

# Scour near Engineering Structures under Stratified Bed Conditions

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## Abstract

One of the reasons for failure of engineering structures due to scour near the foundations is stratification of the river bed. Although the problem is very topical, the scour in the stratified bed conditions has not yet been investigated sufficiently well. In the present study, the scour at abutments and elliptical and straight guide banks with a stratified bed under steady and clear-water conditions was investigated. Tests were carried out with uniform sands, standard deviation, and with two layers of different grain sizes and different sequence and thickness. A new method for computing the scour development in time at elliptical guide banks under stratified bed conditions is presented. The method is confirmed by tests results. Stratification of the river bed considerably affects the value of scour depth near the foundations. According to the results obtained in test sand by the method presented, the depth of scour is always greater when a fine-sand layer is under a coarse-sand layer(s). Using for calculations of scour depth the grain size on the top of the river bed and neglecting stratification can lead to wrong results and possible damages and losses.

## Introduction

Damages of engineering structures in the river flow because of scour at foundations always lead to considerable economic and environmental losses. Although the problem is of great practical importance, the scour under stratified bed conditions near the guide banks has not yet been studied.

The influence of stratification on the scour depth near bridge piers is confirmed by Ettema (1980), Raudkivi and Ettema (1983), Kothyari (1990), Kothyari et al. (1992), Garde and Kothyari (1998), FHWA-RD-99-188 (1999), and Gjunsburgs et al. (2006,2010).

The aim of the present study is to elucidate the influence of the river bed stratification on the scour depth at elliptical guide banks under clear water conditions. Tests were made in different hydraulic conditions, with uniform sand, two mean-size diameters, and layers, with different thickness and sequence.

The differential equation of equilibrium for the bed sediment movement in clear water is used, and a method for

calculating the scour development in time at the head of elliptical guide banks under stratified bed conditions is elaborated and confirmed by experimental data. This method allows one to calculate the scour depth in layers with different mean grain size, thickness, and sequence combination of the river bed layers. The most critical conditions for the structures in flow occur when a fine-sand layer lies under a coarse-sand one. As soon as the coarse layer becomes scoured, the scour is rapidly developing in the next fine-sand layer. In this case, the dominant parameter for computing the depth of scour at foundations under stratified bed conditions is the mean diameter of grains of the second layer or of the next one, where the scour stops. The depth of scour is always greater when a fine-sand layer is under the coarse-sand layer. The calculation of scour depth near hydraulic structures in flow by using only data on the grain size on the top of the river bed and neglecting the stratification of the river bed can lead to wrong results and finally to considerable damages and losses.

## Experimental setup

Tests were carried out in a flume 3.5 m wide and 21 m long. Experimental data obtained in the flume in the open flow conditions are presented in Table 1.

Table 1. Tests data for open flow conditions

Test	L cm	$h_f$ cm	V cm/s	Q l/s	Fr	$Re_c$	$Re_f$
L1	350	7	6.47	16.60	0.078	7500	4390
L2	350	7	8.58	22.70	0.010	10010	6060
L3	350	7	10.30	23.60	0.124	12280	7190
L7	350	13	7.51	35.48	0.066	13700	9740
L8	350	13	8.74	41.38	0.075	16010	11395
L9	350	13	9.90	47.10	0.087	14300	14300

In Table1:  $L$ = width of the flume;  $h_f$  = depth of water on floodplain;  $V$ = velocity of the open flow;  $Q$  = discharge,  $Fr$ = Froude number of the open flow;  $Re_c$  = Reynolds number of the channel flow;  $Re_f$ = Reynolds number of the floodplain flow.

The flow distribution between the channel and floodplain was studied under open flow conditions. The rigid-bed tests

were performed to investigate the changes in velocity and water level in the vicinity of the embankment and at the head of the elliptical guide banks. During the sand-bed tests, we studied the scour development in time at stratified bed conditions, with different grain sizes of the first and second layers. The area 1m up and down at a model had a sand bed for studying the scour process near the head of elliptical guide banks. The tests were performed for the contraction rate  $Q/Q_b = 3.66-4.05$  (where  $Q$  is the flow discharge and  $Q_b$  is the discharge through the bridge opening under open-flow conditions). The water depth in a model floodplain was 7 and 13 cm. The thickness of the layers with different grain sizes 0.24 and 0.67mm, with a standard deviation, was 4, 7, and 10cm.

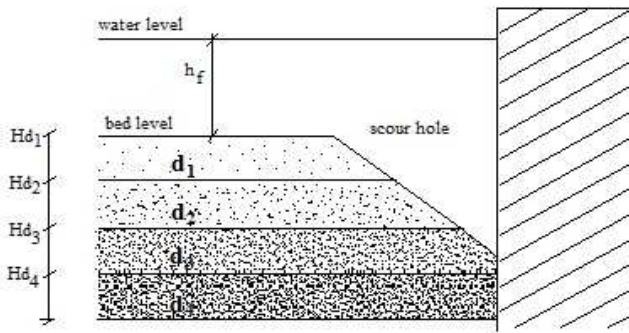


Figure 1: Scour under stratified bed conditions.

The Froude number in the open-flow conditions varied from 0.078 to 0.1243 and the densimetric Froude numbers – from 0.62 to 1.65. The slope of the flume was 0.0012. The opening of the bridge model was 80cm. The condition that  $Fr_R = Fr_f$  was fulfilled, where  $Fr_R$  and  $Fr_f$  are the Froude numbers for the plain river and for the flume, respectively. The tests in the flume lasted for 7 hours. The development of scour was examined for different flow parameters in time intervals within one 7-h step and within two steps, 7 hours each. The tests were carried out with one floodplain model, one side contraction of the flow, one identical floodplain width, and one side contraction. The dimension of the upper part of an elliptical guide banks, namely the length, is calculated according to the Latishenkov (1960) method: it was found to be dependent on the flow contraction rate and the main channel width. The length of the lower part of the elliptical guide banks was assumed to be half of the upper part

The local flow (the local: velocity, Froude number, densimetric Froude number, flow intensity and so on) and the local river bed conditions at the head of the elliptical guide banks are the among main parameters which influence on the local scour development.

## Method

### Scour development in time under stratified river bed conditions

The differential equation for an equilibrium bed sediment movement under clear water conditions at the head of an elliptical guide bank has the form

$$\frac{dv}{dt} = Q_s \quad (1)$$

where  $v = 1/5\pi m^2 h_s^3$  = volume of the scour hole,  $t$  = time,  $Q_s$  = sediment discharge out of the scour hole,  $h_s$  = depth of scour, and  $m$  = scour hole steepness.

The left-hand part of equation (1) can be written as:

$$\frac{dv}{dt} = \frac{3}{5}\pi m^2 h_s^3 \frac{dh_s}{dt} = a h_s^2 \frac{dh_s}{dt} \quad (2)$$

where  $a = 3/5 \pi m^2$ .

The sediment discharge at the initial stage was determined by the Levi (1969) formula

$$Q_s = AB \cdot V_l^4 \quad (3)$$

where  $B = mh_s$  = width of the scour hole,  $V_l$  = local velocity at the head of the guide bank with a plain bed, and  $A$  = parameter in the Levi (1969) formula.

The local velocity at the head of elliptical guide banks is determined by the formula (Gjunsburgs et al., 2006)

$$V_{lel} = \varphi \sqrt{2g\Delta h} \quad (4)$$

where  $\varphi$  = coefficient depending on the contraction rate of the flow and  $\Delta h$  = backwater value (Rotenburg et al., 1965).

The parameter  $A$  at the plain river bed was determined as

$$A = \frac{5.62}{\gamma} \left( 1 - \frac{\beta V_0}{V_{lel}} \right) \frac{1}{d_i^{0.25} \cdot h_f^{0.25}} \quad (5)$$

where  $\gamma$  = specific weight of sediments,  $V_0 = 3.6 d_i^{0.25} h_f^{0.25}$  = critical velocity,  $\beta$  = reduction coefficient of the critical velocity on the bends of open flow determined by using the Rozovski (1957) approach,  $d_i$  = grain size of the bed materials, and  $h_f$  = water depth in the floodplain.

The sediment discharge upon development of the scour is

$$Q_{si} = A_i \cdot m h_s \cdot V_{lt}^4 = b \frac{h_s}{\left(1 + \frac{h_s}{2h_f}\right)^4} \quad (6)$$

where  $V_{lt}$  = local velocity at the scour depth  $h_s$  (Gjunsburgs et al., 2004) and  $b = A_i m V_{lel}^4$ .

The parameter  $A_i$  is changing with increasing  $h_s$  and with changes in the local and critical velocities.

$$A_i = \frac{5.62}{\gamma} \left[ 1 - \frac{\beta V_0}{V_{lel}} \left( 1 + \frac{h_s}{2h_f} \right)^{1.25} \right] \cdot \frac{1}{d_1^{0.25} \cdot h_f^{0.25} \left( 1 + \frac{h_s}{2h_f} \right)^{0.25}} \quad (7)$$

In modelling the development of the local velocity in time, it was found that the discharge across the width of a scour hole before and after its development is  $Q_f = Q_{se}$ , where  $Q_f$  is the discharge across the width of the scour hole with a plain bed and  $Q_{se}$  is that with the scour depth  $h_s$ .

$$m h_s h_f V_{lel} = \left( m h_s h_f + \frac{m h_s}{2} h_s \right) \cdot V_{lt} \quad (8)$$

where  $m$  = steepness and  $m h_s$  = width of the scour hole,  $V_{lel}$  = local velocity with a plain bed,  $h_f$  = water depth in the floodplain, and  $V_{lt}$  = local velocity at scour depth  $h_s$ .

The local velocity  $V_{lt}$  can be determined from equation (8) at any value of  $h_s$ .

$$V_{lt} = \frac{V_{lel}}{1 + \frac{h_s}{2h_f}} \quad (9)$$

The critical velocity  $V_{0t}$  at scour depth  $h_s$  is found by the formula (using mean depth of scour hole)

$$V_{0t} = \beta \cdot 3.6 \cdot d_i^{0.25} \cdot h_f^{0.25} \left( 1 + \frac{h_s}{2h_f} \right)^{0.25} \quad (10)$$

where  $h_m = \frac{\omega}{B} = \frac{h_f m h_s + \frac{1}{2} h_s m h_s}{m h_s} = 1 + \frac{h_s}{2h_f}$  = mean

depth of the scour hole,  $\omega$  = cross section area;  $B$  = width of the scour hole.

Equation (1) can be presented in a different way by using equations (2) and (6).

$$a h_s^2 \frac{dh_s}{dt} = b \frac{h_s}{\left( 1 + \frac{h_s}{2h_f} \right)^4} \quad (11)$$

Separating and integrating the variables yields

$$t = D_i \int_{x_1}^{x_2} h_s \left( 1 + \frac{h_s}{2h_f} \right)^4 dh_s \quad (12)$$

where  $D_i = \frac{a}{b} = \frac{\pi \cdot m}{1.67 A_i \cdot V_{lt}^4}$ .

After integration with new variables,  $x = 1 + h_s/2h_f$ ,  $h_s = 2h_f(x - 1)$ , and  $dh_s = 2h_f dx$ , we obtain:

$$N_i = \frac{t_i}{4 D_i h_f^2} + N_{i-1} \quad (13)$$

where  $N_i = 1/6 x_i^6 - 1/5 x_i^5$  and  $t_i$  = time interval.

Calculating the value of  $N_i$ , we find  $x_i$  and the scour depth:

$$h_s = 2h_f (x - 1) k_m k_\alpha \quad (14)$$

where  $k_m$  = coefficient depending on the side-wall slope of the guide bank and  $k_\alpha$  = coefficient depending on the angle of flow crossing.

At stratified bed conditions parameter  $A_{i2}$  on the second layer is determined by the formula.

$$A_{i2} = \frac{5.62}{\gamma} \left[ 1 - \frac{\beta V_{02}}{V_{lt}} \left( 1 + \frac{H_{d1}}{2h_f} \right)^{1.25} \right] \cdot \frac{1}{d_2^{0.25} \cdot h_f^{0.25} \left( 1 + \frac{H_{d1}}{2h_f} \right)^{0.25}} \quad (15)$$

The local velocity  $V_{lt2}$  on the top of the second layer is found by the formula.

$$V_{lt2} = \frac{V_{lel}}{1 + \frac{H_{d1}}{2h_f}} \quad (16)$$

where  $H_{d1}$  = depth of the layer with grain size  $d_1$ . The critical velocity  $V_{ot2}$  on the top of the second layer can be determined from equation.

$$V_{ot2} = \beta \cdot 3.6 \cdot d_i^{0.25} \cdot h_f^{0.25} \left( 1 + \frac{H_{d1}}{2h_f} \right)^{0.25} \quad (17)$$

Under stratified bed conditions, when the scour depth  $h_s$  is greater than that of the first layer  $H_{d1}$  with grain size  $d_1$ , it is necessary to go back to the top of the second layer with grains of size  $d_2$  and calculate the local velocity  $V_{lt}$ , critical velocity  $V_{ot}$ , and parameter  $Ai_2$ ,  $D_2$ ,  $N_2$ ,  $N_1$ ,  $x$  and  $h_s$  in the second layer  $H_{d2}$  with  $d_2$ . The local velocity on the surface of the next layers can be found by equation (16), and the critical velocity can be determined by equation (17).

## Results

The conditions of modelling the flow pattern at the elliptical guide banks were modified. At the head of the elliptical guide bank, the concentration of streamlines, a sharp drop in water level, and a local increase in velocity are observed. The local velocities form the scour hole. Figure 2 shows a decrease in the local  $V_{lt}$  and increase in the critical  $V_{ot}$  velocities because of scour in steady flow, one layer, and uniform sand.

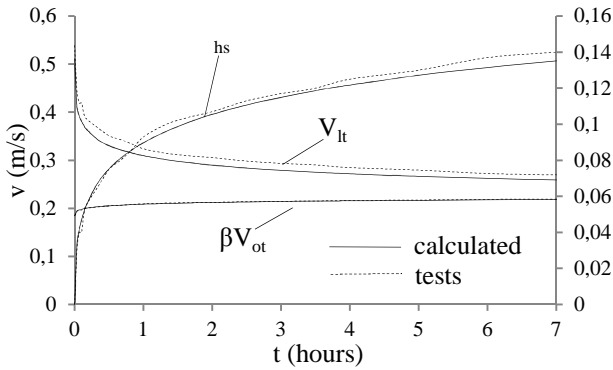


Figure 2: Changes in scour depth, in the local and critical velocities  $V_{lt}$  and  $V_o$  varying with time under steady flow; one sand layer-test EL 6.

Under stratified bed conditions, when the depth of scour exceeds that of the first layer, the scour continues in the second layer at local and critical velocities whose values differ from those calculated in the first layer.

Depending on the sequence of layers, the critical velocity  $V_{ot}$  either increases, when the grains of the second layer are coarser, or reduces, when the grains of the second layer are finer than those of the first layer. The local velocity  $V_{lt}$

reduces more rapidly if the second layer consists of grains of a smaller size. Figures 3 and 4 show the local velocity

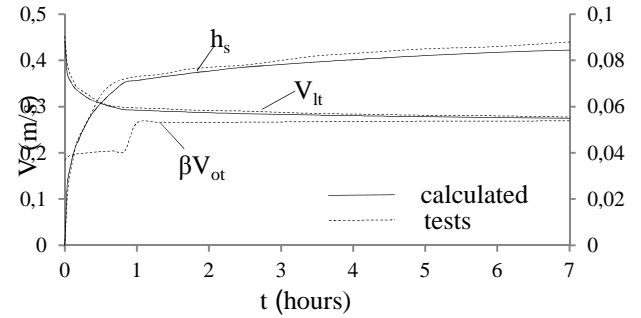


Figure 3: Changes in scour depth, in the local  $V_{lt}$  and critical  $V_{ot}$  velocities;  $d_1=0.24\text{mm}$  in the first layer and  $d_2=0.67\text{mm}$  in the second one; test EUL 5.

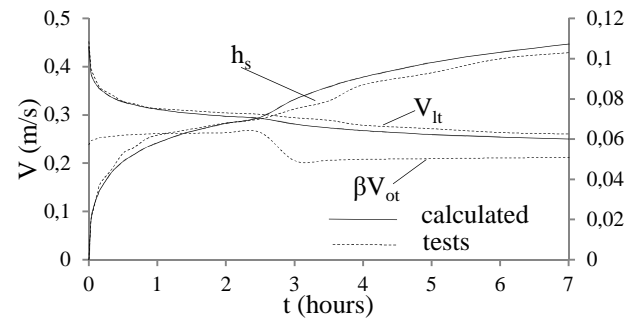


Figure 4: Changes in scour depth, in the local  $V_{lt}$  and critical  $V_{ot}$  velocities, with  $d_1=0.67\text{mm}$  in the first layer and  $d_2=0.24\text{mm}$  in the second one; test EUL 2.

$V_{lt}$  and critical velocity  $V_{ot}$  changing due to scour, with a different sequence of layers — the layer with finer sand lying on the coarser one and vice versa.

If the scour depth exceeds the depth of the second layer, we have to determine the velocities on the top of the next layer where the scour is currently forming. The scour development in time at a steady flow and uniform sand is presented in Figure 5. The depth of scour is known, to depend on the mean grain size, and this was confirmed by our tests; for example, at a grain size  $d=0.24\text{mm}$ , it is greater than at  $d=0.67\text{mm}$ .

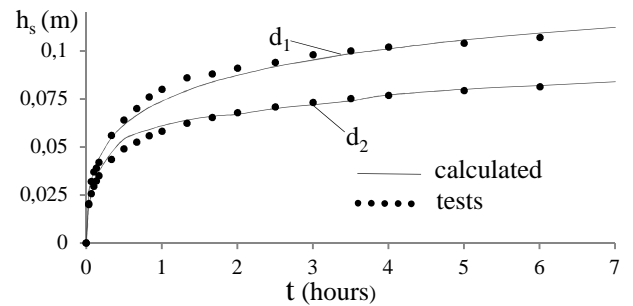


Figure 5: Scour development in time in one layer, with grain sizes  $d_1=0.24\text{mm}$  or with  $d_2=0.67\text{mm}$ ; tests EL17 and EL5.

In the stratified bed conditions, the sequence of layers affects significantly the value and development of scour. Table 2 presents a comparison between some test results and calculated data at different depths of water in the floodplain and different sequence and thickness of layers.

Table 2. Comparison between experimental and calculated values of scour depth under stratified bed conditions

Test	$h_f$ cm	$d_1$ mm	$d_2$ mm	$H_{d1}$ cm	$H_{d2}$ cm	$h_{s \text{ test}}$ cm	$h_{s \text{ calc}}$ cm	$h_s \text{ test}$ $h_s \text{ alc}$
EUL1	7	0.67	0.24	4	46	8.0	8.46	0.86
EUL4	7	0.24	0.67	4	46	5.6	5.74	0.98
EUL2	7	0.67	0.24	7	43	10.3	10.73	0.96
EUL5	7	0.24	0.67	7	43	8.6	8.44	1.04
EUL3	7	0.67	0.24	10	40	12.4	12.13	1.02
EUL6	7	0.24	0.67	10	40	11.4	11.12	1.02
EUL7	13	0.24	0.67	4	46	6.6	6.97	0.89
EUL10	13	0.67	0.24	4	46	10.0	10.88	0.92
EUL8	13	0.24	0.67	7	43	9.4	9.99	0.95
EUL11	13	0.67	0.24	7	43	12.6	13.38	0.94
EUL9	13	0.24	0.67	10	40	13.6	14.58	0.93
EUL12	13	0.67	0.24	10	40	17.6	17.82	0.99

In Table 2:  $d_1$  and  $d_2$  is mean grain size of the first and the second layers;  $H_{d1}$  and  $H_{d2}$  is depth of the first and the second layer;  $h_{s \text{ test}}$  is depth of scour measured in tests;  $h_{s \text{ calc}}$  is calculated depth of scour by method presented.

A comparison of the depth of scour measured in the test and computed by the method suggested gave satisfactory results. The scour development in two layers with different grain sizes,  $d_1/d_2$  or  $d_2/d_1$ , changes its intensity on the border between two layers, namely it develops rapidly if the second layer is fine or slows down if the second layer is coarse. The scour development with time under stratified bed conditions in two layers of different sequence is illustrated in Figure 6.

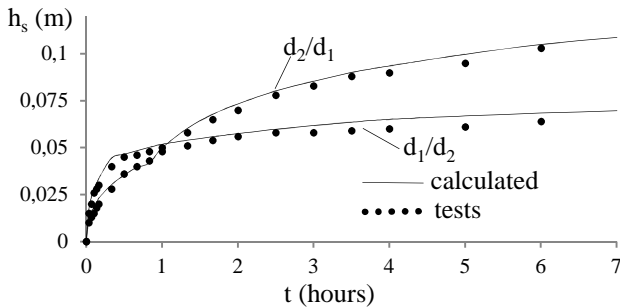


Figure 6: Scour development with time under stratified bed conditions in two layers of different sequence,  $d_1/d_2$  and  $d_2/d_1$ , where  $d_1=0.24\text{mm}$  and  $d_2=0.67\text{mm}$ ; tests EUL 9 and EUL 12.

The data obtained in tests and computed by the method presented show that the most important grain size for

determination of scour depth at elliptical guide banks under stratified bed conditions is the mean diameter of sand particles of the second layer or of any layer where the scour stops (Table2).

In Table3:  $V_l/\beta V_o$  is the local flow intensity,  $V_{ot}/\beta V_o$  is the local increase of the critical velocity,  $Fr$  is the Froude number of the open flow in flume,  $Fr_{Vl}$  is the local Froude number, with the local velocity, at the head of the elliptical guide bank,  $Fr_{Vlt}$  is that at the end of the tests, with a scour depth  $h_s$ ,  $Fr_d$  is the densimetric Froude number for the open flow conditions, and  $Fr_{dl}$  is the local densimetric Froude number at the guide banks with the local velocity

The local flow (the local: velocity, Froude number, densimetric Froude number, flow intensity and so on) and the local river bed conditions at the head of the elliptical guide banks are the among main parameters which influence on the local scour development (Table3).

Table 3. Variations in the relative velocities and Froude numbers at scour under stratified bed conditions

Test	$\frac{d_1}{d_2}$	$\frac{V_l}{\beta V_o}$	$\frac{V_{ot}}{V_o}$	$Fr$	$Fr_{Vl}$	$Fr_{Vlt}$	$Fr_d$	$Fr_{dL}$
EUL1	$\frac{0.67}{0.24}$	1.55	1.09	0.078	0.445	0.234	0.62	3.54
EUL2	$\frac{0.67}{0.24}$	1.85	1.12	0.1035	0.531	0.215	0.82	4.24
EUL3	$\frac{0.67}{0.24}$	2.14	1.16	0.1245	0.617	0.204	0.99	4.91
EUL4	$\frac{0.24}{0.67}$	2.00	1.15	0.078	0.445	0.162	1.04	5.90
EUL5	$\frac{0.24}{0.67}$	2.40	1.19	0.1035	0.531	0.152	1.37	7.06
EUL6	$\frac{0.24}{0.67}$	2.77	1.23	0.1245	0.617	0.146	1.65	8.20
EUL7	$\frac{0.24}{0.67}$	1.37	1.06	0.066	0.340	0.209	0.72	3.67
EUL8	$\frac{0.24}{0.67}$	1.56	1.09	0.075	0.384	0.197	0.84	4.17
EUL9	$\frac{0.24}{0.67}$	1.89	1.13	0.087	0.466	0.183	0.95	5.06
EUL10	$\frac{0.67}{0.24}$	1.77	1.12	0.066	0.340	0.145	1.20	6.11
EUL11	$\frac{0.67}{0.24}$	2.02	1.15	0.075	0.384	0.138	1.40	6.96
EUL12	$\frac{0.67}{0.24}$	2.45	1.19	0.087	0.466	0.128	1.59	8.45

In scour development in time ratio between local and critical velocities ratio is approaching to one, parameter  $A_i$  is reducing and at the final stage will be equal to zero and parameter  $D_i$  is increasing and approaching to infinity. When ratio between local and critical velocities is becomes

equal to 1, parameter  $A_i$  is equal to 0, sediment discharge from the scour hole becomes equal to 0 and scour process stops. The local flow (the local: velocity, Froude number, densimetric Froude number, flow intensity and so on) and the local river bed conditions at the head of the elliptical guide banks are among the main parameters which influence on the local scour development (Table3).

According to experimental results and the method proposed, the scour depth is greater if the coarse-grain layer lies on the top of the river bed and a fine-grain layer goes after it, and the depth is smaller if the fine-grain layer lies on the surface of the river bed.

Calculating the depth of scour by using only the data on grain size on the top of the river bed and neglecting complex geological conditions can lead to wrong results and possible failure of hydraulic structures.

## Conclusions

The scour at the foundations of engineering structures in flow under stratified bed conditions can be the reason for their damage and failure.

In the present study, the scour at abutments and elliptical and straight guide banks with a stratified bed, under steady and unsteady clear water conditions was studied. The tests were performed for different hydraulic conditions, contraction rate, Froude number of the open flow, densimetric Froude numbers, and depth of the flow model.

The local flow (the local: velocity, Froude number, densimetric Froude number, flow intensity and so on) and the local river bed conditions at the head of the elliptical guide banks are among the main parameters which influence on the local scour development (Table3).

A new method for computing the depth of a scour developing with time at elliptical guide banks under stratified bed conditions is presented. The method is confirmed by test results. It is found that the scour depth depends on the sequence and thickness of the river bed layers. The most critical conditions for structures occur when a fine-sand layer lies under a coarse-sand layer. As soon as the coarse layer has been scoured, the scour precedes its rapid development in the next fine-sand layer. In this case, in computing the depth of scour at the foundations under stratified bed conditions, the dominant grain size value is the mean diameter of grains of the second layer or of the next one, where the scour stops. According to the tests and method presented, the depth of scour is always greater when the fine-sand layer is under the coarse-sand layer(s). The calculation of scour depth near hydraulic structures in flow by using the values of grain size obtained on the top of the river bed and neglecting the stratification of the river bed can lead to wrong results and finally to considerable damages and losses.

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