the fixed 2-D dune obtained with the use of the model described above. It is easy to notice that the mean velocity distributions indicate the reverse flow in the recirculation zone after the bedforms for the symmetric (Fig. 3a) as well as the anti-symmetric (Fig. 3d) dune. In Fig. 3b the velocity field behind the dune is presented by multiplying the velocity vectors 4 times. It is done for comparison purpose and for the better visualization of the vortex behind the bedform. Figure 3c shows the velocity profiles in two specific points: on the top and in the lee side of the dune. It is easy to see that on the top of the dune the mean velocity distributions indicate the logarithmic velocity profile close to the bottom and the classical reduction of velocities in the lee of bedform crest.

Model for sediment particles transport

The transport of the particles and the deformation of the bedform are controlled by the physical processes included in the discrete model for sediment particles transport that is also over a triangular lattice (see Fig. 4).

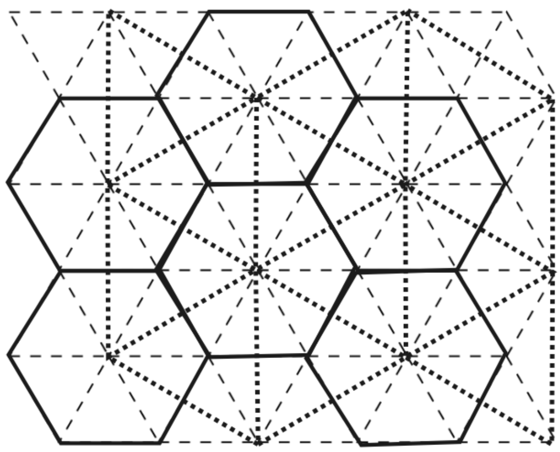

Figure 4: Triangular lattice used in the sediment transport model (dashed lines denote the lattice for the flow velocity model, dotted lines stand for the lattice for the particles transport model and solid lines represent the area in which the particles are placed).

The distance between the nodes in this lattice is equal to $a\sqrt{3}$, where a is the distance between the nodes in the lattice used in the FHP1 model. Moreover, the main assumption of this model is that the particles have spherical shapes and are placed in the area bounded by the hexagon as shown in Fig. 4

We also consider five physical processes that are included in this model:

1. Initiation of the particle movement by the collision with the different particle or the influence of the flow velocity. In the first case the particle that lies on the bed received the part of the momentum from another particle during its collision with the bottom and then starts to move. In the second case the particles' entrainment depends on the bed shear stress τ_* that is calculated as the difference of the flow velocities from two neighboring cells. If the value of the bed shear stress is higher than the Shields number then the particle starts to move. This process is taken into account by the following procedure. Firstly, the friction velocity u_* is calculated from the well-known expression:

$$u_* = \sqrt{\frac{\tau_*}{\rho}} \tag{1}$$

where ρ denotes water density. If the shear velocity is know then the particle has the following initial conditions proposed by Abbott and Francis (1977): $u_p(i)=2u_*$ and $v_p(i)=2u_*$, where u_p and v_p are the particle velocities in the longitudinal and vertical directions, respectively and i stands for the numerical simulation counter.

2. Transport of particles based on the balance of fundamental forces exerting on the spherical particle in the fluid, (i.e. drag, lift and gravity). These forces are explicitly taken into account and are described by the equations similar to those given for example by Bialik (2011).

3. In the deposition we follow the Einstein's (1950) theory (probability) in which the particle is stopped again after traveling the distance that is equal to around 100 of its diameter. The probability of deposition P_d is compared with the probability from random numbers uniformly distributed between 0 and 1 and if the P_d > P(RNG) then the particle is stopped.

4. The next process that is included in the model is the inter-particles collision that is described by the impulse equation and is considered if two particles are in the same cell.

5. The repose of the particle is proceeded if the position of the particle during the simulations is like in Fig. 5.

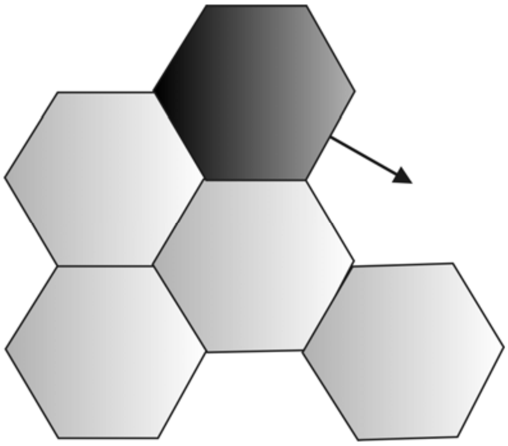

Figure 5: Repose process (grey particles represent the fixed bed and the black one is the moving particle)

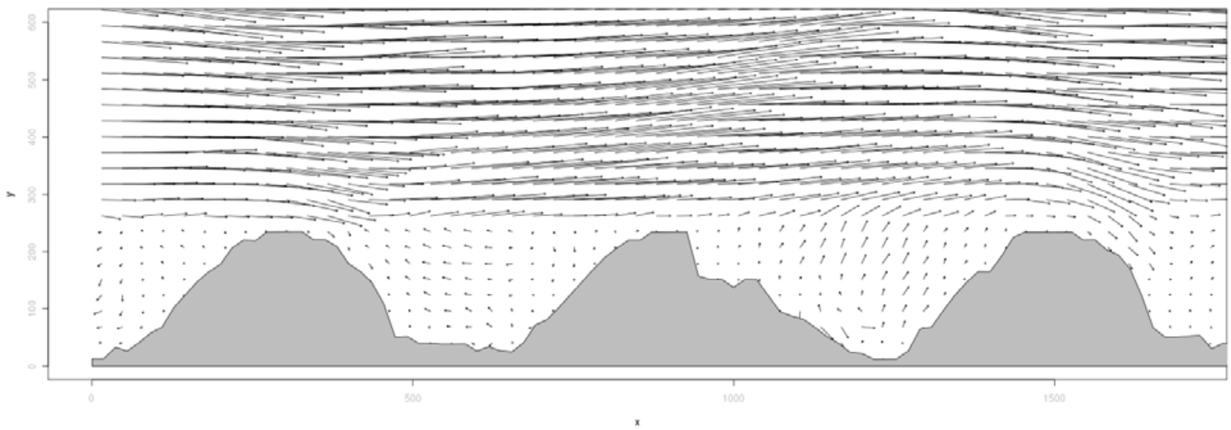

Figure 6: Result of the numerical simulation of the bedform evolution with the flow velocity field

Discussion of preliminary numerical results

Figure 6 presents the final result of the numerical simulation of the bedforms' deformation and the flow velocity field over the erodible 2-D symmetric dunes. It is assumed that in the first step of computation, the bedforms are fixed and the velocity field is generated from the FHP1. This field is similar to that one presented in the Figs. 4 (a-b). The next step is to run the model of particles motion for the small number of iterations and then the velocity field is updated again. This procedure is repeated sequentially until the end of the simulations. Two main processes are clearly seen in Fig. 6: (1) erosion of particles in the lee sides of the dunes; and (2) deposition in the troughs of the dunes. It should also be noted that the fluid velocity field adjusts to the new river bed geometry obtained from the model of sediment transport. The latter result shows the most important advantages of the presented approach.

Concluding remarks

In this preliminary work, we would like to present the model that can be a useful tool for the investigation of the evolution of the river bedforms. This model consists of the Lattice Gas Cellular Automaton Model for flow velocity field over the erodible bed and the model for sediment particles transport that takes into account five main physical processes of particles behavior (entrainment, transport, deposition, particle-particle collisions and particle repose). It is important to note here, that the presented model requires further improvements such as: (1) including more realistic boundary conditions for the description of the free surface; (2) better parameterization of the flow velocity due to the different time and length scales; and (3) validation of the model based on the laboratory or field measurements. All of these are the goals of the forthcoming papers.

Acknowledgements

Funding for this research was provided by the Institute of Geophysics of Polish Academy of Sciences through the project for Young Scientists No. 16/IGF PAN/ 2011/Mł.

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