ON THE PIANO KEY WEIR HYDRAULICS

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Abstract

The Piano Key Weir (PKW) is a kind of labyrinth weir with a geometry that uses overhangs to reduce the base length. The PKW can thus be directly placed on the crest of an existing dam. Together with its high discharge capacity for low heads (up to four times as high as an ogee-crested weir of same length), this geometric feature makes the PKW an interesting solution for dam rehabilitation and for new dam projects with a high level of constraints (design discharge, available space, reservoir storage, ...). PKW has been initially designed in 2001 and built for the first time in 2006 by "*Electricité de France (EDF)*". Even if the first experimental studies confirmed its appealing discharge capacities, the flow upstream, over and downstream of this complex structure is still poorly described.

Following a 3 years intensive experimental and numerical study of PKW hydraulics, the paper presents a general description of the hydraulic behavior of the PKW. It aims to explain the influence of the large set of geometric parameters on the discharge capacity. The assumptions of the study link various phenomenons depicted in the literature about PKW and an evaluation of their relative influence on the PKW efficiency is given. The role of the crests shape, the crests submersion, the nappes interactions, the crests approach conditions, the position of the control section and the head losses are analyzed. Design advices are also given.

Introduction

The Piano Key Weir (PKW) is a kind of labyrinth weir with a geometry that uses overhangs to reduce the base length. The PKW can thus be directly placed on the crest of an existing dam. Following Pralong et al. (2011), the "PKW-unit" can be defined as the basic structure of a PKW, composed of an inlet, two transversal walls and two halves of outlets. The main geometric parameters of a PKW are the weir height P, the number of PKW-units N_u, the lateral crest length B, the inlet and the outlet widths W_i and W_o, the up- and downstream overhang lengths B_o and B_i, and the side wall thickness T_s, as defined in Figure 1.

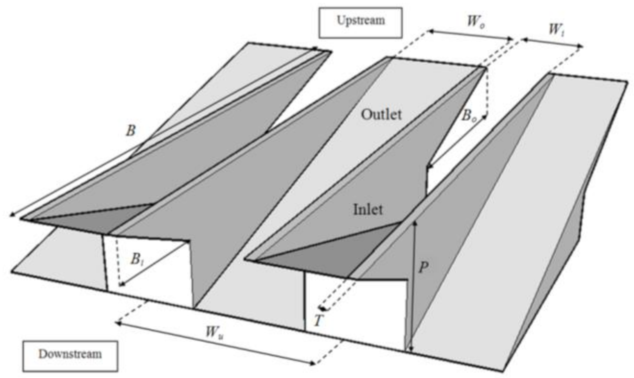

Figure 1: 3D sketch of a PKW and main parameters

Piano key weirs (PKW) are interesting structures for dam rehabilitation as they allow discharges as much as four times higher than ogee-crested weirs for the same water head (Blanc and Lempérière 2001). This discharge capacity is mainly due to its enhanced crest length created by the labyrinth geometry. However, if we focus on the discharge coefficient of a unit crest length, a quick decrease on crest efficiency can be observed compared with sharp or ogee-crested weirs (Anderson 2011, Machiels et al. 2011b).

In the scope of enhancement of PKW geometries, it is of prime importance to clearly explain the reasons of this crest efficiency decrease. This paper analyzes the various theories proposed in literature and attempts to combine their contributions to develop a general understanding of PKW working.

Various approaches of the efficiency decrease

Several authors have focused on the PKW efficiency decrease for increasing heads and three different approaches can be summerized: the first one developed from parametric as well as prototype experiments led at the "*Ecole Polytechnique Fédérale de Lausanne (EPFL)*", the second one based on large scale, parametric and prototype experimental results obtained at the "*University of Liège (Ulg)*", and the third one issued from experimental comparison of PKW and labyrinth weirs performed at the "*Utah State University*".

EPFL approach

The first proposed approach, described by Leite Ribeiro et al. (2007), explains the crest efficiency decrease as a reduction of the effective crest length with increasing heads, due to submersion of the frontal weirs, as well upstream as downstream, and to the overcrossing of jets.

This approach is well correlated to the experimental observation of the flow surface shape variation with head. With increasing heads, a progressive submersion of the upstream part of the outlet key can be observed combined with an overcrossing of jets coming from the lateral crests, limiting the effective crest length (Leite Ribeiro et al. 2007). It explains the interest of keeping the outlet sufficiently large and of providing a sufficient outlet slope to avoid outlet submersion. Indeed, it has been shown that a PKW with a too small outlet key width is less efficient (Machiels et al. 2010a), and that increasing the outlet key height P_o, keeping the inlet key one P_i constant, increases the discharge efficiency (Le Doucen et al. 2009). However, this approach doesn't highlight the interest in increasing the inlet key height (Le Doucen et al. 2009, Machiels et al. 2011a). Worse, it is opposed to the experimental results showing the interest of increasing the ratio between inlet and outlet key widths (Ouamane and Lempérière 2006, Le Doucen et al. 2009, Machiels et al. 2010b).

This approach doesn't distinguish between lateral or longitudinal crests influence and doesn't take into account the approach conditions (approaching slope, flow direction, velocities). These approximations may cause the previously described limitations.

Ulg approach

The Liège approach (Machiels et al. 2011b) completes the Lausanne one's by explaining the reduction of the effective crest length in the downstream part of the inlet. Indeed, apparition of a critical section in the downstream part of the inlet has been observed on a large scale model. For increasing heads, this critical section moves upstream along the inlet key limiting the effective crest length. For highest heads, the critical section is assumed to stay at the inlet entrance, and the weir efficiency is so supposed to be stabilized.

These observations seem to enhance the Lausanne approach, explaining the interest in increasing the inlet cross section, by increasing inlet height (Le Doucen et al. 2009, Machiels et al. 2011a) or/and width (Ouamane and Lempérière 2006, Le Doucen et al. 2009, Machiels et al. 2010b), to avoid the critical section apparition or limit its progression along the inlet key. However, no direct relation between the critical section position and the upstream head has been suggested. It is thus very difficult to quantify the real impact of this phenomenon on the global PKW discharge capacity.

Therefore, a calculation of the global weir discharge based on the separation of the different parts of the crest has been proposed (Machiels et al. 2011a). The specific discharge on each part of the crest is calculated combining the SIA formulation for sharp-crested weirs (Swiss Soc. Eng. Architects 1926), along with a correction coefficient to consider sloping approach for both up- and downstream crests and a correction coefficient to take into account the parallel approach of the lateral crest.

The formulation fits with less than 10% the measurements performed on various scale models in Liège and Lausanne (Machiels et al. 2011a, Machiels et al. 2012a). However, it doesn't distinguish between the effects either of lateral crests approach conditions or of critical section apparition, even if the two phenomenons have been observed. An extensive use remains unavailable for wide variations of geometrical parameters values.

Utah State University approach

In a third approach, Anderson (2011) explains the decrease in global PKW efficiency, for increasing heads, by an increase of the head loss at the inlet entrance. Indeed, the inlet entrance can led to a progressive flow contraction along the upstream overhang before a sudden vertical contraction at inlet entrance. This double contraction may involve a head loss function of the section variation and of the flow velocity.

This theory explains well the interest in increasing the W_i/W_o ratio (Ouamane and Lempérière 2006, Le Doucen et al. 2009, Machiels et al. 2010b) to limit the contraction as well as the flow velocity, but it fails to explain the decrease in PKW efficiency for highest values of this ratio (Machiels et al. 2010a). It also explains the interest in increasing the inlet height to decrease the flow velocity and so the head loss (Le Doucen et al. 2009, Machiels et al. 2011a). Furthermore, it confirmed that the use of parapets, for an inlet height constant, maintained the PKW discharge constant (Le Doucen et al. 2009, Vermeulen et al. 2011). The use of fillets under the upstream overhangs, providing a smoothly flow contraction, also helps to decrease this head loss (Anderson 2011, Vermeulen et al. 2011).

In his experiments, Anderson (2011) has compared PKW with similar geometries of rectangular labyrinth weirs with or without sloped floors in both inlet and outlet keys. He found a higher efficiency for PKW compared with rectangular labyrinth weirs and explained that the upstream overhangs provide a more progressive variation of the flow section limiting the head loss at inlet entrance. However, he also highlighted smaller differences in efficiency between the various rectangular labyrinth weirs considering or not sloped floors in inlet and/or outlet keys. This last behavior cannot to be explained by the head loss theory.

Flow behavior

The entire factors at the root of the three approaches presented before have been observed on the several experimental models tested and have an influence on the global discharge capacity. However, other factors may play a role in the decrease of efficiency with increasing heads. The observation of the flow over PKW enables to highlight the influence on the global discharge capacity of the crest shape and thickness (Leite Ribeiro et al. 2007, Laugier et al. 2011), the submersion of the upstream part of the outlet (Leite Ribeiro et al. 2007), the nappe interferences (Leite Ribeiro et al. 2007), the approach flow conditions (slope, flow direction, velocity) of the crests (Machiels et al. 2011a), the critical section development along the inlet (Machiels et al. 2011b), the head losses at the inlet entrance (Anderson 2011).

Crests shape and thickness

As for thick-crested weirs, the crest shape and thickness influence the discharge capacity of PKW.

The influence of the crest shape on the discharge capacity of thick-crested weirs has been studied by several authors. In a general way, Hager (1987) proposed to correct the C_d coefficient of the Poleni equation, equal to 0.429 for sharp-crested weirs, by a coefficient c function of the upstream head and the crest shape. The Figure 2 shows the evolution of the discharge coefficients proposed by Hager for finite thick weirs as a function of the crest shape.

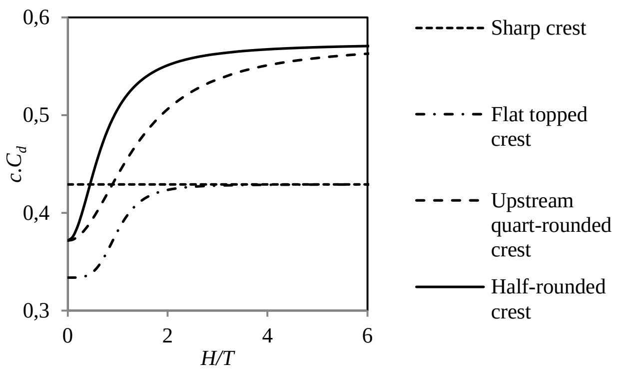

Figure 2: Variation of the discharge coefficient of a thick weir function of the crest shape and thickness

For low heads, C_d coefficient for flat topped crest varies from 0.33 to 0.429 for H/T values between 0.45 and 1.8, as experimentally observed by Johnson (2000). The use of rounded crest enables to increase the discharge capacity up to 30%. The half-rounded crest is more efficient than the quart-rounded as showed on labyrinth weirs (Tullis et al. 2005). The transition zone can be correlated to the variation of the nappe shape moving from a leaping nappe for low heads to a clinging nappe on rounded crests and to springing nappe on sharp and flat-topped crests (Lakshmana Rao 1975, Johnson 2000).

For high heads, no more influence of H/T is observed on C_d values. Half and quart rounded crest provide similar discharge capacity, as observed on labyrinths (Tullis et al. 2005), about 30% higher than flat and sharp crested weirs.

On PKW, similar transition zones of the nappe shape are observed for low heads (Leite Ribeiro et al. 2007, Machiels et al. 2009, Anderson 2011). These transition zones enable explaining the increase in PKW discharge efficiency measured for very low heads (Anderson 2011, Machiels et al. 2011b).

However, for high heads, no more difference is obtained in-between the various crest shapes (Leite Ribeiro et al. 2007, Anderson 2011) traducing the low influence of crest shape on the observed efficiency decrease. From the experience of the first PKW buildings, the quart-rounded or chamfered crest shapes have to be preferred for efficiency and lifetime considerations (Vermeulen et al. 2011).

Regarding Figure 2, an increase of the thickness of the crest reduces the discharge capacity of the weir. However, a decrease of the crest thickness under 0.4H and 0.25H respectively for a flat topped and a rounded crests, does not change the discharge coefficient of more than 0.5%.

However, regarding PKW side walls, the crest thickness also influences the effective section of the inlet and outlet keys. An investigation of the influence of an increase of the side crest thickness, considering an equivalent decrease of both inlet and outlet key widths, has been performed by Laugier et al. (Laugier et al. 2011). The results highlight a large decrease of efficiency for low heads up to 22% in comparison with thin crested configuration. For increasing heads, the lost of efficiency decreases, first quickly, then slowly. Laugier et al. explain the observed decrease in efficiency by a reduction of the effective crest length due to the side wall thickness.

However, as it is the developed length that contributes to the total discharge capacity, the effective geometrical loss observed is defined equal to $2T/L_u$, with L_u the developed crest length of a PKW-unit. It reaches a maximal value only up to 3.5% on the tested configurations. Furthermore, the first quick decrease may be attributed to the transition from a leaping to a springing nappe over the flat topped crest. Indeed this decrease is observed for H/T values between 1 and 5 and is not enhanced on the configuration with lowest thickness for which the studied values of H/T are higher than 5. Using curves of Figure 2 to evaluate the loss in efficiency compared to a sharp crest, the maximal value of the loss due to the crest shape is equal to 12%. The residual losses in efficiency when the effective geometrical loss ($2T/L_u$) and the losses induced by the crest shape (1-c) are deducted from the losses observed by Laugier et al. could reach 14%.

The residual losses may arise with the reduction of the keys sections induced by the crest thickness. For a given head, a

diminution of the inlet key section induces higher velocities. This higher velocity along the inlet key decreases the lateral flow, favours the apparition of a control section in the key, and increases the head loss at the inlet key entrance (see below).
Tests performed with the 1D-PKW numerical model, developed at Liège University (Erpicum et al. 2010), considering thin and thick side walls, provide losses in efficiency in the same range than in the former study of Laugier et al. (Figure 3). The observed losses in efficiency could directly be linked to the free surface profile variation (Figure 4). For low specific discharges on a high PKW ($P/(B\text{-}B_o)$ = 1), the free surface profiles are close whatever the crest thickness, and the losses in efficiency are reduced (< 3%). This is mainly due to the relatively low velocities observed in these specific conditions. On the other hand, as the flow velocity increases by an increase of the specific discharge or a decrease of the weir height, the loss in efficiency increases significantly. In the same time, a variation can be seen on the free surface profile along the inlet key. Due to the increase of the flow velocity in the inlet key direction, the discharge capacity of the side crest decreases. To ensure a fixed specific discharge capacity with a thick crest configuration, an increase of the water height along the inlet key is thus observed.

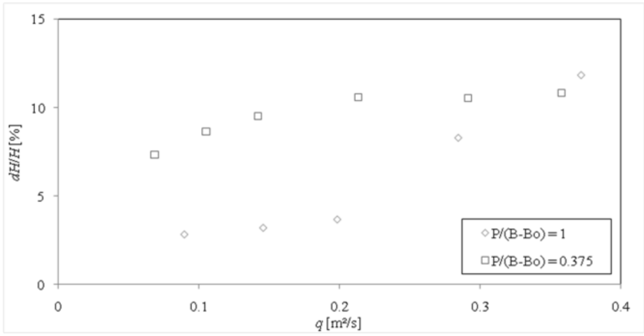

Figure 3: Losses in efficiency measured on the 1D-PKW results for T/L_u passing from 0.0007 to 0.01

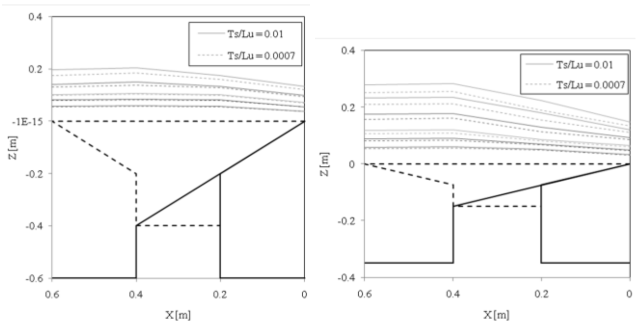

Figure 4: Free surfaces profiles computed with the 1D-PKW model ($P/(B\text{-}B_o)$ = left - 1; right - 0.375)

The main influence of the crest thickness is to reduce the keys widths, increasing therefore the flow velocities. It is thus of main interest to limit the crest thickness to the one needed for structural considerations.

Crests submersion

The crest submergence by the flow in the outlet key decreases the PKW efficiency of around 10%, considering a downstream free surface elevation corresponding to the crest one (Hien et al. 2006, Belaabed and Ouamane 2011). The influence of the submergence of the outlet key could be split in two parts: the influence on the upstream crest of the outlet key and the influence on the side wall crest efficiency.
The discharge capacity of the upstream crest is modified as long as the critical section moves upstream of its position for free flow conditions. To avoid upstream crest submergence it is thus convenient to impose a downstream slope at least equal to the critical slope (Eq. (1)) directly downstream of the upstream crest of the outlet key.

$$S_{o\,cr} = \frac{2}{3} g^{\frac{4}{3}} Q_o^{-\frac{2}{3}} K^{-2} \left(1 + 6.75 \frac{Q_o^2}{gW_o}\right)^{\frac{4}{3}} \qquad (1)$$

K is the Strickler coefficient of the outlet key and Q_o is the discharge along the outlet key.
As the discharge coefficient of the upstream crest is close to 0.41 (see over), the minimal outlet key slope avoiding the upstream crest submergence effect is largely under the usual slopes observed on PKW prototypes.
The effect of submergence of the side wall is observed as soon as the free surface elevation becomes higher than the crest one. Indeed, for thick-crested weirs, while downstream level stays under the crest one, no influence of the downstream level on the discharge capacity is observed (Lakshmana Rao 1975). For higher water level, the discharge coefficient has to be decreased by a submergence coefficient mainly dependant of the ratio between down- and upstream heads (Sinniger and Hager 2004).
To avoid effects of crest submergence, the free surface level along the outlet key must still remain under the crest level. That may be tuned using slopes over the critical one all along the key. Use of parapet walls in the outlet may also help. If the PKW is properly designed, the outlet key flow has no influence on the weir efficiency.

Nappes interaction

Two different types of nappe interference are present on PKW: a lateral interference between nappes coming from lateral and upstream crests and a frontal interference between nappes coming from the two opposite lateral crests along the outlet.
The first one can be compared to a lateral contraction of the nappe, as observed around piles. This contraction is generally taken into account by a reduction of the effective crest length. However, the reduction for triangular pile

configurations, what is the most similar to the observed contraction on PKW, is only equal to the half width of the pile, corresponding to the opposite crest thickness in this case. The reduction of effective length due to lateral interaction is thus in the range of $2T$. That represents generally less than 5% of the developed length.

The frontal interference influences the side wall crest efficiency only if the interaction zone, situated in the middle of the outlet key, is upstream of the control section in free conditions. From the definition of a standard weir (Sinniger and Hager 2004), the distance between the control section and the weir crest for sharp-crest weir must be close to the one situated between upstream wall and crest for standard one, $0.28H$. There is thus no effect of the frontal interference of nappe while the H/W_o ratio stays under 1.8.

Crests approach conditions

The crest of PKW can be decomposed into three parts with different approach conditions: the downstream crest of the inlet key, which can be seen as a downstream inclined sharp-crest, the upstream crest of the outlet key, which can be seen as an upstream inclined sharp-crest, and the side wall crest, which can be seen as a lateral thick-crest.

As a result of an experimental study led on downstream inclined sharp-crested weirs (Machiels et al. 2011a), the discharge coefficient must be corrected from the one of a traditional sharp-crested weir until bottom slopes of 1.2. Over this limit, the discharge coefficient remains constant around 0.46, whatever the bottom slope or the upstream head. As the bottom slopes of traditional PKW are largely under 1.2, the downstream crest approach condition doesn't influence the observed decrease of discharge capacity.

A similar experiment led on an upstream inclined weir has shown a decrease of the discharge coefficient of sharp-crested weir until 0.41 (Karelle 2008). Since this coefficient is not modified with upstream head variations, approach conditions of the upstream crest don't influence the observed efficiency decrease of PKW.

The lateral crest discharge is mainly influenced by the Froude number along the main flow direction (Hager 1987). Hager distinguishes between three influences of the approach conditions on the side weir efficiency: the effect of flow depth, the effect of upstream velocity and the effect of lateral outflow angle. Regarding the coefficients proposed by Hager to modify the usual discharge coefficient for thick-crested weir, a variation of the Froude number along the main channel (inlet key) from 0 to 1 decreases the discharge coefficient of at least 58%, depending on upstream head and weir height. The approach flow conditions of the side wall crest seems thus to be the main reason of the PKW efficiency decrease observed on the various tested models.

Critical section apparition

As a result of the study performed at the University of Liege on the large scale model of PKW, the appearance of a control section in the inlet key has been highlighted for sufficiently high heads (Machiels et al. 2011b). This control section reduces the effective crest length and then the global discharge capacity of the weir. The effective crest length depends on the PKW geometry as well as on the upstream head. Indeed, it has been verified on the scale model that the control section moves upstream along the inlet key for increasing heads until it reaches the inlet key entrance for a limit head (Machiels et al. 2011b).

As defined, the effective length is completely determined, for a given PKW geometry, by the position of the control section over the downstream crest of the inlet key *B'*. As the pressure measurements on the large scale model confirmed hydrostatic pressure profiles along the inlet key (Machiels et al. 2011b), the position of the control section can be determined considering a Froude number equal to 1 in this section. Considering at once no head variation along the lateral crest (see below), and the continuity between flow through the control section and flow over the weir crest downstream, it becomes possible to define the control section position *B'* as a function of the weir geometry and the upstream head:

$$0.385W_i\left(1+\frac{B'S_i}{H}\right)^{\frac{3}{2}} = 0.482W_i + 0.283 \times\left(1-\frac{2}{9\left(1+\left(\frac{H}{T}\right)^4\right)}\right) \times B'\frac{\left(2-\frac{B'S_i}{H}\right)^{\frac{3}{2}}}{\left(5+2\frac{B'S_i}{H}\right)^{\frac{1}{2}}} \quad (2)$$

The existence of a critical section on the 1:10 scale studied in Liège (Machiels et al. 2011b) between upstream heads ratio H/P from 0.1 to 0.5 decreases the effective crest length of 8% following Eq (2). In the same time the discharge through the critical section increases with head. The variation of the critical section position along the inlet key for H/P ratios between 0.1 and 0.5 decreases thus the global PKW efficiency of 5.5%, to be compared with the loss in efficiency of 40% observed on the experimental model.

The influence of the critical section observed along the inlet key cannot be neglected but seems to be of minor importance.

Head losses

The head loss induced by the flow contraction at the inlet entrance can be compared to the head loss induced by a variable flow contraction. Considering head losses formulae proposed by Idel'Cik (1969) for convergent flows, applied to the tested PKW in Liège (Machiels et al. 2011a, Machiels et al. 2011b, Machiels et al. 2012a, b), it appears that the head loss still remain under 3% of the upstream head even for low weir height or low weir width. The head loss at inlet entrance is thus of minor importance and the use of noses must only improve the weir capacity of few percents.

Conclusions

Piano Key Weir is a complex geometric structure inducing a large set of flow conditions along the different parts of the weir. Besides the difficulty of a separate observation of the different flow features, the common or opposite influence of the geometrical parameters variation on the efficiency of the weir, makes the understanding of the PKW working complex. From former researches and observations, six factors have been identified to affect the global discharge capacity of PKW: the crest shape and thickness, the crest submersion, the nappes interactions, the approach conditions, the critical section position along the inlet key and the head losses at inlet entrance.

From the studies discussed before, the main parameter influencing the weir efficiency is the approach conditions of the side wall crest. The decrease of the effective head on the side wall and the inclination of the lateral flow, due to the flow longitudinal velocity along the inlet key, allow to explain the main part of the stage-discharge curves observed on tested models. It is thus of prime importance to decrease the flow velocity along the inlet key for the best PKW design. The crest shape mainly influences the PKW working at low heads. In order to limit the observed decrease in efficiency for low heads, rounded crest have to be preferred on prototype designed on flow mitigation. Effects of the critical section appearance along the inlet key seem to be of secondary importance, modifying only of around 5% the PKW efficiency. In the same way, the consideration of the crest thickness and the lateral nappe interferences only decrease the effective crest length in the range of 3%. In the same way, head losses affect the upstream head of less than 3% on studied models. Finally, effects of crest submergence and frontal nappe interaction can be avoided by the design of sufficient outlet key slope and width.

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