A new approach for morphodynamic modeling using integrating ensembles of artificial neural networks

Keivan Kaveh, Minh Duc Bui and Peter Rutschmann

Abstract

The evolution of bed level is an important process that occurs in rivers, estuaries, and coastal regions. State-of-art morphological models use classical lower order Lax-Wendroff or modified Lax-Wendroff schemes for morphology which are not very stable for long time sediment transport simulation. This paper describes an architecture for ensembles of ANN with emphasis on its application to the prediction of time series morphological changes, where the goal is to minimize the prediction error. To evaluate the prediction qualities of the ensembles of ANN models, a comparative study has been carried out between this proposed model and common ANN model by evaluating several statistical parameters that describe the errors associated with the model in terms of statistical measures of goodness-of-fit between the estimated bed change and analytical approximation. The predicted results showed that for the simple case of 1D morphological problems considered in this study, ensembles of artificial neural networks could provide better results compared to common ANN models in most test cases.

Keywords: Artificial Neural Network, Ensembles, Hydromorphological modeling, Sediment transport

1. Introduction

Numerical morphological models involve coupling between a hydrodynamic model, which provides a description of the flow field leading to a specification of local sediment transport rates, and an equation for bed level change which expresses the conservative balance of sediment volume and its continual redistribution with time. Recent examples of the development of such models include the work of Nicholson et al. (1997), Kobayashi and Johnson (2001), and Hudson et al. (2005). As reviewed in Nicholson et al. (1997), many state-of-art morphodynamic models use classical shock capturing schemes for bed level simulation. For example, Johnson and Zyserman (2002) applied a second order accurately modified Lax-Wendroff scheme (Abbott, 1979). The HR Wallingford model PISCES (Chesher et al. (1993)) uses a one-step Lax-Wendroff scheme. In Hudson et al. (2005), a variety of numerical schemes are discussed including versions of the Lax-Friedrichs scheme (Lax, 2005), the classical Lax-Wendroff scheme, the MacCormack scheme (MacCormack (2003)) and Roe scheme (Osher and Solomon (1982)) based on shallow water equations for hydrodynamics and simple power law for sediment transport rate. A flux-limited version of the Roe scheme is found to be much more stable than Lax-Wendroff and Lax-Friedrichs type schemes. The disadvantage is that the Roe scheme involves calculations of eigenvectors for so-called Roe averaged Jacobian matrix of the entire hydrodynamics and morphology system. This is feasible for shallow water systems and simple power law sediment transport rates for 1D problems. The numeric becomes tedious and complex for a coupled system of more comprehensive hydrodynamic and sediment transport models. In the conventional hydromorphological models the bed level changes are governed by the equation for conservation of sediment mass (Exner equation). Neglecting the suspended load, this equation can be written in a 1D case as follows:

$$\frac{\partial z_b}{\partial t} = -\frac{1}{1-n_p} \cdot \frac{\partial q_b}{\partial x} \qquad [1]$$

where z_b is the bed elevation (defined positive upward relative to a fixed datum) at each horizontal position x and time t; n_p is the bed porosity, q_b is the volumetric sediment transport rate per unit width. The bed load transport rate q_b is a complicated function of various hydrodynamic quantities such as currents and water depth as well as quantities associated with sediment properties such as sediment density and grain size. Many empirical functions are available to calculate bed load transport. Most of the formulae available in the literature have been developed based on the analysis of laboratory and field data using statistical methods such as the regression method, and there are drastic differences between them. No uniformly valid formulation for qb exists at present. Yet, we are still unable to select the most accurate for a particular problem and the accuracy of computational sediment transport models remains a challenging question. The artificial neural networks are a form of artificial intelligence, which attempts to mimic the function of the human brain and nervous system at a sub symbolic level. ANNs learn from data examples presented to them in order to capture the subtle functional relationships within the data. The majority of hydromorphological processes are highly nonlinear in nature and, in many cases, modeling these variables with conventional models may be limited by a poor understanding of the complex interactions that are involved in the process. In such cases, ANN are often viewed as an appealing alternative, as they have the ability to extract a nonlinear relationship from data without requiring an in depth knowledge of the physics occurring within the system. However, there are some basic aspects of this approach, which are in need of better understanding. More specially: (1) No standard methods exist for transforming human knowledge or experience into the system. (2) There is a need of effective methods for tuning the transfer functions so as to minimize the output error measure or maximize a performance index.

In this perspective, the aim of this paper is to develop a new architecture for ensembles of ANN models which can serve as a basic for constructing a set of ANN models with appropriate transfer functions to generate the stipulated input-output pairs. The objective is to predict the morphological changes in a straight alluvial channels under steady flow discharge and uniform bed material, where the bed level changes are calculated directly from the defined flow without calculation of the bed load. The prediction qualities of the designed ensemble of network and common ANN networks are studied by evaluating several statistical parameters that describe the errors associated with the model in terms of statistical measures of goodness-of-fit between the estimated bed change and analytical approximate.

2. ANN and ensemble learning

This section presents the basic concepts of ANN, to give us an idea of how the operation of ANN really is. In the second step we consider the ensemble learning process, for better understanding of how they are applied in the proposed method.

2.1. Basic aspects of ANN

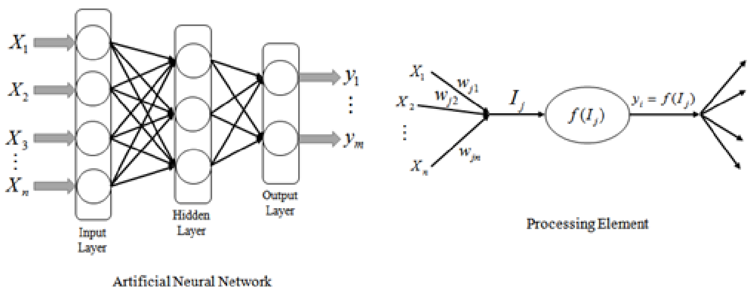

Fig. 1 Typical structure and operation of an ANN model (I_j = activation level of node *j*; w_{ji} = connection weight between nodes *j* and *i*; x_i = input from node *i*; y_j = output of node *j*; and $f(I_j)$ = transfer function)

Artificial neural network is a broad term covering a large variety of network architectures, the most common being a multilayer perceptron (MLP). The parameters to be found by applying the so-called error -back-propagation method are the weight vectors connecting the different nodes of the input, hidden, and output layers of the network. During training the values of the parameters (weights) are varied so that the ANN output becomes similar to the measured output on a known data set.

An ANN model consists of a number of artificial neurons variously known as processing elements or nodes. For multilayer networks, neurons are arranged in layers: an input layer, an output layer and one or more intermediate or hidden layers. The net is formed by these layers of neurons, and each neuron in a specific layer is connected to neurons in other layers via weighted connections. Neurons are defined as mathematical expressions that filter the signal through the net. From the connected neurons in the previous layer, an individual neuron receives its weighted inputs which are usually summed along with a bias unit. The bias unit is used to scale the input to a useful range in order to improve the convergence properties of the network.

The result of this combined summation is passed through a transfer function to produce the output of the neuron. This output is then passed through weighted connections to neurons in the next layer, where the process is repeated. A trained response is achieved by changing the connections weights in the network according to an error minimization criterion. A validation process can be used during the training in order to prevent overfitting. Once the network has been trained to simulate the best response to input data, the configuration of the network is fixed and a test process is conducted to evaluate the performance of the ANN as a predictive tool. According to Shahin et al. (2003), the structure and process for node *j* of an ANN model can be illustrated in Fig. 1.

2.2. Ensemble learning

The ensemble consists of learning paradigm where multiple component learners are trained (Zhou et al. (2002)) for a same task, and the predictions of the component learners are combined for dealing with future instances (Chen and Zhang (2005)). Since an ensemble is often more accurate than its component learners, such a paradigm become a hot topic in recent years and has already been successfully applied to optical character recognition, face recognition, scientific image analysis, medical diagnosis, etc. (Zhou et al. (2002)).

2.3. Structure of the ensemble of ANN

In this paper, the proposed ensemble structure is illustrated in Fig. 2. This structure is divided into 5 parts, where the first part represents the database to simulate in the ensemble of ANN, which in our case is time series data generated by analytical solution of Exner equation. In the second part training and validation is done sequentially in each ANN, where the number of ANN to be trained can be from 1 to *n* depending on what the user wants to test, but in our case we are dealing with a set of 3 ANN in the ensemble. In the third part we have to generate the results of each ANN trained in the previous part and in the fourth part we integrate the overall results of each ANN and finally the outcome or the final prediction of the ensemble ANN learning is obtained.

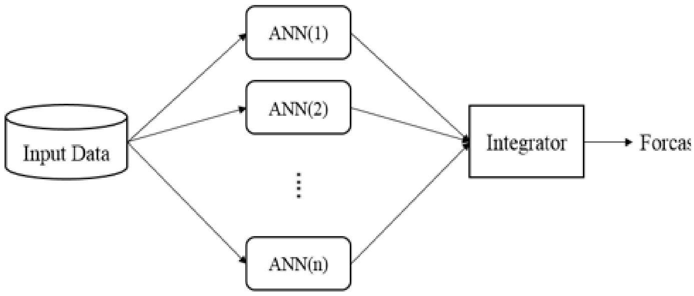

Fig. 2 The structure of the ensemble of ANN

The main idea of ensemble learning in ANN, is that each ANN has different ways to train and thus be simulated. This makes something like an expert system, i.e., give different viewpoints and then predict the time series, then take decisions based on the results of each ANN and reach some conclusion, then integrating the results and obtaining the best prediction of the time series being simulated, and then avoid future unexpected events.

3. Integration methods

There are diversity integration methods or aggregation of information. In this study, we use an integration in the form of the following:

$$Y_i = Int(Y_{i1}, Y_{i2}, \ldots, Y_{in}; \beta_1, \beta_2, \ldots, \beta_n) = \sum_{j=1}^{n} \beta_j Y_{ij} \quad [2]$$

Where Int (.) is the integration function, $Y = (Y_1, Y_2, \ldots, Y_m)^T$ is the output vector of ensemble of ANN, m is the number of data set, Y_{ij} is the output of ANN(j) using data set i which is used as input for the integration function, and $\beta = (\beta_1, \beta_2, \ldots, \beta_n)^T$ are parameters to be determined. Once having several ways to determine β achieving an optimal combination, we mention some of these methods below:

- Integration by average: this method is used in the ensemble of ANN. This integration method is the simplest and most straightforward and consists in the sum of results generated by each ANN divided by the sum of number of ANN, and the disadvantage is that there are cases in which the prognosis is not good.

  $$\beta_i = \frac{1}{n} \quad [3]$$

- Integration by weighted average: this method is an extension of the integration by average, with the main difference that the weighted average assigns importance weights to each of the ANN models. These weights are assigned to a particular ANN based on several factors; the most important is the knowledge product of experience. This integration method belongs to the well-known aggregation operators.

$$\begin{cases} \sum \beta_i = 1 \\ \beta_i = \beta_i(e_i) \end{cases} \quad [4]$$

- Where e_i is training error from ANN(i). Based on these errors weights are manually assigned to each β_i.

- Proposed integration method: in this section we propose linear regression as an integration method for the ensemble of ANN. The goal of this linear regression is to point out the relation between a dependent variable (target) and a great deal of independent variables (output of each ANN) to have minimum error. The general form of the error function is:

$$\varepsilon = Y^* - Y = \varepsilon(\beta) \rightarrow \min \quad [5]$$

- Where $Y^* = (Y_1^*, Y_2^*, \ldots, Y_m^*)^T$ is target vector, $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m)^T$ is error vector which must be minimized. β can be calculated in a matrix form to minimize the error as follows:

$$\beta = (Y^T Y)^{-1} Y^T Y^* \quad [6]$$

4. Time series data generation

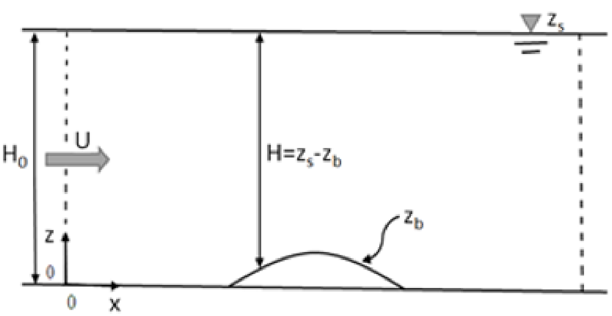

Fig. 3 Bathymetry and coordinate system for the test case

We consider a straight channel with a length of 1000*m* and a finite amplitude perturbation of the bed level near the center of the domain as illustrated in Fig. 3. In this case can represent a sand dune in a river flow. We assume that the bed elevation z_b is very small in comparison to the water free surface level z_s and the bed form movement is only due to bed load. Assuming a steady flow discharge throughout the channel with a rigid lid $H_0 = z_s$ = const, we have

$$H \approx H_0 - Z_b \quad [7]$$

and

$$u \approx \frac{Q}{H} \quad [8]$$

Assuming that transport rate q is a power function of current speed u (Grass (1981); Van Rijn et al. (1993)), we have

$$q = au^m \quad [9]$$

Where *a*, is a given function and *m* is a given positive constant both of which are specific to the particular sediment transport formula. Note that *a*, is typically a function of the mean flow velocity *u*, the total height of the water column *H*, and a number of constants that are based on sediment properties (e.g. sediment type and grain size) and data fitting procedures. The constant *m* is typically in the range of $1 \le m \le 3$. The phase speed of bed form $C(z_b)$ can be now expressed as

$$C(z_b) = \frac{maq^m}{(1-n_p)(H_0 - z_b)^{m+1}} \quad [10]$$

The following quantities are specified according to similar setting in Hudson et al. (2005): $a = 0.001\ s^2/m$, $m = 3.0$, and $n_p = 0.4$. Three different initial conditions $z_b(x, 0)$ are given as Gaussian, Sinusoidal, and Fractional, respectively:

$$z_{b1}(x,0) = Ae^{-W(x-400)^2} \quad [11]$$

$$z_{b2}(x,0) = \begin{cases} A\sin^2\left(\frac{\pi(x-300)}{W}\right); if\ 300 \le x \le 500 \\ 0; otherwise \end{cases} \quad [12]$$

$$z_{b3}(x,0) = \frac{AW}{(x-400)^2 + W} \quad [13]$$

Where *W* and *A*, are the width and height of each hump.

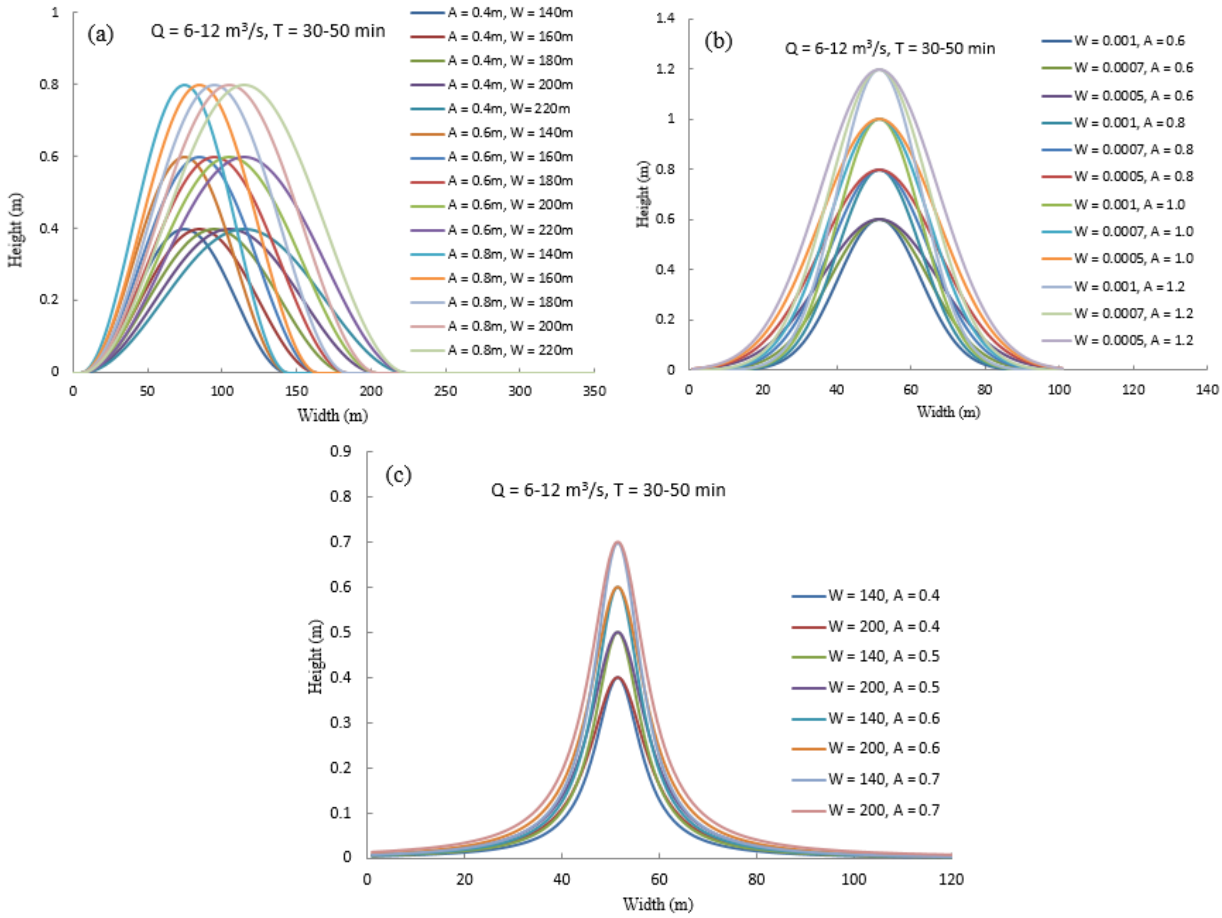

Fig. 4 Initial conditions with different width and height used for data generation: (a) Sinusoidal; (b) Gaussian; (c) Fractional

Fig. 4 shows every type of initial conditions with different width and height. Then Eq. (1) can be solved analytically for each initial condition by the method of characteristic with $0 \le t \le 60000s$ and different Δt to generate the data for ANN model (see Bui et al. (2015)). The quantities mentioned previously are specified for this part as well. Grid spacing is chosen to be $x = 2m$. Tab. 1 presents the ranges of changeable parameters used for data generation.

Tab. 1 Range of parameters used for data generation

Bed shape	Range	Discharge $Q(m^3/s)$	Time step $\Delta t(min)$	Width *(W)*	Height *(A)*
Gaussian	**Lower limit**	6.0	30	0.0005	0.6
	Upper limit	12.0	50	0.0010	1.2
Sinusoidal	**Lower limit**	6.0	30	180	0.6
	Upper limit	12.0	50	240	1.2
Fractional	**Lower limit**	6.0	30	140	0.6
	Upper limit	12.0	50	200	1.2

5. Design of each ANN model

As mentioned before the Eq. (1) is solved analytically for all initial conditions and for all different parameters described in section 4 by the method of characteristic to generate the data for each ANN model. In contrast to numerical schemes where a very small time step has to be chosen to satisfy stability conditions, for the ANN models, we use large time steps (see Bui et al. (2015)). To generate the data set, u has to be evaluated using Eq. (8) at all alternate grid points i at time level n (denoted by uin). Then, zb has to be similarly evaluated using an analytical solution at the same grid points and time level (denoted by $z_{bi}{}^{n}$). Once completed, the process is then repeated at time level (n + 1) and so on. Finally, we have a data set which is then divided randomly into three subsets, whereby the biggest amount of data (70%) is added randomly to the training subset. The remaining data set samples are used for validating (15%) and testing the networks (15%). The training subset is used to design the weights. The validation subset is used additionally to monitor the accuracy of training, while training is ongoing. After each epoch, the validation subset acts as a barometer for determining when the accuracy of the multilayer perceptron is at an acceptable level. After the network is considered optimally trained, the test subset is used to verify its performance. Bui et al. (2015) carried out an investigation to find the optimum ANN model for morphological bed level calculation. Since the performance of ANN model is significantly related to the number of hidden layer nodes, they employed a trial and error approach to choose the appropriate number of nodes in the hidden layer. According to Bui et al. (2015) the ANN configuration with ten hidden neurons shows acceptable accuracy. Further tests have been carried out for different transfer functions used in the hidden layer and output one. As they concluded, applying the log-sigmoid transfer function for hidden layer and the linear function for the output one, can generate the best performance of the ANN model. In this paper, we use the same architecture for each ANN but with different transfer functions in their only hidden layer. They also designed ANN model with eight different inputs combination to find the best input combination. As it can be seen from Eq. (10), the phase speed of bed form is always positive. Hence, in this study case the morphological change at the point i depends mostly on the bed level and water velocity at this point and at the upstream neighbor point (i - 1). It should be noted that in contrast with the study of Bui et al. (2015) where only one time step was used for data generation, in this study different time steps are used, therefore the same input combination plus Δt was considered as input: $z_{bi}^{n}, z_{bi-1}^{n}, u_{bi}^{n}, u_{bi-1}^{n}, \text{and } \Delta t$

6. Simulation results

In this section we make use of the morphodynamic test cases to examine the efficiency of the approaches we outlined in previous sections. First we analyze the capability of the designed ensemble of ANN models for prediction of bed level changes in the new coupling model. To achieve this aim, three different test sets are considered. The test parameters are selected to be in the range of the training data but not the same as the parameters used for training. The characteristic of each test set is listed below:

Test case 1: Sinusoidal shape, Q = 9 m^3/s, Δt = 42 *min*, W = 190, and A = 0.9

Test case 2: Gaussian shape, Q = 7 m^3/s, Δt = 47 *min*, W = 0.0009, and A = 0.7

Test case 3: Fractional shape, Q = 11 m^3/s, Δt = 33 *min*, W = 190, and A = 0.55

In our ensembles we have a set of three ANN for learning, it is noteworthy that the type of transfer functions are assigned differently to each ANN and prediction error of each ANN is calculated by some statistical criteria resorting to *R*, *MAE*, and *RMSE*. In brief, the models predictions are optimal if *R* is found to be close to one, and *MAE* and *RMSE* are close to zero.

6.1. Member networks

After Applying designed ANN models for training data set, the values of the weights and biases have been specified after a successful learning and validating process. They represent the stored knowledge of each ANN for bed level change modeling, which are separated in one input weight matrix $IW^{1,1}$, one hidden-layer weight matrix $LW^{2,1}$, one bias vector b^1 and one bias value b^2 for each data ANN. Using the designed network, we received the following equations for the bed level change:

$$(z_{bi}^{n+1})_1 = LW_1^{2,1} \times \log sig(IW_1^{1,1} \times \begin{bmatrix} z_{bi}^{n} \\ z_{bi-1}^{n} \\ u_i^{n} \\ u_{i-1}^{n} \\ \Delta t \end{bmatrix} + b_1^1) + b_1^2 \quad [14]$$

$$(z_{bi}^{n+1})_2 = LW_2^{2,1} \times \tan sig(IW_2^{1,1} \times \begin{bmatrix} z_{bi}^{n} \\ z_{bi-1}^{n} \\ u_i^{n} \\ u_{i-1}^{n} \\ \Delta t \end{bmatrix} + b_2^1) + b_2^2 \quad [15]$$

$$(z_{bi}^{n+1})_3 = LW_3^{2,1} \times radbas(IW_3^{1,1} \times \begin{bmatrix} z_{bi}^n \\ z_{bi-1}^n \\ u_i^n \\ u_{i-1}^n \\ \Delta t \end{bmatrix} + b_3^1) + b_3^2 \quad [16]$$

6.2. Ensemble ANN

This section presents the methods of integration of average, weighted average, and proposed linear regression which are used for simulation in our experiments of the ensemble of ANN in order to obtain a good forecast of simulated time series. In the case of integration by average, we added the result of each model and divided them by number of ANN models. For integrating the ensemble of ANNs using weighted average method, a weight was given depending on the results obtained from each ANN, these weights were assigned manually, where the lowest error between the ANN outputs had a weight of 0.50, the ANN that had an intermediate error 0.30 and the biggest error was assigned a weight of 0.20, thus obtaining 100 percent of the weights assigned among the models of the ANN ensemble. And finally for integrating the ensemble of ANN using linear regression the Eq. (6) is used to calculate integration weights.

6.3. Coupling flow and sediment computations

Since the characteristic time scale of bed-evolution and bed load transport processes is normally much greater than that of fluid flow, it can be assumed that changes in the bed elevation during one computational time step do not significantly influence the flow field. This assumption leads to the computationally attractive possibility of coupling flow and sediment computations in an iterative manner. Hereby, the flow and sediment-transport modules communicate through a quasi-steady morphodynamic time-stepping mechanism: during the flow computation, the bed level is assumed constant and during the computation of the bed level the flow and sediment transport are assumed invariant to the bed level changes. Based on this coupling concept, the main calculation procedure implemented in this study is shown in Fig. 5.

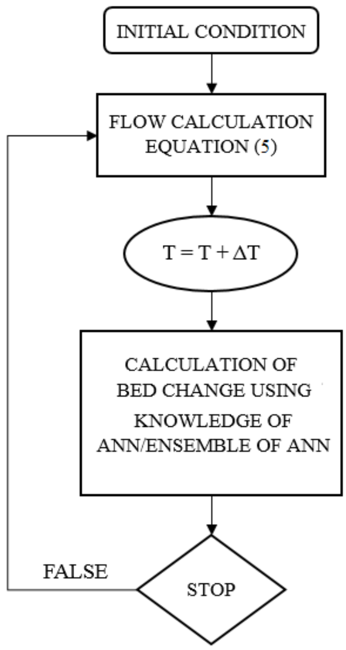

Fig. 5 Flow chart of the coupling system

First, the initial values (at time t = 0) are defined at every grid point i. The bed levels at one time step ahead are calculated using the ensemble of ANN (Eqs. (14), (15), and (16) and obtained weights and biases, then using integration method). The water velocities at this time step are updated using Eq. (8). This procedure is repeated until the last time step is reached (t = 60000s). Tab. 2 presents the statistical performances indices of different ANN models and ensemble of ANN based on a comparison between the predicted bed levels and analytical approximation at different time steps.

Tab. 2 Statistical performance of the designed ensemble of ANN models

Test case	Method	Type	*R*	*RMSE*	*MAE*
1	ANN	1	0.999938	0.004196	0.003382
		2	0.999929	0.004447	0.003523
		3	0.999826	0.006486	0.004511
1	Ensemble of ANN	Integration by average	0.999949	0.003700	0.003132
		Integration of weighted average	0.999954	0.003540	0.002984
		Integration by linear regression	0.999933	0.004663	0.003781
2	ANN	1	0.999946	0.002789	0.001772
		2	0.999873	0.003092	0.001373
		3	0.999984	0.001984	0.001225
2	Ensemble of ANN	Integration by average	0.999978	0.001584	0.001046
		Integration of weighted average	0.999960	0.001970	0.001281
		Integration by linear regression	0.999992	0.001002	0.000732
3	ANN	1	0.997888	0.008145	0.003714
		2	0.995977	0.010668	0.004672
		3	0.999120	0.005052	0.002254
3	Ensemble of ANN	Integration by average	0.998496	0.006513	0.002904
		Integration of weighted average	0.997926	0.007722	0.003432
		Integration by linear regression	0.998922	0.005481	0.002403

The results indicate that the designed ensemble of ANNs perform well the morphological change in the channel almost for all test sets with high values of *R* as well as small values of *RMSE* and *MAE*. Since the integrated ANN model uses the predicted values from the past, it can be shown empirically that multi stage prediction is susceptible to the error accumulation problem, i.e. error committed in the past are propagated into future predictions. According to this table, the ensemble of ANNs integrated by weighted average provides the best performance for first test case in comparison to other models. The values of *MAE* and *RMSE* for this model are significantly smaller than the values of these parameters for other models, especially in comparison with single ANNs. For the second test case, the *RMSE*, and *MAE* values for the ensemble of ANN model integrated with the proposed linear regression are 0.001002 and 0.000732, respectively which are significantly lower than other models. The mentioned statistical parameters are in the ranges of 0.001984 to 0.002789, and 0.001225 to 0.001772, respectively for single ANN models. According to Tab. 2, ANN3 using Radial basis transfer function in its only hidden layer is performing better than the proposed ensemble of ANN for test case 3. However, it should be noted that for this case all proposed ensemble of ANNs have more accuracy than ANN1 and ANN2. Fig. 6 (a), (b), and (c) show a comparison between the best predicted results for each test case with the analytical solution at time t = 6000*s*. As it can be seen from this figure, there is a good agreement between predicted results and analytical solution for all test cases.

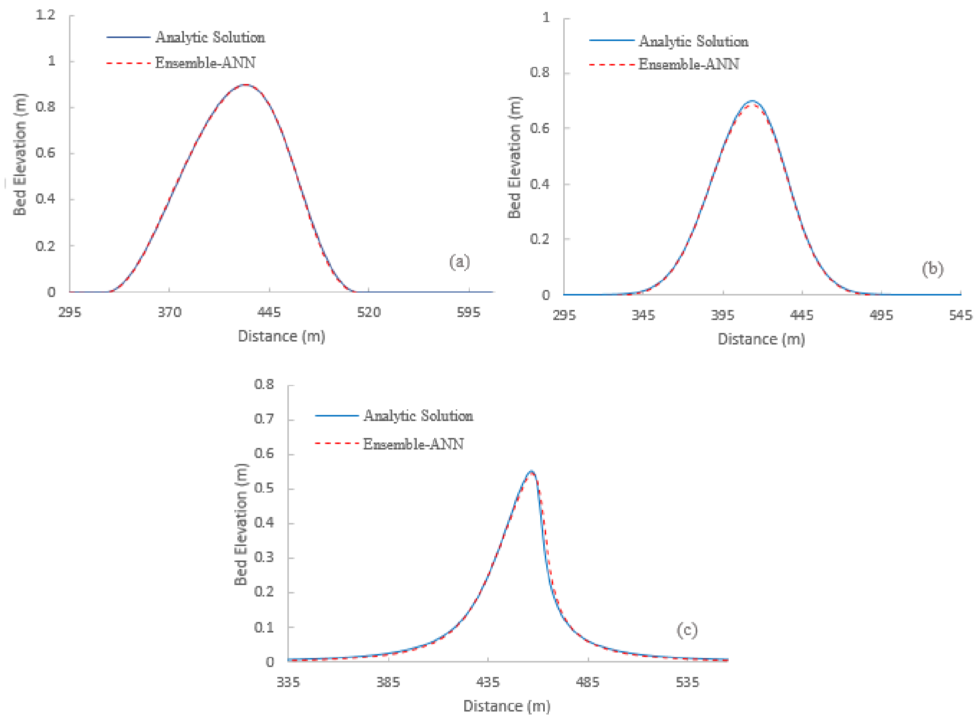

Fig. 6 Comparison of the best predicted results for each test case with analytical solution; (a) Sinusoidal; (b) Gaussian; (c) Fractional

7. Conclusions

The ensemble of artificial neural network was applied for time series morphological bed level changes prediction as an alternative to common ANN models. Three different integrating methods were also applied for the integration of ensemble of ANN. An analytical approximation based on the equation of conservation of sediment has been applied to generate data used for training and testing the proposed ensemble of ANN model. The predicted results showed that for the simple case of 1D morphological problems considered in this study, the proposed ensemble of ANN could provide a good performance for long term time series prediction. The calculated results also showed that the ensemble of ANN could perform better than single ANNs for time series bed level changes prediction in most test cases.

References

Abbot, M. B. (1979). "Computational Hydraulics: Elements of the Theory of Free-Surface Flows", Pitman, London.

Bui, M.D.; K. Kaveh, and P. Rutschmann. (2015). "Integrating artificial neural networks into hydromorphological model for alluvial channels", Proc. of the 36th IAHR World Congress, The Hague, the Netherlands.

Chen, D.W; and J.P. Zhang. (2005). "Time series prediction based on ensemble ANFIS", Proceedings of International Conference on Machine Learning and Cybernetics. Vol. 6.

Chesher, T.J.; H.M. Wallace, I. Meadowcroft, and H.N. Southgate. (1993). Pisces: A morphodynamic coastal area model first annual report.

Grass, A. J. (1981). Sediment transport by waves and currents. (1981). University College, London, Dept. of Civil Engineering.

Hudson, J. et al. (2005). "Numerical approaches for 1D morphodynamic modelling", Coastal Engineering, 52.8. 691-707.

Johnson, H.K.; and J.A. Zyserman. (2002). "Controlling spatial oscillations in bed level update schemes", Coastal Engineering 46.2 (2002): 109-126.

Kobayashi, N.; and B.D. Johnson. (2001). "Sand suspension, storage, advection, and settling in surf and swash zones", Journal of Geophysical Research: Oceans 106.C5 (2001): 9363-9376.

Lax, Peter D. (1954). "Weak solutions of nonlinear hyperbolic equations and their numerical computation", Communications on pure and applied mathematics, 7.1, 159-193.

Mc Cormack, R. (2003). "The effect of viscosity in hypervelocity impact cratering", Journal of spacecraft and rockets, 40.5, 757-763.

Nicholson, J., et al. (1997). "Intercomparison of coastal area morphodynamic models", Coastal Engineering, 31.1, 97-123.

Osher, S.; and F. Solomon. (1982). "Upwind difference schemes for hyperbolic systems of conservation laws", Mathematics of computation, 38.158, 339-374.

Shahin, M. A.; H.R. Maier, and M.B. Jaksa. (2003). "Closure to “Predicting Settlement of Shallow Foundations Using Neural Networks”, Journal of Geotechnical and Geoenvironmental Engineering, 129.12, 1175-1177.

Van Rijn, L.C. (2002). Principles of sediment transport in rivers, estuaries and coastal seas. Vol. 1006. Amsterdam: Aqua publications.

Zhou, Z.H.; J. Wu, and W. Tang. (2002). "Ensembling neural networks: many could be better than all", Artificial intelligence, 137.1, 239-263.

Authors' addresses

Keivan Kaveh MSc.
Lehrstuhl für Wasserbau und Wasserwirtschaft, TU München
Arcisstraße 21, D-80333 München
Keivan.kaveh@tum.de

Dr.-Ing., Dipl.-Math. Minh Duc Bui
Lehrstuhl für Wasserbau und Wasserwirtschaft, TU München
Arcisstraße 21, D-80333 München
bui@tum.de

Prof. Dr. Peter Rutschmann
Lehrstuhl für Wasserbau und Wasserwirtschaft, TU München
Arcisstraße 21, D-80333 München
peter.rutschmann@tum.de